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Fluid Mechanics

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To Theresa

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Principles of Fluid Mechanics

Principles of

Preface

Teaching a course or learning is not exclusively an elegant display of knowledge. The method of teaching or learning, as the case may be, must manifest a logical and rigorous continuity. In general, it is this logical rigor of thought that makes a course interesting and consequently appealing to the mind that has a desire to learn. Logic is essential for reasoning. Furthermore, accurate reasoning requires accurate usage of the rules of logic. Today, every field of study has developed its own specialized operational logic which enables reasoning in specialized terms. To the scientist and the engineer, mathematics is a symbolic or shorthand form of logical thinking enabling him to reason his premise to a satisfactory conclusion. If experimentation of the physical phenomenon is possible to perform, it will yield the same conclusion, provided the same conditions prevail in both cases. However, it would be wise for scientists and engineers to remember the thoughts of Albert Einstein on logical thinking as related to science. He said, "Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experiment and ends in it." Furthermore, he warns us also that "experience remains, of course, the sole criterion of the physical utility of mathematical construction. But the creative principle resides in mathematics."¹

This book is intended for the beginning student in mechanics of fluids. It will provide him with the basic and fundamental concepts so necessary for advancement in the fields of general fluid dynamics (hydro- and aerodynamics). Throughout the book an attempt has been made to present the concepts and notions as simply as possible, without distortion of meaning or significance. First, the most important concepts, variables, constants, and parameters necessary for the formulation of the most basic axioms and theorems are presented in a logical sequence. Without the clear understanding of these concepts, axioms, and theorems, the student cannot hope to grasp clearly the nature,

¹ Albert Einstein, "On the Method of Theoretical Physics," from *Essays in Science*, Philosophical Library, New York, 1934.

mechanism, and scope of applied problems related to the dynamics of flowing fluids.

The same general laws of equilibrium already encountered in the mechanics of solids will be found to apply for fluids as well. Some concepts in kinematics and dynamics pertaining to solids will be used for fluids also. Some concepts will require certain alterations in order to include fluids. Finally, some new concepts will be introduced which will have a meaning for fluids but not for solids. In essence, it may be said that solid mechanics as well as fluid mechanics are limited studies of a broad field in general mechanics. In elementary books it is common practice to cover fluid statics in great length at the beginning of the course. This practice is not followed here for two reasons: Since good treatments of fluid statics are available in many textbooks, any extended effort on the part of the present author would be merely a duplication of material. Furthermore, since the behavior of fluids under static conditions is similar to that of solids, they are generally treated together in an earlier course.

In mechanics of rigid bodies, the relative positions of the elemental masses constituting the body always remain unchanged. This can be accomplished only when the elemental mass behaves as the body itself. In mechanics of deformable bodies, however, the deformation within the entire body must be distributed uniformly in order to say that the behavior of the elemental mass is a small-scale replica of the body's behavior. A *fluid* substance deforms under the influence of external forces. This deformation is often nonuniformly distributed throughout the extent of the fluid without causing separation of mass. Under these circumstances the over-all deformation, being the sum of the nonuniform elemental deformations, will not be a scale replica of the deformation of any elemental mass. In fluid mechanics a knowledge of the behavior of the elemental mass is essential in order to deduce the behavior of larger masses. It was just mentioned that every fluid portion throughout the flow field does not necessarily move in precisely the same amount; therefore, since small fluid portions must be examined in detail, the use of partial and total differentiation is inevitable. In most instances it is not unreasonable to postulate that the fluid properties vary in a smooth, continuous fashion in a given region of consideration. Therefore, provided that these variations and their derivatives are continuous, a property of the fluid at a given point can be expressed in terms of the property at another neighboring point. This interrelation between properties at two neighboring points in a *continuum* comes to us from calculus and is generally identified as Taylor's series expansion. This famous theorem states that if a property P and its derivatives are

known at a given point x_0 , then the same property at a neighboring point x_1 is evaluated from the relation

$$P(x_1) = P(x_0) + (x_1 - x_0) \left(\frac{dP}{dx} \right)_{x_0} + \frac{(x_1 - x_0)^2}{2!} \left(\frac{d^2P}{dx^2} \right)_{x_0} + \dots$$

