PURE AND APPLIED MATHEMATICS
A DEKKER SERIES OF MONOGRAPHS AND TEXTBOOKS

STOCHASTIC VERSUS DETERMINISTIC SYSTEMS OF DIFFERENTIAL EQUATIONS

G. S. LADDE
M. SAMBANDHAM

STOCHASTIC VERSUS DETERMINISTIC SYSTEMS OF DIFFERENTIAL EQUATIONS

G. S. LADDE
The University of Texas at Arlington
Arlington, Texas, U.S.A.

M. SAMBANDHAM
Morehouse College
Atlanta, Georgia, U.S.A.

MARCEL DEKKER

MARCEL DEKKER, INC. NEW YORK • BASEL

Library of Congress Cataloging-in-Publication Data
A catalog record for this book is available from the Library of Congress.

ISBN: 0-8247-4697-X

This book is printed on acid-free paper.

Headquarters
Marcel Dekker, Inc.
270 Madison Avenue, New York, NY 10016, U.S.A.
tel: 212-696-9000; fax: 212-685-4540

Distribution and Customer Service
Marcel Dekker, Inc.
Cimarron Road, Monticello, New York 12701, U.S.A.
tel: 800-228-1160; fax: 845-796-1772

Eastern Hemisphere Distribution
Marcel Dekker AG
Hutgasse 4, Postfach 812, CH-4001 Basel, Switzerland
tel: 41-61-260-6300; fax: 41-61-260-6333

World Wide Web
http://www.dekker.com

The publisher offers discounts on this book when ordered in bulk quantities. For more information, write to Special Sales/Professional Marketing at the headquarters address above.

Current printing (last digit):

