François Axisa &
Philippe Trompette

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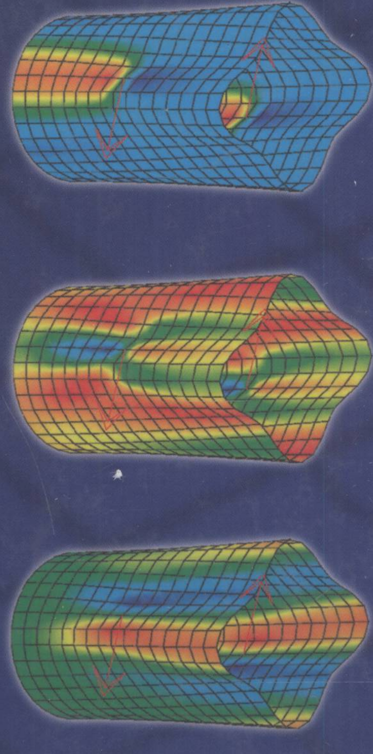
MODELLING OF MECHANICAL SYSTEMS

Structural Elements



VOLUME 2

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AMSTERDAM • BOSTON • HEIDELBERG • LONDON • NEW YORK • OXFORD
PARIS • SAN DIEGO • SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Elsevier Butterworth-Heinemann
Linacre House, Jordan Hill, Oxford OX2 8DP
30 Corporate Drive, Burlington, MA 01803

First published in France 2001 by Hermes Science, entitled '*Modélisation des systèmes mécaniques, systèmes continus, Tome 2*'

First published in Great Britain 2005

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication Data
A catalogue record for this book is available from the Library of Congress

ISBN 0 7506 6846 6

For information on all Elsevier Butterworth-Heinemann publications visit our website at http://books.elsevier.com

Typeset by Newgen Imaging Systems (P) Ltd., Chennai, India
Printed and bound in Great Britain

MODELLING OF MECHANICAL SYSTEMS VOLUME 2

Preface

In mechanical engineering, the needs for design analyses increase and diversify very fast. Our capacity for industrial renewal means we must face profound issues concerning efficiency, safety, reliability and life of mechanical components. At the same time, powerful software systems are now available to the designer for tackling incredibly complex problems using computers. As a consequence, computational mechanics is now a central tool for the practising engineer and is used at every step of the designing process. However, it cannot be emphasized enough that to make a proper use of the possibilities offered by computational mechanics, it is of crucial importance to gain first a thorough background in theoretical mechanics. As the computational process by itself has become largely an automatic task, the engineer, or scientist, must concentrate primarily in producing a tractable model of the physical problem to be analysed. The use of any software system either in a University laboratory, or in a Research department of an industrial company, requires that meaningful results be produced. This is only the case if sufficient effort was devoted to build an appropriate model, based on a sound theoretical analysis of the problem at hand. This often proves to be an intellectually demanding task, in which theoretical and pragmatic knowledge must be skilfully interwoven. To be successful in modelling, it is essential to resort to physical reasoning, in close relationship with the information of practical relevance.

This series of four volumes is written as a self-contained textbook for engineering and physical science students who are studying structural mechanics and fluid–structure coupled systems at a graduate level. It should also appeal to engineers and researchers in applied mechanics. The four volumes, already available in French, deal respectively with Discrete Systems, Basic Structural Elements (beams, plates and shells), Fluid–Structure Interaction in the absence of permanent flow, and finally, Flow-Induced Vibrations. The purpose of the series is to equip the reader with a good understanding of a large variety of mechanical systems, based on a unifying theoretical framework. As the subject is obviously too vast to cover in an exhaustive way, presentation is deliberately restricted to those fundamental physical aspects and to the basic mathematical methods which constitute the backbone of any large software system currently used in mechanical engineering. Based on the experience gained as a research engineer in nuclear engineering at the French Atomic Commission, and on course notes offered to

2nd and 3rd year engineer students from ECOLE NATIONALE SUPERIEURE DES TECHNIQUES AVANCEES, Paris and to the graduate students of Paris VI University, the style of presentation is to convey the main physical ideas and mathematical tools, in a progressive and comprehensible manner. The necessary mathematics is treated as an invaluable tool, but not as an end in itself. Considerable effort has been taken to include a large number of worked exercises, especially selected for their relative simplicity and practical interest. They are discussed in some depth as enlightening illustrations of the basic ideas and concepts conveyed in the book. In this way, the text incorporates in a self-contained manner, introductory material on the mathematical theory, which can be understood even by students without in-depth mathematical training. Furthermore, many of the worked exercises are well suited for numerical simulations by using software like MATLAB, which was utilised by the author for the numerous calculations and figures incorporated in the text. Such exercises provide an invaluable training to familiarize the reader with the task of modelling a physical problem and of interpreting the results of numerical simulations. Finally, though not exhaustive the references included in the book are believed to be sufficient for directing the reader towards the more specialized and advanced literature concerning the specific subjects introduced in the book.