The use of total differentiation in the above equation implies that the property P is a function of x alone. If it were a function of other variables as well, then a similar expression could be written for every variable in terms of the partial derivatives with respect to that variable. For a finite variation $(x_1 - x_0)$ the series expansion for $P(x_1)$ is infinite. This means that the accuracy with which $P(x_1)$ can be evaluated from $P(x_0)$ and its derivatives will depend on the number of terms considered in the expansion. Naturally, the smaller the difference $(x_1 - x_0)$, the smaller will be the contribution of the higher-order terms. It is therefore conceivable that if $(x_1 - x_0)$ approaches an infinitesimally small variation dx , second-order and higher terms are made negligible compared with the first two terms on the right-hand side of the equation. This implies, then, that the variation of property is in the form of a straight line in the interval dx . Actually, the milder the variation of P on x , the stronger the justification to neglect higher-order terms.

The differential equation expressing the motion of a fluid element may, in most instances, involve more than one dependent variable. In that case, as in algebraic equations, as many independent equations are needed as independent variables in order to obtain a solution in terms of the independent variables alone. In mechanics of fluids these additional equations are obtained from axiomatic laws such as the conservation of mass and energy and the equation of state of the substance. This availability of the exact number of equations does not necessarily guarantee a solution; it merely defines the problem. The engineers and scientists are dependent on available mathematical information on how to obtain solutions in a form that is easily handled in practice. Most fluid-dynamic problems are well defined; this means that the physical set of conditions required to define the problem is complete. Some of them are unsolvable today, however, on account of the lack of mathematical tools. Where mathematical tools are not abundant, the scientist and engineer must resort to experimental means. The experimental approach is comparatively slower, more expensive, and generally more tedious. In essence, theory and experiment depend on each other for sensible progress in a given field.

During the past years, extensive contributions have been made in the field of fluid dynamics. A student, after having been exposed to general mechanics of fluids or aerodynamics, will be able to follow the

basic concepts of viscous, nonviscid, rotational, irrotational, compressible, and incompressible states of fluid motion. Needless to say, less prepared students must devote extra effort to grasp the basic fundamentals they lack if they wish to assimilate their graduate courses satisfactorily. In a similar fashion, the less well-prepared student who chooses to work for industry will find it difficult to follow his industrial leaders working on contemporary research, design, or development problems. Generally speaking, the author believes that this book will give the undergraduate student the quality of knowledge in mechanics of fluids that is expected from a prospective graduate student.

Salamon Eskinazi

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S. E.

Symbols

The following is a list of the most commonly used symbols in the book.

a	Acceleration
a	Speed of sound in the fluid
A	Cross-section area
b	Width
B	Integration constant
c	Speed of pressure wave
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
C	Dimensional constant in Newton's law of gravitation
C_p	Pressure coefficient
C_v	Velocity coefficient
C_d	Discharge coefficient
C_c	Contraction coefficient
C_D	Drag coefficient
d	Distance
d	Diameter
D	Diameter of pipe
D	Drag
e	Internal energy per unit mass
E	Modulus of elasticity
E	Energy
f	Frequency
f	Resistance coefficient
F	Force
Fr	Froude number
g	Gravitational acceleration
g_0	Gravitational acceleration at 45° latitude and sea level
g_c	Dimensional constant in Newton's law of inertia
G	Grashoff number
h	Enthalpy

h	Heat transfer coefficient
h	Height
H	Head loss
I_{xy}	Product moment of inertia
I_{xx}	Moment of inertia about x -axis
J	Mechanical equivalent of heat
k	Size of roughness
k	Strength of point source
K	Strength of line source
l	Linear length
L	Characteristic length
L	Lift force
L_e	Equivalent length
m	Mass
M	Moment
M	Mach number
\overline{MG}	Metacentric height
n	Exponent in polytropic process
p	Pressure
p_t	Total pressure
P	Power
P	Péclet number
q	Heat transferred
Q	Rate of mass flow
r	Radius
R	Radius of the earth
R	Dimensional gas constant
R	Radius of curvature
R	Reynolds number
s	Length of path
s	Entropy
S	Surface area
t	Time
T	Surface-tension force
T	Period
T	Temperature
u	x -component of velocity
u'	x -component of turbulent velocity fluctuation
u_*	Shearing velocity
U	Velocity
U_s	Steady-state velocity
U_u	Unsteady-state velocity