10 9 8 7 6 5 4 3 2 1

PRINTED IN THE UNITED STATES OF AMERICA

STOCHASTIC VERSUS DETERMINISTIC SYSTEMS OF DIFFERENTIAL EQUATIONS

PURE AND APPLIED MATHEMATICS

A Program of Monographs, Textbooks, and Lecture Notes

MONOGRAPHS AND TEXTBOOKS IN PURE AND APPLIED MATHEMATICS

1. *K. Yano,* Integral Formulas in Riemannian Geometry (1970)
2. *S. Kobayashi,* Hyperbolic Manifolds and Holomorphic Mappings (1970)
3. *V. S. Vladimirov,* Equations of Mathematical Physics (A. Jeffrey, ed.; A. Littlewood, trans.) (1970)
4. *B. N. Pshenichnyi,* Necessary Conditions for an Extremum (L. Neustadt, translation ed.; K. Makowski, trans.) (1971)
5. *L. Narici et al.,* Functional Analysis and Valuation Theory (1971)
6. *S. S. Passman,* Infinite Group Rings (1971)
7. *L. Dornhoff,* Group Representation Theory. Part A: Ordinary Representation Theory. Part B: Modular Representation Theory (1971, 1972)
8. *W. Boothby and G. L. Weiss, eds.,* Symmetric Spaces (1972)
9. *Y. Matsushima,* Differentiable Manifolds (E. T. Kobayashi, trans.) (1972)
10. *L. E. Ward, Jr.,* Topology (1972)
11. *A. Babakhanian,* Cohomological Methods in Group Theory (1972)
12. *R. Gilmer,* Multiplicative Ideal Theory (1972)
13. *J. Yeh,* Stochastic Processes and the Wiener Integral (1973)
14. *J. Barros-Neto,* Introduction to the Theory of Distributions (1973)
15. *R. Larsen,* Functional Analysis (1973)
16. *K. Yano and S. Ishihara,* Tangent and Cotangent Bundles (1973)
17. *C. Procesi,* Rings with Polynomial Identities (1973)
18. *R. Hermann,* Geometry, Physics, and Systems (1973)
19. *N. R. Wallach,* Harmonic Analysis on Homogeneous Spaces (1973)
20. *J. Dieudonné,* Introduction to the Theory of Formal Groups (1973)
21. *I. Vaisman,* Cohomology and Differential Forms (1973)
22. *B.-Y. Chen,* Geometry of Submanifolds (1973)
23. *M. Marcus,* Finite Dimensional Multilinear Algebra (in two parts) (1973, 1975)
24. *R. Larsen,* Banach Algebras (1973)
25. *R. O. Kujala and A. L. Vitter, eds.,* Value Distribution Theory: Part A; Part B: Deficit and Bezout Estimates by Wilhelm Stoll (1973)
26. *K. B. Stolarsky,* Algebraic Numbers and Diophantine Approximation (1974)
27. *A. R. Magid,* The Separable Galois Theory of Commutative Rings (1974)
28. *B. R. McDonald,* Finite Rings with Identity (1974)
29. *J. Satake,* Linear Algebra (S. Koh et al., trans.) (1975)
30. *J. S. Golan,* Localization of Noncommutative Rings (1975)
31. *G. Klambauer,* Mathematical Analysis (1975)
32. *M. K. Agoston,* Algebraic Topology (1976)
33. *K. R. Goodearl,* Ring Theory (1976)
34. *L. E. Mansfield,* Linear Algebra with Geometric Applications (1976)
35. *N. J. Pullman,* Matrix Theory and Its Applications (1976)
36. *B. R. McDonald,* Geometric Algebra Over Local Rings (1976)
37. *C. W. Groetsch,* Generalized Inverses of Linear Operators (1977)
38. *J. E. Kuczkowski and J. L. Gersting,* Abstract Algebra (1977)
39. *C. O. Christenson and W. L. Voxman,* Aspects of Topology (1977)
40. *M. Nagata,* Field Theory (1977)
41. *R. L. Long,* Algebraic Number Theory (1977)
42. *W. F. Pfeffer,* Integrals and Measures (1977)
43. *R. L. Wheeden and A. Zygmund,* Measure and Integral (1977)
44. *J. H. Curtiss,* Introduction to Functions of a Complex Variable (1978)
45. *K. Hrbacek and T. Jech,* Introduction to Set Theory (1978)
46. *W. S. Massey,* Homology and Cohomology Theory (1978)
47. *M. Marcus,* Introduction to Modern Algebra (1978)
48. *E. C. Young,* Vector and Tensor Analysis (1978)
49. *S. B. Nadler, Jr.,* Hyperspaces of Sets (1978)
50. *S. K. Segal,* Topics in Group Kings (1978)
51. *A. C. M. van Rooij,* Non-Archimedean Functional Analysis (1978)
52. *L. Corwin and R. Szczarba,* Calculus in Vector Spaces (1979)
53. *C. Sadosky,* Interpolation of Operators and Singular Integrals (1979)
54. *J. Cronin,* Differential Equations (1980)
55. *C. W. Groetsch,* Elements of Applicable Functional Analysis (1980)
56. *I. Vaisman,* Foundations of Three-Dimensional Euclidean Geometry (1980)
57. *H. I. Freedan,* Deterministic Mathematical Models in Population Ecology (1980)

