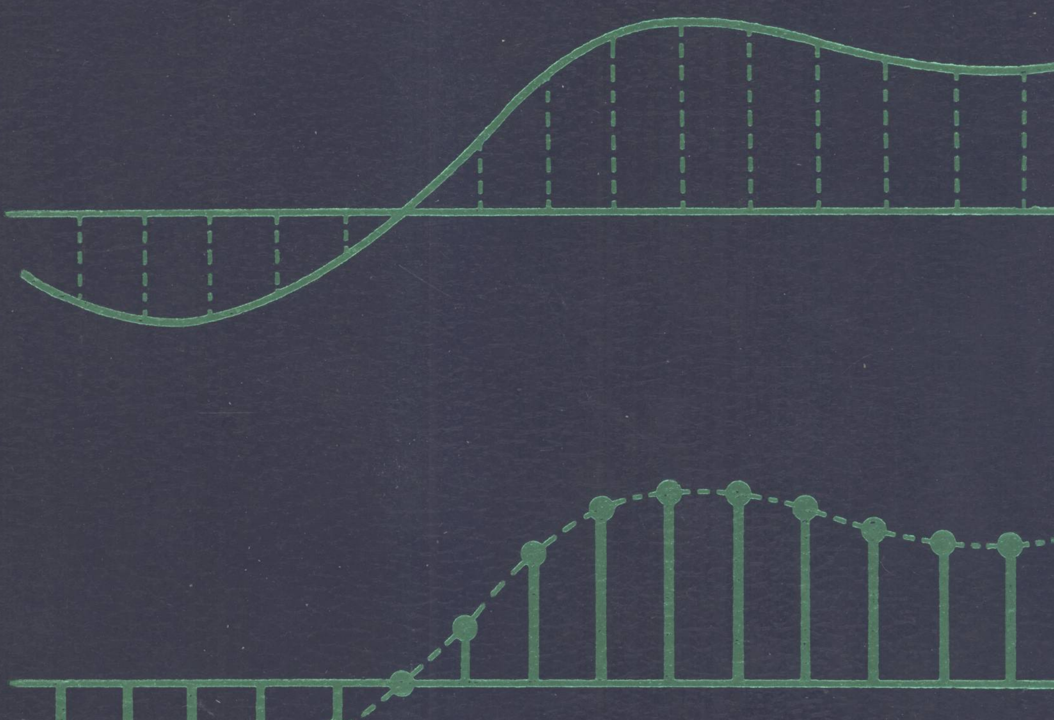
Signals, Systems, and Transforms

Leland B. Jackson



SIGNALS, SYSTEMS, AND TRANSFORMS

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CONTINUOUS-TIME LINEAR TIME-INVARIANT SYSTEMS

Convolution	$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)\, d\tau$
Causality	$h(t) = 0,\ t < 0$, and thus $y(t) = \int_{-\infty}^{t} x(\tau)h(t - \tau)\, d\tau$
Stability	$\int_{-\infty}^{\infty} \lvert h(t)\rvert\, dt < \infty$
Step response	$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$

PROPERTIES OF THE LAPLACE TRANSFORM

Definition	$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\, dt$	
Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(s) + bX_2(s)$	$R' \supset R_1 \cap R_2$
Time shift	$x(t - t_0) \leftrightarrow e^{-st_0}X(s)$	$R' = R$
Modulation	$e^{s_0 t}x(t) \leftrightarrow X(s - s_0)$	$R' = R + \mathrm{Re}\{s_0\}$
Axis scaling	$x(at) \leftrightarrow \frac{1}{\lvert a\rvert} X\left(\frac{s}{a}\right)$	$R' = aR$
Axis reversal	$x(-t) \leftrightarrow X(-s)$	$R' = -R$
Differentiation	$\frac{dx(t)}{dt} \leftrightarrow sX(s)$	$R' \supset R$
	$-tx(t) \leftrightarrow \frac{dX(s)}{ds}$	$R' = R$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau \leftrightarrow \frac{1}{s}X(s)$	$R' \supset R \cap \mathrm{Re}\{s\} > 0$
Convolution	$x_1(t) * x_2(t) \leftrightarrow X_1(s)X_2(s)$	$R' \supset R_1 \cap R_2$
Right-sided	$x(t) = 0,\ t < t_0 \leftrightarrow X(s)$	$R = \mathrm{Re}\{s\} > \sigma_{\mathrm{max}}$
Left-sided	$x(t) = 0,\ t > t_0 \leftrightarrow X(s)$	$R = \mathrm{Re}\{s\} < \sigma_{\mathrm{min}}$
Stable LTI system	$h(t) \leftrightarrow H(s)$	$R \supset \mathrm{Re}\{s\} = 0$

