APPLIED MECHANICS

J Hannah
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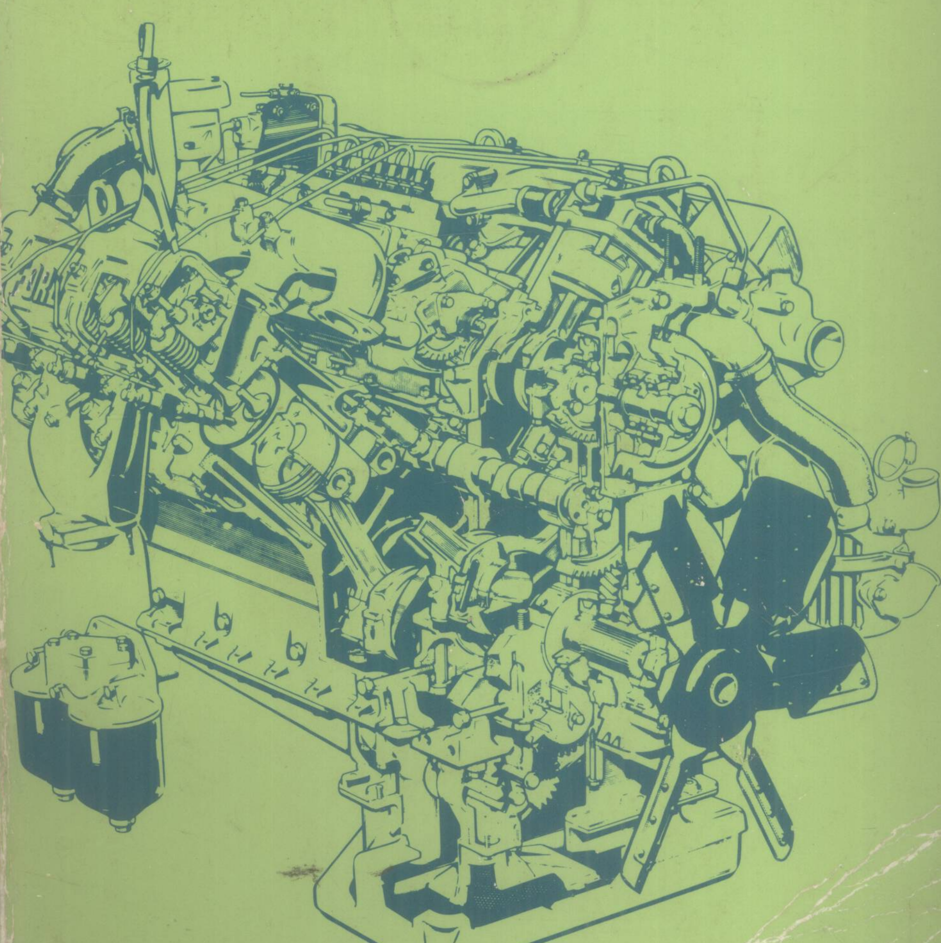
Metric Edition



Applied Mechanics

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First Metric Edition

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Preface to First Metric Edition

Due to the adoption of SI units (Système International d'Unités) as the primary system of weights and measures it has become necessary to bring the book up to date in this respect. The text has been rewritten in SI units and no mention has been made of Imperial or other units. In this way it is hoped that the student will be able to follow the principles of the subject without having to master various unit systems and face the complications caused by conversion from one set of units to another. These other unit systems will exist for some time to come and conversion of units will be a necessary common exercise, but it is felt that these aspects are best dealt with by the teacher or by a different form of book.

Besides meeting the needs of the Ordinary National Certificate course the book has been widely used by students in Diploma and Technician courses. Since some deletions have had to be made because of the change in units the opportunity has been taken to add some material to bring the book more completely into line with the Ordinary National Certificate and Diploma syllabuses. Two main additions are a section on Impact of Jets in the chapter on Fluids in Motion, and some work on the Method of Sections as applied to Frameworks. It has been found necessary also to increase the number of worked and unworked examples in order to illustrate and give practice in the new system of units.

J.H.
M.J.H.

Contents

1

Statics

The following notes on some elementary principles and theorems of statics are intended as a reminder of work which the student should have already covered and which will be required in subsequent chapters.