58. *S. B. Chae,* Lebesgue Integration (1980)
59. *C. S. Rees et al.,* Theory and Applications of Fourier Analysis (1981)
60. *L. Nachbin,* Introduction to Functional Analysis (R. M. Aron, trans.) (1981)
61. *G. Orzech and M. Orzech,* Plane Algebraic Curves (1981)
62. *R. Johnsonbaugh and W. E. Pfaffenberger,* Foundations of Mathematical Analysis (1981)
63. *W. L. Voxman and R. H. Goetschel,* Advanced Calculus (1981)
64. *L. J. Corwin and R. H. Szczarba,* Multivariable Calculus (1982)
65. *V. I. Istrățescu,* Introduction to Linear Operator Theory (1981)
66. *R. D. Järvinen,* Finite and Infinite Dimensional Linear Spaces (1981)
67. *J. K. Beem and P. E. Ehrlich,* Global Lorentzian Geometry (1981)
68. *D. L. Armacost,* The Structure of Locally Compact Abelian Groups (1981)
69. *J. W. Brewer and M. K. Smith, eds.,* Emmy Noether: A Tribute (1981)
70. *K. H. Kim,* Boolean Matrix Theory and Applications (1982)
71. *T. W. Wieting,* The Mathematical Theory of Chromatic Plane Ornaments (1982)
72. *D. B.Gauld,* Differential Topology (1982)
73. *R. L. Faber,* Foundations of Euclidean and Non-Euclidean Geometry (1983)
74. *M. Carmeli,* Statistical Theory and Random Matrices (1983)
75. *J. H. Carruth et al.,* The Theory of Topological Semigroups (1983)
76. *R. L. Faber,* Differential Geometry and Relativity Theory (1983)
77. *S. Barnett,* Polynomials and Linear Control Systems (1983)
78. *G. Karpilovsky,* Commutative Group Algebras (1983)
79. *F. Van Oystaeyen and A. Verschoren,* Relative Invariants of Rings (1983)
80. *I. Vaisman,* A First Course in Differential Geometry (1984)
81. *G. W. Swan,* Applications of Optimal Control Theory in Biomedicine (1984)
82. *T. Petrie and J. D. Randall,* Transformation Groups on Manifolds (1984)
83. *K. Goebel and S. Reich,* Uniform Convexity, Hyperbolic Geometry, and Nonexpansive Mappings (1984)
84. *T. Albu and C. Năstăsescu,* Relative Finiteness in Module Theory (1984)
85. *K. Hrbacek and T. Jech,* Introduction to Set Theory: Second Edition (1984)
86. *F. Van Oystaeyen and A. Verschoren,* Relative Invariants of Rings (1984)
87. *B. R. McDonald,* Linear Algebra Over Commutative Rings (1984)
88. *M. Namba,* Geometry of Projective Algebraic Curves (1984)
89. *G. F. Webb,* Theory of Nonlinear Age-Dependent Population Dynamics (1985)
90. *M. R. Bremner et al.,* Tables of Dominant Weight Multiplicities for Representations of Simple Lie Algebras (1985)
91. *A. E. Fekete,* Real Linear Algebra (1985)
92. *S. B. Chae,* Holomorphy and Calculus in Normed Spaces (1985)
93. *A. J. Jerri,* Introduction to Integral Equations with Applications (1985)
94. *G. Karpilovsky,* Projective Representations of Finite Groups (1985)
95. *L. Narici and E. Beckenstein,* Topological Vector Spaces (1985)
96. *J. Weeks,* The Shape of Space (1985)
97. *P. R. Gribik and K. O. Kortanek,* Extremal Methods of Operations Research (1985)
98. *J.-A. Chao and W. A. Woyczynski, eds.,* Probability Theory and Harmonic Analysis (1986)
99. *G. D. Crown et al.,* Abstract Algebra (1986)
100. *J. H. Carruth et al.,* The Theory of Topological Semigroups, Volume 2 (1986)
101. *R. S. Doran and V. A. Belfi,* Characterizations of C*-Algebras (1986)
102. *M. W. Jeter,* Mathematical Programming (1986)
103. *M. Altman,* A Unified Theory of Nonlinear Operator and Evolution Equations with Applications (1986)
104. *A. Verschoren,* Relative Invariants of Sheaves (1987)
105. *R. A. Usmani,* Applied Linear Algebra (1987)
106. *P. Blass and J. Lang,* Zariski Surfaces and Differential Equations in Characteristic $p > 0$ (1987)
107. *J. A. Reneke et al.,* Structured Hereditary Systems (1987)
108. *H. Busemann and B. B. Phadke,* Spaces with Distinguished Geodesics (1987)
109. *R. Harte,* Invertibility and Singularity for Bounded Linear Operators (1988)
110. *G. S. Ladde et al.,* Oscillation Theory of Differential Equations with Deviating Arguments (1987)
111. *L. Dudkin et al.,* Iterative Aggregation Theory (1987)
112. *T. Okubo,* Differential Geometry (1987)
113. *D. L. Stancl and M. L. Stancl,* Real Analysis with Point-Set Topology (1987)
114. *T. C. Gard,* Introduction to Stochastic Differential Equations (1988)
115. *S. S. Abhyankar,* Enumerative Combinatorics of Young Tableaux (1988)
116. *H. Strade and R. Farnsteiner,* Modular Lie Algebras and Their Representations (1988)