PROPERTIES OF THE CONTINUOUS-TIME FOURIER TRANSFORM

Definition	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$
Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(j\omega) + bX_2(j\omega)$
Time shift	$x(t - t_0) \leftrightarrow e^{-j\omega t_0}X(j\omega)$
Modulation	$e^{j\omega_0 t}x(t) \leftrightarrow X[j(\omega - \omega_0)]$
Axis scaling	$x(at) \leftrightarrow \frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Axis reversal	$x(-t) \leftrightarrow X(-j\omega)$
Duality	$X(jt) \leftrightarrow 2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt} \leftrightarrow j\omega X(j\omega)$
	$-jtx(t) \leftrightarrow \frac{dX(j\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau \leftrightarrow \frac{1}{j\omega}X(j\omega) + \pi X(0)\,\delta(\omega)$
	$-\frac{1}{jt}x(t) + \pi x(0)\,\delta(t) \leftrightarrow \int_{-\infty}^{\omega} X(j\lambda)\,d\lambda$
Convolution	$h(t) * x(t) \leftrightarrow H(j\omega)X(j\omega)$
Multiplication	$x(t)p(t) \leftrightarrow \frac{1}{2\pi}X(j\omega) * P(j\omega)$
Conjugation	$x^*(t) \leftrightarrow X^*(-j\omega)$
Real signals	$\mathrm{Ev}\{x(t)\} \leftrightarrow \mathrm{Re}\{X(j\omega)\}$
	$\mathrm{Od}\{x(t)\} \leftrightarrow j\,\mathrm{Im}\{X(j\omega)\}$
	$\mathrm{Re}\{X(j\omega)\} = \mathrm{Re}\{X(-j\omega)\}$
	$\mathrm{Im}\{X(j\omega)\} = -\mathrm{Im}\{X(-j\omega)\}$
	$\lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert$
	$\angle X(j\omega) = -\angle X(-j\omega)$
Parseval's relation	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$

SIGNALS, SYSTEMS, AND TRANSFORMS

To all my family

Preface

This book is intended as the text for a one-year introductory course in the theory of signals and linear systems at the third-year undergraduate level. It may also serve as the text for a one-semester or two-quarter course with appropriate selection of the topics to be covered. Both continuous-time and discrete-time signals and systems are included.

Two distinct approaches have evolved in the teaching of this subject area: (1) sequential coverage of continuous-time signals and systems and then of the corresponding discrete-time material and (2) integrated coverage of continuous-time and discrete-time material. In an effort to accommodate both approaches, Chapters 2 and 3 of this text are divided into Part A on continuous-time topics and Part B on discrete-time topics. Hence, to cover only continuous-time material at first, the instructor can utilize the Part A sections of Chapters 2 and 3 and then take up continuous-time Fourier and Laplace transforms in Chapters 4 and 5. The Part B sections of Chapters 2 and 3 on the corresponding discrete-time topics can then be assigned and covered before proceeding on to the discrete-time z and Fourier transforms in Chapters 6 and 7. On the other hand, if the earlier material is to be presented in an integrated manner, both Parts A and B of Chapters 2 and 3 can be assigned and covered before continuing on to subsequent chapters.

The Part A and Part B treatments are exactly parallel so that the student can refer back and forth between the continuous- and discrete-time material, thereby reinforcing what is common in the topics, while noting the differences.