1.1. Mass, Force and Weight

The *mass* of a body is the quantity of matter it contains.

A *force* is simply a push or a pull and may be measured by its effect on a body. A force may change or tend to change the shape or size of a body; if applied to a body at rest the force will move or tend to move it; if applied to a body already moving the force will change the motion.

A particular force is that due to the effect of gravity on a body, i.e. the *weight* of a body.

These three quantities—mass, force and weight—are dealt with fully in Chapter 5, but it is necessary here to specify the units and the essential relationship between mass and weight.

The basic SI unit of mass is the *kilogramme* (**kg**); other units of mass are—

1 *megagramme* (**Mg**) or *tonne* (**t**) $= 10^3$ kg
1 *gramme* (**g**) $= 10^{-3}$ kg
1 *milligramme* (**mg**) $= 10^{-6}$ kg

The basic SI unit of force is the *newton* (**N**) defined as *that force which, when applied to a body having a mass of one kilogramme, gives it an acceleration of one metre per second squared.* From Newton's second law of motion (*see* page 83) we have

$$\text{force} = \text{mass} \times \text{acceleration}$$
$$P = Mf$$

where P is the applied force, M the mass of a body, and f the acceleration produced in the body. Thus in the SI system $P = 1$ N, $M = 1$ kg and $f = 1$ m/s², i.e.

$$1\ (\text{N}) = 1\ (\text{kg}) \times 1\ (\text{m/s}^2)$$

Other units of force used are—

1 *kilonewton* (**kN**) $= 10^3$ N
1 *meganewton* (**MN**) $= 10^6$ N
1 *giganewton* (**GN**) $= 10^9$ N

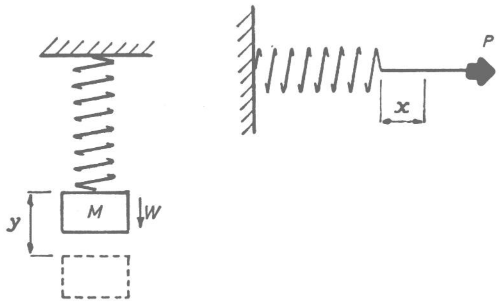

FIG. 1.1

The acceleration of any body towards earth in free fall is $g = 9{\cdot}8$ m/s², hence the weight W of a body of mass M is—

$$\text{force} = \text{mass} \times \text{acceleration}$$
$$W = Mg$$

If the mass M is in kilogrammes, then

$$W = M \times 9{\cdot}8\ \text{N}$$

If the mass M is in megagrammes (tonnes), then

$$W = M \times 1{,}000 \times 9{\cdot}8\ \text{N}$$
$$= M \times 9{\cdot}8\ \text{kN}$$

(Fig. 5.5, Chapter 5, shows the relationship between the weight W of a body and its mass M.)

Although defined in dynamic terms, a force may also be measured statically by the weight of the mass it will just support or by comparing its effect with the weight of a standard mass. Thus if a mass M is suspended from a spring (Fig. 1.1) the extension is said to be due to the force of gravity on the mass, i.e. to its weight $W = Mg$. If y is the extension produced by this force W and a force P on the same spring produces an extension x then the value of P is measured in terms of W by simple proportion: thus

$$\frac{P}{W} = \frac{x}{y}$$

Proper specification of a force requires knowledge of three quantities—

1. its magnitude
2. its point of application
3. its line of action.

Since a force has magnitude, direction and sense it is a *vector quantity* and may be represented by a straight line of definite length.

DEAD LOADS

In statics the vertically downwards force due to the weight of "dead loads" must always be taken into account, and this force of gravity acts through the centre of gravity of the load. A load may be given in force units, i.e. N, kN, MN, or GN. A "load" may also be specified in mass units, i.e. kg or Mg (tonne), and in this case the corresponding *weight* of the mass must be found before carrying out calculations involving forces.