117. *J. A. Huckaba*, Commutative Rings with Zero Divisors (1988)
118. *W. D. Wallis*, Combinatorial Designs (1988)
119. *W. Więsław*, Topological Fields (1988)
120. *G. Karpilovsky*, Field Theory (1988)
121. *S. Caenepeel and F. Van Oystaeyen*, Brauer Groups and the Cohomology of Graded Rings (1989)
122. *W. Kozlowski*, Modular Function Spaces (1988)
123. *E. Lowen-Colebunders*, Function Classes of Cauchy Continuous Maps (1989)
124. *M. Pavel*, Fundamentals of Pattern Recognition (1989)
125. *V. Lakshmikantham et al.*, Stability Analysis of Nonlinear Systems (1989)
126. *R. Sivaramakrishnan*, The Classical Theory of Arithmetic Functions (1989)
127. *N. A. Watson*, Parabolic Equations on an Infinite Strip (1989)
128. *K. J. Hastings*, Introduction to the Mathematics of Operations Research (1989)
129. *B. Fine*, Algebraic Theory of the Bianchi Groups (1989)
130. *D. N. Dikranjan et al.*, Topological Groups (1989)
131. *J. C. Morgan II*, Point Set Theory (1990)
132. *P. Biler and A. Witkowski*, Problems in Mathematical Analysis (1990)
133. *H. J. Sussmann*, Nonlinear Controllability and Optimal Control (1990)
134. *J.-P. Florens et al.*, Elements of Bayesian Statistics (1990)
135. *N. Shell*, Topological Fields and Near Valuations (1990)
136. *B. F. Doolin and C. F. Martin*, Introduction to Differential Geometry for Engineers (1990)
137. *S. S. Holland, Jr.*, Applied Analysis by the Hilbert Space Method (1990)
138. *J. Okninski*, Semigroup Algebras (1990)
139. *K. Zhu*, Operator Theory in Function Spaces (1990)
140. *G. B. Price*, An Introduction to Multicomplex Spaces and Functions (1991)
141. *R. B. Darst*, Introduction to Linear Programming (1991)
142. *P. L. Sachdev*, Nonlinear Ordinary Differential Equations and Their Applications (1991)
143. *T. Husain*, Orthogonal Schauder Bases (1991)
144. *J. Foran*, Fundamentals of Real Analysis (1991)
145. *W. C. Brown*, Matrices and Vector Spaces (1991)
146. *M. M. Rao and Z. D. Ren*, Theory of Orlicz Spaces (1991)
147. *J. S. Golan and T. Head*, Modules and the Structures of Rings (1991)
148. *C. Small*, Arithmetic of Finite Fields (1991)
149. *K. Yang*, Complex Algebraic Geometry (1991)
150. *D. G. Hoffman et al.*, Coding Theory (1991)
151. *M. O. González*, Classical Complex Analysis (1992)
152. *M. O. González*, Complex Analysis (1992)
153. *L. W. Baggett*, Functional Analysis (1992)
154. *M. Sniedovich*, Dynamic Programming (1992)
155. *R. P. Agarwal*, Difference Equations and Inequalities (1992)
156. *C. Brezinski*, Biorthogonality and Its Applications to Numerical Analysis (1992)
157. *C. Swartz*, An Introduction to Functional Analysis (1992)
158. *S. B. Nadler, Jr.*, Continuum Theory (1992)
159. *M. A. Al-Gwaiz*, Theory of Distributions (1992)
160. *E. Perry*, Geometry: Axiomatic Developments with Problem Solving (1992)
161. *E. Castillo and M. R. Ruiz-Cobo*, Functional Equations and Modelling in Science and Engineering (1992)
162. *A. J. Jerri*, Integral and Discrete Transforms with Applications and Error Analysis (1992)
163. *A. Charlier et al.*, Tensors and the Clifford Algebra (1992)
164. *P. Biler and T. Nadzieja*, Problems and Examples in Differential Equations (1992)
165. *E. Hansen*, Global Optimization Using Interval Analysis (1992)
166. *S. Guerre-Delabrière*, Classical Sequences in Banach Spaces (1992)
167. *Y. C. Wong*, Introductory Theory of Topological Vector Spaces (1992)
168. *S. H. Kulkarni and B. V. Limaye*, Real Function Algebras (1992)
169. *W. C. Brown*, Matrices Over Commutative Rings (1993)
170. *J. Loustau and M. Dillon*, Linear Geometry with Computer Graphics (1993)
171. *W. V. Petryshyn*, Approximation-Solvability of Nonlinear Functional and Differential Equations (1993)
172. *E. C. Young*, Vector and Tensor Analysis: Second Edition (1993)
173. *T. A. Bick*, Elementary Boundary Value Problems (1993)
174. *M. Pavel*, Fundamentals of Pattern Recognition: Second Edition (1993)
175. *S. A. Albeverio et al.*, Noncommutative Distributions (1993)
176. *W. Fulks*, Complex Variables (1993)
177. *M. M. Rao*, Conditional Measures and Applications (1993)