Even for the integrated approach, a major departure here from several competing texts is that the Laplace transform is covered immediately following the Fourier transform for continuous-time signals, rather than after the discrete-time Fourier transform has also been covered. The author believes from his teaching experience that it is somewhat artificial to make the students wait for the Laplace transform when it follows so naturally after the Fourier transform. He has also found that the multiplicity of different Fourier representations for periodic and aperiodic discrete-time signals (DFS, DTFT, DFT) and the periodicity and aliasing of these transforms are quite confusing to students before they have studied the z transform, but not after. Hence the sequence of topics in Chapters 4 through 7 is the Fourier representation of continuous-time signals, the Laplace transform, the z transform, and the Fourier representation of discrete-time signals, in that order.

Another feature of the book is the considerable effort made to prepare the students to understand convolution and to implement it correctly. Most students can readily repeat that convolution involves "flipping one signal around and sliding it along," but too often they don't really know what that means. Therefore in Chapter 2 we spend substantial time studying modification of an independent variable and stress the difference between the functionals $x(\ )$ and $x[\ ]$, the functions $x(t)$ and $x[n]$, and the values $x(t_0)$ and $x[n_0]$. The student is then introduced to convolution purely as a mathematical operation, with many examples given for both continuous- and discrete-time signals. The application of convolution to linear time-invariant systems follows in Chapter 3 and subsequent chapters, with many additional examples.

Other notable features of the book include the following:

- Among the examples presented are several that are given special emphasis because of their significance. These examples are called Applications and are listed as such in the Table of Contents.
- The fundamental applications of filtering, sampling, and modulation are integrated into the chapters on Fourier, Laplace, and z transforms, rather than being isolated in separate chapters.
- A chapter is included on state variables, with a section devoted to operational-amplifier networks that gives the students a chance to synthesize many useful analog filter networks and ties in nicely with their electronics courses.
- Problems are included to extend the concepts presented as well as to provide an opportunity to practice specific techniques. For instance, correlation functions are introduced and examined in Problems 2.21, 3.22, 4.24, 5.40, and 6.40.

The author is grateful to all those who have made suggestions and otherwise assisted him in this project, including Profs. Faye Boudreaux-Bartels, Steven Kay, Ramdas Kumaresan, Allen Lindgren, John Spence, Peter Swaszek, and Richard Vaccaro at the University of Rhode Island and Gerald Lemay at Southeastern Massachusetts University. The suggestions of Prof. Weiping Li of Lehigh University have been particularly helpful. I also appreciate the support and assistance of Tom Robbins, Eileen Bernadette Moran, Bette Aaronson, and Sarah Hallet at Addison-Wesley Publishing Company. Careful proofreading was contributed by Haiguang Chen and Su-Hsiu Wu at URI. Finally, my students in ELE313 and ELE314 have motivated this book and provided continuing feedback and support.

L. B. J.

DISCRETE-TIME LINEAR TIME-INVARIANT SYSTEMS

Convolution	$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
Causality	$h[n] = 0,\ n < 0$, and thus $y[n] = \sum_{k=-\infty}^{n} x[k]h[n-k]$
Stability	$\sum_{n=-\infty}^{\infty} \lvert h[n]\rvert < \infty$
Step response	$s[n] = \sum_{k=-\infty}^{n} h[k]$

PROPERTIES OF THE z TRANSFORM

Definition	$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$	
Linearity	$ax_1[n] + bx_2[n] \leftrightarrow aX_1(z) + bX_2(z)$	$R' \supset R_1 \cap R_2$
Time shift	$x[n-n_0] \leftrightarrow z^{-n_0}X(z)$	$R' \supset R \cap 0 < \lvert z\rvert < \infty$
Modulation	$z_0^n x[n] \leftrightarrow X(z/z_0)$ $e^{j\Omega_0 n}x[n] \leftrightarrow X(ze^{-j\Omega_0})$	$R' = \lvert z_0\rvert R$ $R' = R$
Time reversal	$x[-n] \leftrightarrow X(1/z)$	$R' = 1/R$
Differentiation	$nx[n] \leftrightarrow -z\dfrac{dX(z)}{dz}$	$R' = R$
Convolution	$x_1[n] * x_2[n] \leftrightarrow X_1(z)X_2(z)$	$R' \supset R_1 \cap R_2$
Accumulation	$\sum_{k=-\infty}^{n} x[k] \leftrightarrow X(z)\dfrac{1}{1-z^{-1}}$	$R' \supset R \cap \lvert z\rvert > 1$
Causal signal	$x[n] = 0,\ n < 0 \leftrightarrow X(z)$	$R = \lvert z\rvert > r_{max}$
Anticausal signal	$x[n] = 0,\ n > 0 \leftrightarrow X(z)$	$R = \lvert z\rvert < r_{min}$
Stable LTI system	$h[n] \leftrightarrow H(z)$	$R \supset \lvert z\rvert = 1$

PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Definition	$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$
Linearity	$ax_1[n] + bx_2[n] \leftrightarrow aX_1(e^{j\Omega}) + bX_2(e^{j\Omega})$
Time shift	$x[n - n_0] \leftrightarrow e^{-j\Omega n_0}X(e^{j\Omega})$
Modulation	$e^{j\Omega_0 n}x[n] \leftrightarrow X(e^{j(\Omega-\Omega_0)})$
Time reversal	$x[-n] \leftrightarrow X(e^{-j\Omega})$
Conjugation	$x^*[n] \leftrightarrow X^*(e^{-j\Omega})$
Differentiation	$nx[n] \leftrightarrow j\dfrac{dX(e^{j\Omega})}{d\Omega}$
Convolution	$x_1[n] * x_2[n] \leftrightarrow X_1(e^{j\Omega})X_2(e^{j\Omega})$
Multiplication	$x_1[n]x_2[n] \leftrightarrow \dfrac{1}{2\pi}X_1(e^{j\Omega}) \otimes X_2(e^{j\Omega})$
Accumulation	$\sum_{k=-\infty}^{n} x[k] \leftrightarrow \dfrac{1}{1 - e^{-j\Omega}}X(e^{j\Omega}) + \pi X(e^{j0})\,\delta(\Omega)$
Real signals	$\mathrm{Ev}\{x[n]\} \leftrightarrow \mathrm{Re}\{X(e^{j\Omega})\}$ $\mathrm{Od}\{x[n]\} \leftrightarrow j\,\mathrm{Im}\{X(e^{j\Omega})\}$ $\mathrm{Re}\{X(e^{j\Omega})\} = \mathrm{Re}\{X(e^{-j\Omega})\}$ $\mathrm{Im}\{X(e^{j\Omega})\} = -\mathrm{Im}\{X(e^{-j\Omega})\}$ $\lvert X(e^{j\Omega})\rvert = \lvert X(e^{-j\Omega})\rvert$ $\angle X(e^{j\Omega}) = -\angle X(e^{-j\Omega})$
Parseval's relation	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \dfrac{1}{2\pi}\int_{2\pi} \lvert X(e^{j\Omega})\rvert^2\, d\Omega$

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Contents

CHAPTER 5 209

The Laplace Transform

CHAPTER 6 285

The *z* Transform

CHAPTER

1

Overview of Signals and Systems

INTRODUCTION

The theory, analysis, and design of signals and systems play a major role in almost all branches of electrical engineering and in many other engineering and scientific fields as well. Examples of electronic *systems* include radio and television, telephone networks, sonar and radar, guidance and navigation, laboratory instrumentation, industrial control, biomedical instrumentation, remote (satellite) sensing, communications intelligence, military surveillance and fire-control, seismic analysis, radio astronomy, and on and on. Mechanical system examples include vibration analyzers, suspension systems, microphones and hydrophones, loudspeakers, accelerometers, and so forth. *Signals* are the input, output, and internal functions that these systems process or produce, such as voltage, pressure, displacement, or intensity. Often the independent variable for these signal functions is time, but it can also be distance, angle, etc., especially for *two-dimensional* signals such as images.

Our study in this text thus has two major components—corresponding to *signals* and *systems,* respectively—but there are many points of commonality, or overlap, between them. Traditionally the corresponding