1.2. Forces in Equilibrium: Triangle of Forces

Statics is the study of forces in *equilibrium* ("in balance"). A single force cannot exist alone and is unbalanced. For equilibrium it must be balanced by an equal and opposite force acting along the same straight line. Thus in Fig. 1.2 the load of 1 kN *on* the tie is

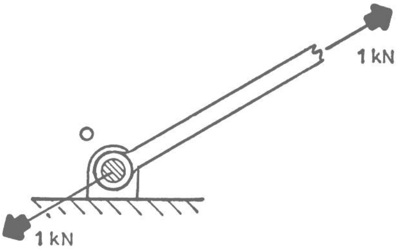

FIG. 1.2

balanced at the joint O by an equal and opposite force of 1 kN exerted *by* the joint *on* the tie. Thus forces may be said to exist in pairs. Nevertheless a single force may also be balanced by any number of other forces.

For three forces in the same plane to be in equilibrium—

(*a*) they must have their lines of action all passing through one point, i.e. they must be *concurrent*;

(*b*) they may be represented in magnitude and direction by the three sides of a triangle *taken in order*, i.e. by a *triangle of forces*.

The condition that all three forces must pass through one point is particularly useful in solving mechanics problems. For example, the light jib crane shown in Fig. 1.3 (*a*) is in equilibrium under the action of three forces. The jib carries a load *W* at A; the free end is supported by a cable in which the tension is *T*; the end C is

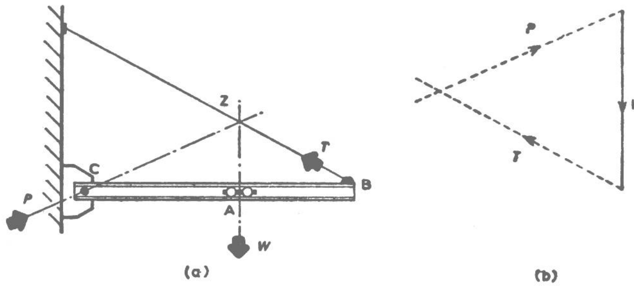

FIG. 1.3

pinned to the wall by a joint which allows free rotation of the jib at C. The reaction *P* of the joint on the jib is completely unknown; the magnitude of *T* is unknown but its direction must be that of the cable. Since the three forces are in balance their lines of action must pass through one point, i.e. where the lines of action of *W* and *T* intersect (point Z, Fig. 1.3 (*a*)). The line of action of *P* is therefore found by joining C to Z.

Since the magnitude of *W* is known and the directions of the three forces have been determined the triangle of forces can now be drawn, Fig. 1.3 (*b*).

1.3. Resultant and Equilibrant: Parallelogram of Forces

The forces *W* and *T* of Fig. 1.3 may also be represented by the two sides **ab** and **ad**, respectively, of the parallelogram **abcd**, Fig. 1.4. The diagonal **ac**, taken in the sense **a** to **c**, is the *resultant R* of the two forces *W* and *T* acting together. This resultant force is equivalent to, and may replace completely, these two forces. The resultant **ac** may be balanced by an equal and opposite force **ca** called the *equilibrant*. This is in fact the force *P* at the pin-joint, Fig. 1.3 (*a*). This construction, by which two forces are replaced by a single equivalent force, is known as the *parallelogram of forces*. It can only be used if the two forces are specified in both magnitude *and* direction.

1.4. Resolution of Forces

Since the forces represented by **ab** and **ad** in Fig. 1.4 may be replaced completely by a single force **ac**, it is often useful to carry out the reverse process, i.e. to replace a single force by two other forces in any two convenient directions. These two forces are then known as the *components* of the single force. Physically this is equivalent to finding the effects of the single force in the two chosen directions.

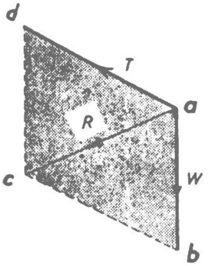

FIG. 1.4

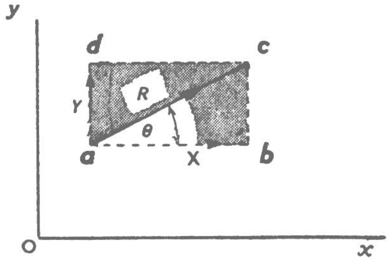

FIG. 1.5

The most convenient choice of directions in which to *resolve* a force is in two directions at right angles. Fig. 1.5 shows a force $R = \mathbf{ac}$ resolved into forces $X = \mathbf{ab}$ and $Y = \mathbf{ad}$ along the two perpendicular directions Ox and Oy, respectively. Since the three forces shown do not represent independent forces the components of R are shown in broken lines. Let R make an angle θ with the Ox direction, then

$$\mathbf{ab} = \mathbf{ac} \cos \theta$$

i.e.