178. *A. Janicki and A. Weron*, Simulation and Chaotic Behavior of α-Stable Stochastic Processes (1994)
179. *P. Neittaanmäki and D. Tiba*, Optimal Control of Nonlinear Parabolic Systems (1994)
180. *J. Cronin*, Differential Equations: Introduction and Qualitative Theory, Second Edition (1994)
181. *S. Heikkilä and V. Lakshmikantham*, Monotone Iterative Techniques for Discontinuous Nonlinear Differential Equations (1994)
182. *X. Mao*, Exponential Stability of Stochastic Differential Equations (1994)
183. *B. S. Thomson*, Symmetric Properties of Real Functions (1994)
184. *J. E. Rubio*, Optimization and Nonstandard Analysis (1994)
185. *J. L. Bueso et al.*, Compatibility, Stability, and Sheaves (1995)
186. *A. N. Michel and K. Wang*, Qualitative Theory of Dynamical Systems (1995)
187. *M. R. Darnel*, Theory of Lattice-Ordered Groups (1995)
188. *Z. Naniewicz and P. D. Panagiotopoulos*, Mathematical Theory of Hemivariational Inequalities and Applications (1995)
189. *L. J. Corwin and R. H. Szczarba*, Calculus in Vector Spaces: Second Edition (1995)
190. *L. H. Erbe et al.*, Oscillation Theory for Functional Differential Equations (1995)
191. *S. Agaian et al.*, Binary Polynomial Transforms and Nonlinear Digital Filters (1995)
192. *M. I. Gil'*, Norm Estimations for Operation-Valued Functions and Applications (1995)
193. *P. A. Grillet*, Semigroups: An Introduction to the Structure Theory (1995)
194. *S. Kichenassamy*, Nonlinear Wave Equations (1996)
195. *V. F. Krotov*, Global Methods in Optimal Control Theory (1996)
196. *K. I. Beidar et al.*, Rings with Generalized Identities (1996)
197. *V. I. Arnautov et al.*, Introduction to the Theory of Topological Rings and Modules (1996)
198. *G. Sierksma*, Linear and Integer Programming (1996)
199. *R. Lasser*, Introduction to Fourier Series (1996)
200. *V. Sima*, Algorithms for Linear-Quadratic Optimization (1996)
201. *D. Redmond*, Number Theory (1996)
202. *J. K. Beem et al.*, Global Lorentzian Geometry: Second Edition (1996)
203. *M. Fontana et al.*, Prüfer Domains (1997)
204. *H. Tanabe*, Functional Analytic Methods for Partial Differential Equations (1997)
205. *C. Q. Zhang*, Integer Flows and Cycle Covers of Graphs (1997)
206. *E. Spiegel and C. J. O'Donnell*, Incidence Algebras (1997)
207. *B. Jakubczyk and W. Respondek*, Geometry of Feedback and Optimal Control (1998)
208. *T. W. Haynes et al.*, Fundamentals of Domination in Graphs (1998)
209. *T. W. Haynes et al., eds.*, Domination in Graphs: Advanced Topics (1998)
210. *L. A. D'Alotto et al.*, A Unified Signal Algebra Approach to Two-Dimensional Parallel Digital Signal Processing (1998)
211. *F. Halter-Koch*, Ideal Systems (1998)
212. *N. K. Govil et al., eds.*, Approximation Theory (1998)
213. *R. Cross*, Multivalued Linear Operators (1998)
214. *A. A. Martynyuk*, Stability by Liapunov's Matrix Function Method with Applications (1998)
215. *A. Favini and A. Yagi*, Degenerate Differential Equations in Banach Spaces (1999)
216. *A. Illanes and S. Nadler, Jr.*, Hyperspaces: Fundamentals and Recent Advances (1999)
217. *G. Kato and D. Struppa*, Fundamentals of Algebraic Microlocal Analysis (1999)
218. *G. X.-Z. Yuan*, KKM Theory and Applications in Nonlinear Analysis (1999)
219. *D. Motreanu and N. H. Pavel*, Tangency, Flow Invariance for Differential Equations, and Optimization Problems (1999)
220. *K. Hrbacek and T. Jech*, Introduction to Set Theory, Third Edition (1999)
221. *G. E. Kolosov*, Optimal Design of Control Systems (1999)
222. *N. L. Johnson*, Subplane Covered Nets (2000)
223. *B. Fine and G. Rosenberger*, Algebraic Generalizations of Discrete Groups (1999)
224. *M. Väth*, Volterra and Integral Equations of Vector Functions (2000)
225. *S. S. Miller and P. T. Mocanu*, Differential Subordinations (2000)
226. *R. Li et al.*, Generalized Difference Methods for Differential Equations: Numerical Analysis of Finite Volume Methods (2000)
227. *H. Li and F. Van Oystaeyen*, A Primer of Algebraic Geometry (2000)
228. *R. P. Agarwal*, Difference Equations and Inequalities: Theory, Methods, and Applications, Second Edition (2000)
229. *A. B. Kharazishvili*, Strange Functions in Real Analysis (2000)
230. *J. M. Appell et al.*, Partial Integral Operators and Integro-Differential Equations (2000)
231. *A. I. Prilepko et al.*, Methods for Solving Inverse Problems in Mathematical Physics (2000)