To complete this work I largely benefited from the input and help of many people. Unfortunately, it is impossible to properly acknowledge here all of them individually. However, I wish to express my gratitude to Alain Hoffmann head of the Department of Mechanics and Technology at the Centre of Nuclear Studies of Saclay and to Pierre Sintes, Director of ENSTA who provided me with the opportunity to be Professor at ENSTA. A special word of thanks goes to my colleagues at ENSTA and at Saclay – Ziad Moumni, Laurent Rota, Emanuel de Langre, Ianis Politopoulos and Alain Millard – who assisted me very efficiently in teaching mechanics to the ENSTA students and who contributed significantly to the present book by pertinent suggestions and long discussions. Acknowledgements also go to the students themselves whose comments were also very stimulating and useful. I am also especially grateful to Professor Michael Païdoussis from McGill University Montreal, who encouraged me to produce an English edition of my book, which I found quite a challenging task afterwards! Finally, without the loving support and constant encouragement of my wife Françoise this book would not have materialized.

François Axisa
August 2003

Introduction

To understand what is meant by structural elements, it is convenient to start by considering a whole structure made of various components assembled together with the aim to satisfy various functional and cost criteria. Depending on the domain of application, the terminology used to designate such assemblies varies; they are referred to as buildings, civil engineering works, machines and devices, vehicles etc. In most cases, the shapes of such structures are so complicated that the appropriate way to make a mathematical model feasible, is to identify simpler structural elements, defined according to a few generic response properties. Such a theoretical approach closely follows the common engineering practice of selecting a few appropriate generic shapes to build complex structures. Since the architects and engineers of the Roman Empire, two geometrical features have been recognized as key factors to save material and weight in a structure. The first one is to design slender components, that is, at least one dimension of the body is much less than the others. From the analyst standpoint this allows to model the actual 3D solid by using an equivalent solid of reduced dimension. Accordingly, one is led to distinguish first between 1D and 2D structural elements. The second geometrical property of paramount importance to optimise the mechanical resistance of structural elements is the curvature of the equivalent solid. Based on these two properties structural elements can be identified as:

1. Straight beams, modelled as a one-dimensional and rectilinear equivalent solid.
2. Plates, modelled as a two-dimensional and planar equivalent solid.
3. Curved beams, modelled as a one-dimensional and curved equivalent solid.
4. Shells, modelled as a two-dimensional and curved equivalent solid.

The second volume of this series deals with modelling and analysis of the mechanical responses of such structural elements. However, this vast subject is restricted here, essentially, to the linear elastodynamic domain, which constitute the cornerstone of mathematical modelling in structural mechanics. Moving on from discrete systems to deformable solids, as material is assumed to be continuously distributed over a bounded domain defined in a 3D Euclidean space, two new salient points arise. First, motion must be described in terms of continuous functions of space and then appropriate boundary conditions have to be specified in order to describe the mechanical equilibrium of the solid boundary. That mastering the consequences of these two features in structural modelling is by far not a simple task can be amply asserted by recalling that it progressed, along with the necessary mathematics, step by step over a long period lasting essentially from the eighteenth to the first half of the twentieth century. Apart from the concepts and methods inherent to the continuous nature of the problem, those already described in Volume 1, to deal with discrete systems keep all their interest, in particular the concept of natural modes of vibration and the methods of modal analysis. Actually, in practice, to analyse most of the engineering structures, it is necessary to build first a finite element model, according to which the structure is discretized into a finite number of parts, leading to a finite set of time differential equations. The latter can be solved numerically on the computer, either by using a spectral or a time stepping method.

Chapter 1 reviews the fundamental concepts and results of continuum mechanics used as a necessary background for the rest of the book. Major points concern the concepts of strain and stress tensors, the formulation of equilibrium equations, using the Newtonian approach and Hamilton's principle, successively. Then, they are particularized to the case of linear elastodynamics, producing the Navier's equation which govern the elastic waves in a solid. The concept of natural modes of vibration in a solid is introduced by solving the Navier's equations in terms of harmonic waves and accounting for the reflection conditions at the solid boundary. Finally, the Saint-Venant's principle is used as a guiding line to model a solid as a structural element.