$$X = R \cos \theta$$

and

$$\mathbf{ad} = \mathbf{ac} \sin \theta$$

or

$$Y = R \sin \theta$$

1.5. Polygon of Forces

If more than three forces act at the same point and are in equilibrium, they may be represented in magnitude and direction by the sides of a polygon *taken in order*. "Taken in order" refers to the order of drawing the sides of the polygon and not to the order in which the forces are taken from the space diagram.

Suppose the four forces 1, 2, 3 and 4 acting at the joint shown in Fig. 1.6 (*a*) to be in balance; they may then be represented by the four sides of the polygon **abcd**, Fig. 1.6 (*b*). This is a closed polygon since the forces are in equilibrium. If the forces are not in balance the polygon will not close and the required closing line gives the equilibrant or the equal and opposite resultant force, depending on the sense in which it is taken. Fig. 1.6 (*c*) shows the force polygon assuming unbalance. The forces 1, 2, 3, 4 are represented by lines **ab**, **bc**, **cd** and **de**, respectively. To close the polygon and maintain a balance of forces requires the equilibrant **ea** taken in the sense **e** to **a**. The resultant of the original set of four unbalanced forces is given by the line **ae** and acts in the sense **a** to **e**.

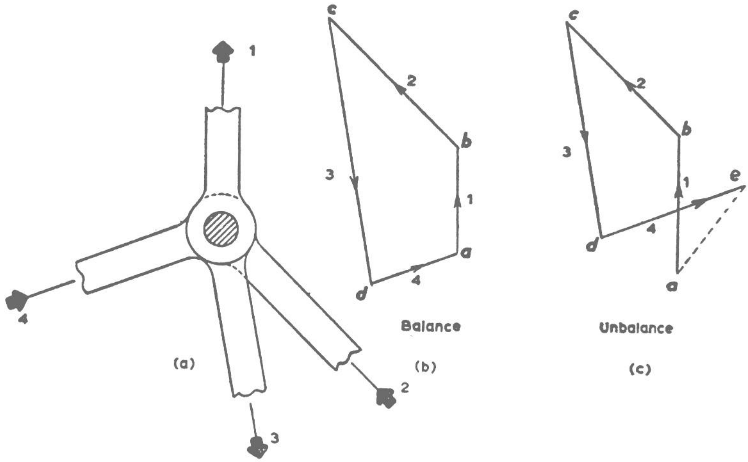

FIG. 1.6

1.6. Moment of a Force

The *moment* of a force P about a point O (Fig. 1.7) is the product of the force and the perpendicular distance x of its line of action from O, thus—

$$\text{moment of } P \text{ about O} = Px$$

If the force P is in newtons and the distance x in metres then the units for moment are written *newton-metres* (**N-m**). For larger values it is usual to retain the metre for distance and use the higher multiples of the newton, e.g. MN-m.

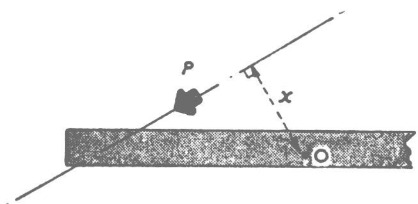

FIG. 1.7

1.7. Couple

A pair of equal and opposite parallel forces, which do not act in the same straight line, form a *couple*, Fig. 1.8. The total moment M of the two forces P about any point O in the plane of the couple is

$$\begin{aligned} M &= P(a + x) - Px \\ &= Pa \end{aligned}$$

This moment is independent of the distance x and is therefore *the same about any point in the same plane.* The turning effect of the couple is also the same wherever it may be placed in the plane.