232. *F. Van Oystaeyen,* Algebraic Geometry for Associative Algebras (2000)
233. *D. L. Jagerman,* Difference Equations with Applications to Queues (2000)
234. *D. R. Hankerson et al.,* Coding Theory and Cryptography: The Essentials, Second Edition, Revised and Expanded (2000)
235. *S. Dăscălescu et al.*, Hopf Algebras: An Introduction (2001)
236. *R. Hagen et al.*, C*-Algebras and Numerical Analysis (2001)
237. *Y. Talpaert*, Differential Geometry: With Applications to Mechanics and Physics (2001)
238. *R. H. Villarreal*, Monomial Algebras (2001)
239. *A. N. Michel et al.,* Qualitative Theory of Dynamical Systems: Second Edition (2001)
240. *A. A. Samarskii,* The Theory of Difference Schemes (2001)
241. *J. Knopfmacher and W.-B. Zhang,* Number Theory Arising from Finite Fields (2001)
242. *S. Leader,* The Kurzweil-Henstock Integral and Its Differentials (2001)
243. *M. Biliotti et al.,* Foundations of Translation Planes (2001)
244. *A. N. Kochubei,* Pseudo-Differential Equations and Stochastics over Non-Archimedean Fields (2001)
245. *G. Sierksma,* Linear and Integer Programming: Second Edition (2002)
246. *A. A. Martynyuk,* Qualitative Methods in Nonlinear Dynamics: Novel Approaches to Liapunov's Matrix Functions (2002)
247. *B. G. Pachpatte,* Inequalities for Finite Difference Equations (2002)
248. *A. N. Michel and D. Liu,* Qualitative Analysis and Synthesis of Recurrent Neural Networks (2002)
249. *J. R. Weeks,* The Shape of Space: Second Edition (2002)
250. *M. M. Rao and Z. D. Ren,* Applications of Orlicz Spaces (2002)
251. *V. Lakshmikantham and D. Trigiante,* Theory of Difference Equations: Numerical Methods and Applications, Second Edition (2002)
252. *T. Albu,* Cogalois Theory (2003)
253. *A. Bezdek,* Discrete Geometry (2003)
254. *M. J. Corless and A. E. Frazho,* Linear Systems and Control: An Operator Perspective (2003)
255. *I. Graham and G. Kohr,* Geometric Function Theory in One and Higher Dimensions (2003)
256. *G. V. Demidenko and S. V. Uspenskii,* Partial Differential Equations and Systems Not Solvable with Respect to the Highest-Order Derivative (2003)
257. *A. Kelarev,* Graph Algebras and Automata (2003)
258. *A. H. Siddiqi,* Applied Functional Analysis (2004)
259. *F. W. Steutel and K. van Harn,* Infinite Divisibility of Probability Distributions on the Real Line (2004)
260. *G. S. Ladde and M. Sambandham,* Stochastic Versus Deterministic Systems of Differential Equations (2004)
261. *B. J. Gardner and R. Wiegandt,* Radical Theory of Rings (2004)
262. *J. Haluška,* The Mathematical Theory of Tone Systems (2004)

