Principles of Fluid Mechanics

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Fluid Mechanics

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Preface

Teaching a course or learning is not exclusively an elegant display of knowledge. The method of teaching or learning, as the case may be, must manifest a logical and rigorous continuity. In general, it is this logical rigor of thought that makes a course interesting and consequently appealing to the mind that has a desire to learn. Logic is essential for reasoning. Furthermore, accurate reasoning requires accurate usage of the rules of logic. Today, every field of study has developed its own specialized operational logic which enables reasoning in specialized terms. To the scientist and the engineer, mathematics is a symbolic or shorthand form of logical thinking enabling him to reason his premise to a satisfactory conclusion. If experimentation of the physical phenomenon is possible to perform, it will yield the same conclusion, provided the same conditions prevail in both cases. However, it would be wise for scientists and engineers to remember the thoughts of Albert Einstein on logical thinking as related to science. He said, "*Pure* logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experiment and ends in it." Furthermore, he warns us also that "experience remains, of course, the sole criterion of the physical utility of mathematical construction. But the creative principle resides in mathematics."[1]

This book is intended for the beginning student in mechanics of fluids. It will provide him with the basic and fundamental concepts so necessary for advancement in the fields of general fluid dynamics (hydro- and aerodynamics). Throughout the book an attempt has been made to present the concepts and notions as simply as possible, without distortion of meaning or significance. First, the most important concepts, variables, constants, and parameters necessary for the formulation of the most basic axioms and theorems are presented in a logical sequence. Without the clear understanding of these concepts, axioms, and theorems, the student cannot hope to grasp clearly the nature,

[1] Albert Einstein, "On the Method of Theoretical Physics," from *Essays in Science*, Philosophical Library, New York, 1934.

mechanism, and scope of applied problems related to the dynamics of flowing fluids.

The same general laws of equilibrium already encountered in the mechanics of solids will be found to apply for fluids as well. Some concepts in kinematics and dynamics pertaining to solids will be used for fluids also. Some concepts will require certain alterations in order to include fluids. Finally, some new concepts will be introduced which will have a meaning for fluids but not for solids. In essence, it may be said that solid mechanics as well as fluid mechanics are limited studies of a broad field in general mechanics. In elementary books it is common practice to cover fluid statics in great length at the beginning of the course. This practice is not followed here for two reasons: Since good treatments of fluid statics are available in many textbooks, any extended effort on the part of the present author would be merely a duplication of material. Furthermore, since the behavior of fluids under static conditions is similar to that of solids, they are generally treated together in an earlier course.

In mechanics of rigid bodies, the relative positions of the elemental masses constituting the body always remain unchanged. This can be accomplished only when the elemental mass behaves as the body itself. In mechanics of deformable bodies, however, the deformation within the entire body must be distributed uniformly in order to say that the behavior of the elemental mass is a small-scale replica of the body's behavior. A *fluid* substance deforms under the influence of external forces. This deformation is often nonuniformly distributed throughout the extent of the fluid without causing separation of mass. Under these circumstances the over-all deformation, being the sum of the nonuniform elemental deformations, will not be a scale replica of the deformation of any elemental mass. In fluid mechanics a knowledge of the behavior of the elemental mass is essential in order to deduce the behavior of larger masses. It was just mentioned that every fluid portion throughout the flow field does not necessarily move in precisely the same amount; therefore, since small fluid portions must be examined in detail, the use of partial and total differentiation is inevitable. In most instances it is not unreasonable to postulate that the fluid properties vary in a smooth, continuous fashion in a given region of consideration. Therefore, provided that these variations and their derivatives are continuous, a property of the fluid at a given point can be expressed in terms of the property at another neighboring point. This interrelation between properties at two neighboring points in a *continuum* comes to us from calculus and is generally identified as Taylor's series expansion. This famous theorem states that if a property P and its derivatives are known at a given point x_0, then the same property at a neighboring point x_1 is evaluated from the relation

$$P(x_1) = P(x_0) + (x_1 - x_0)\left(\frac{dP}{dx}\right)_{x_0} + \frac{(x_1 - x_0)^2}{2!}\left(\frac{d^2P}{dx^2}\right)_{x_0} + \cdots$$

The use of total differentiation in the above equation implies that the property P is a function of x alone. If it were a function of other variables as well, then a similar expression could be written for every variable in terms of the partial derivatives with respect to that variable. For a finite variation $(x_1 - x_0)$ the series expansion for $P(x_1)$ is infinite. This means that the accuracy with which $P(x_1)$ can be evaluated from $P(x_0)$ and its derivatives will depend on the number of terms considered in the expansion. Naturally, the smaller the difference $(x_1 - x_0)$, the smaller will be the contribution of the higher-order terms. It is therefore conceivable that if $(x_1 - x_0)$ approaches an infinitesimally small variation dx, second-order and higher terms are made negligible compared with the first two terms on the right-hand side of the equation. This implies, then, that the variation of property is in the form of a straight line in the interval dx. Actually, the milder the variation of P on x, the stronger the justification to neglect higher-order terms.