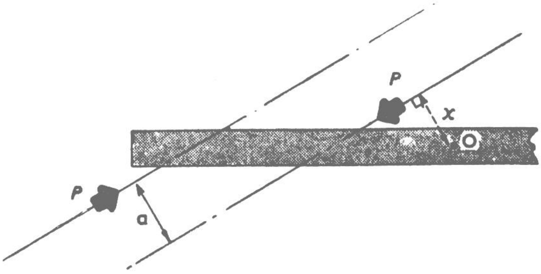

FIG. 1.8

The magnitude Pa of a couple is known as its *moment* or *torque,* although the term torque is usually restricted to a moment tending to twist a shaft.

1.8. Balance of Moments

Consider any system of forces which do not act at a point. Take moments about any arbitrary point and let clockwise moments be positive and anticlockwise moments be negative. Then for balance of moments we must have—

clockwise moments = anticlockwise moments

or *the algebraic sum of the moments of all the forces about the same point is zero.*

1.9. Resolution of a Force into a Force and a Couple

Consider the arm OA pivoted at a bearing at O and subjected to a force P at A which acts perpendicularly to OA (Fig. 1.9). We

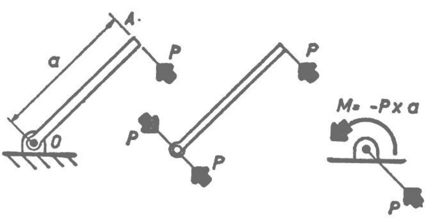

FIG. 1.9

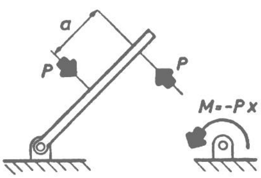

FIG. 1.10

wish to know the effect of force P at the bearing O. Suppose therefore two equal and opposite forces P to be introduced at O acting parallel to the existing force P at A. The system of forces is not upset and the resultant force is unaltered since the two new forces are self-cancelling. However, it can now be seen that the effect of P at A is equivalent to a single force P at O, together with a couple of moment Pa tending to turn the arm anticlockwise. The effect therefore of a single force P on a body at a point offset from the line of action of the force is to produce at that point both a force *and* a couple.

A pure couple on the other hand will not introduce this single out-of-balance force at any point. In Fig. 1.10 the two forces P form a pure couple; they are self-balancing and no force is produced at the bearing (unlike the previous case). The couple due to the two forces is of course out of balance and can only be balanced by an equal and opposite couple in the same plane. This result is independent of the position of the couple.

1.10. The General Conditions of Equilibrium

We now require to consider the balance of *any* system of forces, in a plane, which do *not* all act through the same point. Since any force may be replaced by a similar force at any other point, together with a couple, then each of the forces may be considered as acting at any one point, provided that allowance is made for all the couples

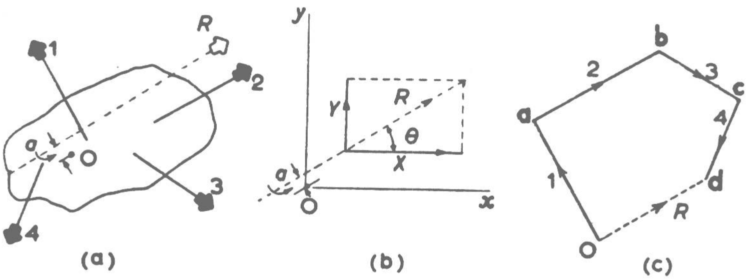

FIG. 1.11

produced. For complete equilibrium of the system, therefore, there must be no unbalanced force *or* couple. *For force balance a polygon of forces may be drawn and must close for equilibrium. For couple balance the algebraic sum of the moments of all the forces about any point must be zero.*

Alternatively it is often convenient to resolve all the forces in the same two mutually-perpendicular directions. Consider the force system shown in Fig. 1.11. The forces may be resolved in the two directions parallel to Ox and Oy. Let X and Y be the algebraic sum of the components of all the forces in the Ox- and Oy-directions, respectively; and let M be the algebraic sum of the moments of all the forces about any chosen point O. Then the resultant force R is given by Fig. 1.11 (b)—

$$R^2 = X^2 + Y^2$$

and the line of action of the resultant is at an angle θ to Ox given by—

$$\tan \theta = \frac{Y}{X}$$

The resultant couple M is the same about any point in its plane hence it may be obtained by calculating the algebraic sum of the moments of all the forces about any point O.