\bar{U}	Temporal mean velocity
U_{\perp}	Velocity component normal to s
U_{∞}	Velocity of undisturbed flow
v	y -component of velocity
v_r	Radial component of velocity
v_{θ}	Peripheral component of velocity
v'	y -component of turbulent velocity fluctuation
V	Velocity
w'	z -component of turbulent velocity fluctuation
w	z -component of velocity
W	Work
W	Weight
W	Weber number
x	Cartesian coordinate
x_p	Coordinate of center of pressure
\bar{x}	Coordinate of centroid
y	Cartesian coordinate
y_p	Coordinate of center of pressure
\bar{y}	Coordinate of centroid
z	Cartesian coordinate
z_p	Coordinate of center of pressure
\bar{z}	Coordinate of centroid
α	Angle
$\ddot{\alpha}$	Angular acceleration
β	Coefficient of compressibility
β_1	Coefficient of thermal expansion
β_2	Coefficient of tension
β	Angle
γ	Ratio of specific heats
Γ	Circulation
δ	Boundary-layer thickness
δ^*	Displacement thickness
ϵ	Resultant small pressure
ϵ	Ratio of bearing width
ζ	Vorticity
η	Dimensionless variable
θ	Angle of deformation
θ	Angle
θ	Momentum thickness
λ	Temperature-lapse rate
μ	Absolute viscosity
ν	Kinematic viscosity

Π	Dimensionless grouping
ρ	Density
σ	Surface-tension coefficient
σ	Normal viscous stress
τ	Shear stress
ϕ	Dimensionless pressure gradient
ϕ	Potential function
ϕ_l	Load factor
ϕ_d	Drag factor
ψ	Stream function
ω	Angular velocity

Subscripts

x	Pertaining to x -direction
y	Pertaining to y -direction
z	Pertaining to z -direction
i	Inlet or input
o	Outlet or output
s	Along streamline
max	Maximum value
av	Spatial average
w	Pertaining to wall
L	Along length
δ	Across boundary layer

Superscript

$^{\circ}$	Pertaining to stagnation point
$*$	Property at sonic point

Contents

Preface v
 Symbols xv



I Fundamental Concepts 1

- 1.1 Solid, Liquid, and Gas 1
- 1.2 System, Property, and State 3
- 1.3 Properties in a Continuum 5
- 1.4 Mass and Force 7
- 1.5 Dimensions in Newton's Law of Inertia 11
- 1.6 Density, Specific Weight, Specific Gravity, and Specific Volume 13
- 1.7 The Equation of State 14
- 1.8 Coefficient of Compressibility, Coefficient of Thermal Expansion, and Coefficient of Tension 17
- 1.9 Compressible and Incompressible Fluids 19
- 1.10 Viscosity and Shearing Stress 21
- 1.11 Frictionless Motion 28
- 1.12 Perfect Fluid 29
- 1.13 Real Fluids 30
- 1.14 Surface Tension of Liquids 32
- Problems 33

II Statics of Fluids 37

- 2.1 Static Equilibrium 37
- 2.2 Types of Forces on Fluid Systems 38
- 2.3 Concept of Pressure in Fluids in Equilibrium 39

- 2.4 Relationship Between the Body Forces and the Surface Forces for a Fluid in Static Equilibrium 41
- 2.5 The Hydrostatic Equation 43
- 2.6 Stability of Static Equilibrium 45
- 2.7 Equilibrium of the Atmosphere 47
- 2.8 The Polytropic Atmosphere 48
- 2.9 The Isothermal Atmosphere 49
- 2.10 Forces on Submerged Flat Surfaces 51
- 2.11 Forces on Submerged Curved Surfaces 55
- 2.12 Buoyant Force on Submerged Bodies 58
- 2.13 Stability of Floating Bodies—Metacenter and Metacentric Height 60
- 2.14 The Period of Rolling 64
- 2.15 Surface Tension 65
- 2.16 Wetting and Nonwetting Fluids 68
- 2.17 Capillarity 69
- Problems 71

- III Similarity of Motion in Fluid Mechanics 75**
 - 3.1 Basic Concepts and Dimensions 75
 - 3.2 Geometric, Kinematic, and Dynamic Similitude 76
 - 3.3 The Drag on a Ship 83
 - Problems 86