The differential equation expressing the motion of a fluid element may, in most instances, involve more than one dependent variable. In that case, as in algebraic equations, as many independent equations are needed as independent variables in order to obtain a solution in terms of the independent variables alone. In mechanics of fluids these additional equations are obtained from axiomatic laws such as the conservation of mass and energy and the equation of state of the substance. This availability of the exact number of equations does not necessarily guarantee a solution; it merely defines the problem. The engineers and scientists are dependent on available mathematical information on how to obtain solutions in a form that is easily handled in practice. Most fluid-dynamic problems are well defined; this means that the physical set of conditions required to define the problem is complete. Some of them are unsolvable today, however, on account of the lack of mathematical tools. Where mathematical tools are not abundant, the scientist and engineer must resort to experimental means. The experimental approach is comparatively slower, more expensive, and generally more tedious. In essence, theory and experiment depend on each other for sensible progress in a given field.

During the past years, extensive contributions have been made in the field of fluid dynamics. A student, after having been exposed to general mechanics of fluids or aerodynamics, will be able to follow the basic concepts of viscous, nonviscid, rotational, irrotational, compressible, and incompressible states of fluid motion. Needless to say, less prepared students must devote extra effort to grasp the basic fundamentals they lack if they wish to assimilate their graduate courses satisfactorily. In a similar fashion, the less well-prepared student who chooses to work for industry will find it difficult to follow his industrial leaders working on contemporary research, design, or development problems. Generally speaking, the author believes that this book will give the undergraduate student the quality of knowledge in mechanics of fluids that is expected from a prospective graduate student.

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S. E.

Symbols

The following is a list of the most commonly used symbols in the book.

a	Acceleration
a	Speed of sound in the fluid
A	Cross-section area
b	Width
B	Integration constant
c	Speed of pressure wave
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
C	Dimensional constant in Newton's law of gravitation
C_p	Pressure coefficient
C_v	Velocity coefficient
C_d	Discharge coefficient
C_c	Contraction coefficient
C_D	Drag coefficient
d	Distance
d	Diameter
D	Diameter of pipe
D	Drag
e	Internal energy per unit mass
E	Modulus of elasticity
E	Energy
f	Frequency
f	Resistance coefficient
F	Force
$\boldsymbol{Fr}$	Froude number
g	Gravitational acceleration
g_0	Gravitational acceleration at 45° latitude and sea level
g_c	Dimensional constant in Newton's law of inertia
$\boldsymbol{G}$	Grashoff number
h	Enthalpy

h	Heat transfer coefficient
h	Height
H	Head loss
I_{xy}	Product moment of inertia
I_{xx}	Moment of inertia about x-axis
J	Mechanical equivalent of heat
k	Size of roughness
k	Strength of point source
K	Strength of line source
l	Linear length
L	Characteristic length
L	Lift force
L_e	Equivalent length
m	Mass
M	Moment
$\boldsymbol{M}$	Mach number
$\overline{MG}$	Metacentric height
n	Exponent in polytropic process
p	Pressure
p_t	Total pressure
P	Power
$\boldsymbol{P}$	Péclet number
q	Heat transferred
Q	Rate of mass flow
r	Radius
R	Radius of the earth
R	Dimensional gas constant
R	Radius of curvature
$\boldsymbol{R}$	Reynolds number
s	Length of path
s	Entropy
S	Surface area
t	Time
T	Surface-tension force
T	Period
T	Temperature
u	x-component of velocity
u'	x-component of turbulent velocity fluctuation
u_*	Shearing velocity
U	Velocity
U_s	Steady-state velocity
U_u	Unsteady-state velocity

$\bar{U}$	Temporal mean velocity
$U_{\perp}$	Velocity component normal to s
U_{∞}	Velocity of undisturbed flow
v	y-component of velocity
v_r	Radial component of velocity
v_{θ}	Peripheral component of velocity
v'	y-component of turbulent velocity fluctuation
V	Velocity
w'	z-component of turbulent velocity fluctuation
w	z-component of velocity
W	Work
W	Weight
$\mathbf{W}$	Weber number
x	Cartesian coordinate
x_p	Coordinate of center of pressure
$\bar{x}$	Coordinate of centroid
y	Cartesian coordinate
y_p	Coordinate of center of pressure
$\bar{y}$	Coordinate of centroid
z	Cartesian coordinate
z_p	Coordinate of center of pressure
$\bar{z}$	Coordinate of centroid
α	Angle
$\ddot{\alpha}$	Angular acceleration
β	Coefficient of compressibility
β_1	Coefficient of thermal expansion
β_2	Coefficient of tension
β	Angle
γ	Ratio of specific heats
Γ	Circulation
δ	Boundary-layer thickness
δ^*	Displacement thickness
ϵ	Resultant small pressure
ϵ	Ratio of bearing width
ζ	Vorticity
η	Dimensionless variable
θ	Angle of deformation
θ	Angle
θ	Momentum thickness
λ	Temperature-lapse rate
μ	Absolute viscosity
ν	Kinematic viscosity

Π	Dimensionless grouping
ρ	Density
σ	Surface-tension coefficient
σ	Normal viscous stress
τ	Shear stress
ϕ	Dimensionless pressure gradient
ϕ	Potential function
ϕ_l	Load factor
ϕ_d	Drag factor
ψ	Stream function
ω	Angular velocity

Subscripts

x	Pertaining to x-direction
y	Pertaining to y-direction
z	Pertaining to z-direction
i	Inlet or input
o	Outlet or output
s	Along streamline
max	Maximum value
av	Spatial average
w	Pertaining to wall
L	Along length
δ	Across boundary layer

Superscript

$^{\circ}$	Pertaining to stagnation point
$*$	Property at sonic point

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