The conditions of equilibrium are therefore—

$$R = 0 \quad \text{(i.e. } X = 0, \text{ and } Y = 0)$$

and $$M = 0$$

Otherwise, if R and M are not zero, to find the *position* of the resultant R we calculate the distance a of its line of action from any chosen point O, Fig. 1.11 (a). The resultant R may be replaced by a force acting at O, together with a couple of moment $R \times a$ about O. This couple is equal in magnitude and sense to the resultant couple M, i.e.

$$R \times a = M$$

This determines the distance a from O of the line of action of R.

If $M = 0$, then $a = 0$ and the resultant R passes through the chosen point O.

If M is not zero, then as R becomes very small the distance a becomes very large so that $R \times a$ is always equal to M.

Note—If the force polygon is drawn, Fig. 1.11 (c), the closing line **od** gives the resultant R in magnitude, sense and direction but *not* in position. To obtain the position we must take moments about some point O.

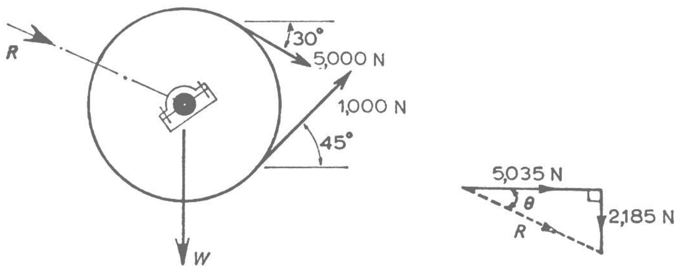

FIG. 1.12

Example. Fig. 1.12 shows the tensions in the tight and slack sides of a rope passing round a pulley of mass 40 kg. Calculate the resultant force on the bearings, and its direction.

Solution

Weight of pulley is

$$W = Mg = 40 \times 9{\cdot}8 = 392 \text{ N}$$

Resolving horizontally—

$$\text{unbalanced force, } X = 5{,}000 \cos 30° + 1{,}000 \cos 45°$$
$$= 5{,}035 \text{ N (to the right)}$$

Resolving vertically—

$$\text{unbalanced force, } Y = 5{,}000 \sin 30° - 1{,}000 \sin 45° + 392$$
$$= 2{,}185 \text{ N (downwards)}$$

Resultant force R is given by

$$R^2 = X^2 + Y^2$$

i.e.

$$R = \sqrt{(5{,}035^2 + 2{,}185^2)}$$
$$= 5{,}500 \text{ N}$$
$$= \mathbf{5{\cdot}5 \text{ kN}}$$

The line of action of R makes an angle θ with the horizontal given by

$$\tan \theta = \frac{2{,}185}{5{,}035} = 0{\cdot}433$$

thus

$$\theta = \mathbf{23° \, 26'}$$

Fig. 1.12 shows the base of the bearing inclined to this resultant thrust in order that the cap bolts shall not carry any lifting load.

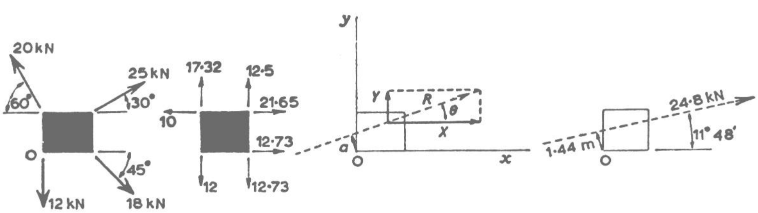

FIG. 1.13

Example. Find the magnitude, direction and position of the resultant of the system of forces shown in Fig. 1.13. The forces act at the four corners of a square of 3 m side.