- IV Kinematics of Fluid Motion 89**
 - 4.1 Introduction 89
 - 4.2 Classification of Types of Motion 90
 - 4.3 Steady and Unsteady Motion 95
 - 4.4 Representation of Turbulent Motion 96
 - 4.5 Uniform Flow. One-, Two-, and Three-Dimensional Flow 99
 - 4.6 Streamline, Path Line, and Streak Line 100
 - 4.7 Stream Filament, Stream Tube, and Stream Surface 102
 - 4.8 The Integral Form of the Continuity Equation Applied to Any Arbitrary Control Surface in the Fluid 106
 - 4.9 Stream Function in Two-Dimensional Flow 106
 - 4.10 Linear Combination of Two-Dimensional Perfect Fluids 110

4.11	Existence and Visualization of Streamlines	117
4.12	The Concept of Fluid Rotation	120
4.13	Vorticity in Cylindrical Coordinates	124
4.14	Conditions for Irrotational Plane Motion	127
4.15	The Concept of Circulation	128
4.16	The Differential Form of Conservation of Mass	132
	Problems	137

V Dynamics of an Ideal Motion 141

5.1	Methods of Representing the Motion of Fluids	141
5.2	Mathematical Representation of the Eulerian Method	142
5.3	Acceleration in the Natural Coordinates	144
5.4	Uniformly Accelerated Fluids	146
5.5	Axisymmetric Circulatory Motion	150
5.6	Hydrostatic Accelerometers	153
5.7	Considerations on the Frictionless Motion of a Fluid	155
5.8	Euler's Equations of Motion. Frictionless Motion	156
5.9	Integration of Euler's Equations	159
5.10	Special Applications of Bernoulli's Equation	162
5.11	Bernoulli's Equation for Nonuniform Flows in Conduits	172
5.12	Fluid Motion with Internal Losses	175
5.13	Losses in Metering Systems	176
5.14	Flow Measurements in Open Channels	181
5.15	Similarity in Ideal Nonviscous Motion	183
	Problems	185

VI Momentum Considerations in Fluid Dynamics 193

6.1	Introduction	193
6.2	Momentum Equations Applied to a Finite Fluid Volume	194
6.3	Derivation of Steady-Flow Momentum Equation by Integration Through a Stream Filament in an Arbitrary Control Volume	198
6.4	The Stream Tube as a Control Volume	200
6.5	Applications of the Momentum Equations	203
6.6	Moment of Momentum or Angular Momentum	219
6.7	Application of Angular Momentum to Turbomachines	226
	Problems	228

VII Energy Considerations in Fluid Dynamics 233

- 7.1 Introduction 233
- 7.2 Energy Transfer Within a System 234
- 7.3 Energy Transfer Within a Control Volume 237
- 7.4 Comparison Between Bernoulli's Equation and the Energy Equation 241
- 7.5 Energy Equation Applied to an Arbitrary Control Volume 243
- Problems 244

VIII The Motion of a Two-Dimensional Perfect Fluid 247

- 8.1 Introduction 247
- 8.2 The Stream Function 248
- 8.3 The Velocity Potential 250
- 8.4 The Geometrical Relationship Between the Stream Function and the Velocity Potential 253
- 8.5 The Point Source 254
- 8.6 Superposition of Point Sources on a Finite Line Segment 258
- 8.7 The Two-Dimensional Rectilinear Source. The Line Source 262
- 8.8 Flow from Two Line Sources 264
- 8.9 A Line Source Near a Wall 265
- 8.10 A Line Source and a Line Sink Placed at a Finite Distance Apart 270
- 8.11 The Uniform Stream 271
- 8.12 A Line Source in a Uniform Stream 272
- 8.13 The Point Source in a Uniform Stream 277
- 8.14 Line Source and Line Sink in a Uniform Stream 282
- 8.15 The Point Source and the Point Sink in a Uniform Stream 284
- 8.16 Uniform Flow Around a Cylinder 286
- 8.17 Flow Around a Sphere 290
- 8.18 The Free Vortex 294
- 8.19 Flow Around the Cylinder with Circulation 295
- 8.20 The Lift on the Cylinder. Kutta-Joukowski's Theorem 297
- 8.21 The Apparent or Virtual Mass 300
- Problems 303