Chapter 2 presents the basic ideas to model beam-like structures as a 1D solid; the starting point is to assume that the beam cross-sections behave as rigid bodies. Here, modelling is restricted to the case of straight beams and the 1D equilibrium equations, including boundary conditions, are derived by using the Newtonian approach, i.e. by balancing directly the forces and moments acting on a beam element of infinitesimal length. Study is further particularized to the case of linear elastodynamics producing the so called vibration equations. Presented here in their simplest and less refined form, they comprise three uncoupled equations which govern stretching, torsion and bending, respectively. The lateral contraction induced by stretching, due to the Poisson ratio is neglected, which is a realistic assumption in most engineering applications. According to the Bernoulli–Euler model, coupling of bending with transverse shear strains is negligible, which is a reasonable assumption if the beam is slender enough. Concerning torsion, in the case of noncircular cross-sections they are found to warp in such a way that torsion rigidity can be considerably lowered with respect to the value given by a pure torsion model. Warping induced by torsion is classically described based on the Saint-Venant model. The chapter is concluded by presenting a few problems of thermoelasticity and plasticity to illustrate further the modelling process required to approximate a 3D solid as an equivalent 1D solid.

In Chapter 3, the problem of modelling straight beams is revisited and completed by presenting a few distinct topics of theoretical and practical importance. At first, Hamilton's principle is used to improve the basic beam models established in Chapter 2, by accounting for the deformation of the cross-sections and the effect of axial preloads on beam bending. Then, the weighted integral equations of motion are introduced as a starting point to introduce various mathematical concepts and techniques. They are used first together with the singular Dirac distribution, already introduced in Volume 1, to express the equilibrium equations in a unified manner, independently from the continuous or discrete nature of the physical quantities involved in the system. As a second application of the weighted integral equations, the symmetry properties of the stiffness and mass operators are demonstrated, based on the beam operators. Finally, weighted integral equations together with Hamilton's principle give us a good opportunity to present an introductory description of the finite element method.

Chapter 4 is devoted to the modal analysis method, which is a particularly elegant and efficient tool for modelling a large variety of problems in mechanics, independently of their discrete or continuous nature. At first, the natural modes of vibration of straight beams are described. Then they are used as convenient structural examples to present several aspects of modal analysis, focusing on those specific to the case of continuous systems. In particular, the criteria to truncate suitably the modal series are established and illustrated by several examples. Finally, the substructuring method using truncated modal bases for describing each substructure is introduced and illustrated by solving a few linear and nonlinear problems involving intermittent contacts.

Chapters 5 and 6 deal with thin plates described as 2D solids by assuming that strains in the thickness direction can be neglected. Plates are characterized by a plane geometry bounded by edges comprising straight and/or curved lines. Chapter 5 is concerned with the in-plane solicitations and responses, where the part is played by the so called membrane components solely. Chapter 6 is concerned with the out-of-plane, or transverse, solicitations and responses, where the part is played by the flexure and torsion components and the in-plane preloads. Modelling is based on the so called Kirchhoff–Love hypotheses which extend to the 2D case the Bernoulli–Euler model of straight beam bending. Solution of a few problems help to concretize the major features of plate responses to various load conditions. Amongst others, enlightening results concerning the Saint-Venant principle invoked in Chapter 1, are obtained by using the modal analysis method to the response of a rectangular plate to an in-plane point load. On the other hand, the Rayleigh–Ritz discretization method is described and applied to the semi-analytical calculation of the natural modes of vibration of rectangular plates.

Chapters 7 and 8 are devoted to curved structures, namely arches and thin shells. In curved beams and shells, tensile or compressive stresses can resist transverse loads, even in the absence of a prestress field. This can be conveniently emphasized by considering first simplified arch and shell models where bending and torsion terms are entirely discarded, which is the object of Chapter 7. Though the range of validity of the equilibrium equations obtained by using such a simplifying assumption, is clearly limited to certain load conditions, it is believed appropriate to present and discuss them in a rather detailed manner before embarking on the more elaborate models presented in Chapter 8, which account for string or membrane stresses as well as for bending and torsion stresses. Solution of a few problems concerning circular arches or rings and then shells of revolution, brings out that transverse loads cannot be exactly balanced by tensile or compressive stresses in the case of beams but they can in the case of shells. In any case, to deal with general loading conditions, it is necessary to include bending and torsion into the equilibrium equations of arches and shells which is the object of Chapter 8, the last of this volume. As illustrated by the solution of a few problems, the relative importance of the various coupling terms arising in the arch and shell equations, largely depend on the geometry of the structure and on the space distribution of the loads.

The content of the English version of the present volume is basically the same as that of the first edition in French. However, it benefited from various significant improvements and complements, concerning in particular the reflection and the guided propagation of elastic waves and the presentation of the finite element method. Finally, a special word of thanks goes again to Philip Kogan, for checking and rechecking every part of the manuscript. His professional attitude has contributed significantly to the quality of this book. Any remaining errors and inaccuracies are purely the author's own.

François Axisa and Philippe Trompette
November 2004

Contents