Solution

Force (kN)	Vertical component (kN)	Moment of vertical component about O (kN-m)
20	$+20 \sin 60° = 17{\cdot}32$	0
12	-12	0
18	$-18 \sin 45° = -12{\cdot}73$	$+12{\cdot}73 \times 3 = 38{\cdot}2$
25	$+25 \sin 30° = 12{\cdot}5$	$-12{\cdot}5 \times 3 = -37{\cdot}5$
Totals	$Y = +5{\cdot}09$	$+0{\cdot}7$

Force (kN)	Horizontal component (kN)	Moment of horizontal component about O (kN-m)
20	$-20 \cos 60° = -10$	$-10 \times 3 = -30$
12	0	0
18	$+18 \cos 45° = +12{\cdot}73$	0
25	$+25 \cos 30° = +21{\cdot}65$	$+21{\cdot}65 \times 3 = 64{\cdot}95$
Totals	$X = +24{\cdot}38$	$+34{\cdot}95$

The horizontal and vertical components of each force together with the moments of these components about point O are shown in the above table. Upward vertical forces and horizontal forces to the right are positive, clockwise moments are positive.

$$R^2 = X^2 + Y^2$$

therefore
$$R = \sqrt{(24{\cdot}38^2 + 5{\cdot}09^2)}$$
$$= \mathbf{24{\cdot}8\ kN}$$
$$\tan\theta = \frac{5{\cdot}09}{24{\cdot}38} = 0{\cdot}209$$

thus
$$\theta = \mathbf{11°\ 48'} \text{ above the horizontal}$$

Total moment about O $= +\ 0{\cdot}7 + 34{\cdot}95$
$$= +\ 35{\cdot}65 \text{ kN-m (clockwise)}$$

This moment is equal to that of the resultant force R about O. If a is the perpendicular distance of the line of action of R from O then

$$R \times a = 35{\cdot}65$$
$$a = \frac{35{\cdot}65}{24{\cdot}8} = \mathbf{1{\cdot}44\ m}$$

Therefore R must act along a line 1·44 m from O as shown in Fig. 1.13, such that it produces a *clockwise* moment about O and is inclined at 11° 48′ to the horizontal. This solution may be checked by drawing the force polygon to obtain the magnitude and direction of R. To find the total moment about O, measure from a scale drawing the perpendicular distance of the line of action of each force from O.

Example. The jib crane shown in Fig. 1.14 (*a*) carries a mass M tonne at C. If the maximum permissible loads in the tie and jib are 25 and 36 kN respectively find the safe value of the load M.

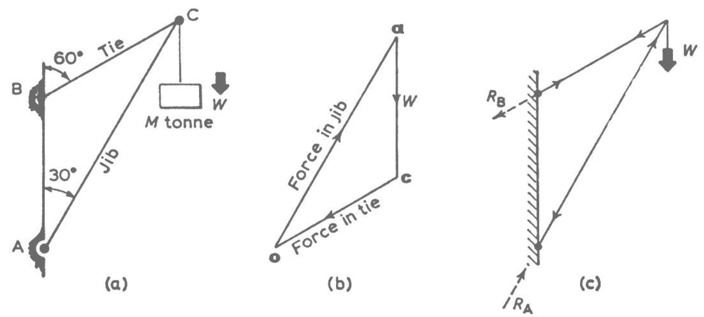

FIG. 1.14

Solution

M tonne = $M \times 1{,}000$ kg. The weight of this mass is

$$W = M \times 1{,}000 \times 9{\cdot}8 \text{ N}$$
$$= 9{\cdot}8M \text{ kN}$$

Fig. 1·14 (*b*) shows the triangle of forces for joint C where **oc** represents the force in the tie, **oa** the force in the jib, and **ac** the load W. If **oc** is drawn to represent 25 kN (the maximum permitted force in the tie) then, by construction or calculation—

$$\mathbf{ac} = 25 \text{ kN}$$

hence

$$W = 9{\cdot}8\, M = 25 \text{ kN}$$
$$M = 2{\cdot}55 \text{ tonne}$$

If **oa** is drawn to represent 36 kN, the maximum permitted force in the jib, then, by construction or calculation (take AB = 1 m)—

$$\mathbf{ac} = 20{\cdot}8 \text{ kN}$$

hence

$$W = 9{\cdot}8\, M = 20{\cdot}8 \text{ kN}$$
$$M = 2{\cdot}13 \text{ tonne}$$

The safe value of the load is therefore **2·13 tonne** since the permitted forces in the tie and jib are not exceeded.