Additional Volumes in Preparation

E. Hansen and G. W. Walster, Global Optimization Using Interval Analysis: Second Edition, Revised and Expanded (2004)

PREFACE

The classical random flow and Newtonian mechanics approaches are the most extensively studied stochastic modeling methods for dynamic processes in biological, engineering, physical and social sciences. Both of these approaches lead to differential equations.

In the classical stochastic modeling approach, the state of a dynamic process is considered to be a random flow or process satisfying a certain probabilistic law such as Markov or diffusion. From these types of probabilistic assumptions, one then needs to determine the state transition probability distribution and density functions (STPDF). The determination of the unknown STPDF leads to the study of deterministic problems in the theory of ordinary, partial or integro-differential equations. These types of equations are referred to as master equations in the literature. The solution processes of such systems of differential equations are used to find the higher moments and other statistical properties of dynamic processes described by random flows.

On the other hand, the classical Newtonian mechanics type of stochastic modeling approach deals with a stochastic calculus to formulate stochastic mathematical models of dynamic processes. This approach leads directly to a system of stochastic differential equations, and its solution processes provide the description of the states of the dynamic processes as stochastic or random processes. This method of stochastic modeling generates three basic problems:

(i) Concepts of solution processes depending on modes of convergence and the fundamental properties of solutions: existence, uniqueness, measurability, continuous dependence on system parameters.

(ii) Probabilistic and statistical properties of solution process: probability distribution and density function, variance, and moments of solution processes and the qualitative/quantitative behavior of solutions.

(iii) Deterministic versus stochastic modeling of dynamic processes: If the deterministic mathematical model is available, then why do we need a stochastic mathematical model? If a stochastic mathematical model provides a better description of a dynamic process than the deterministic model, then the second question is to what extent the stochastic mathematical model differs from the corresponding deterministic model in the absence of random disturbances or fluctuations and uncertainties.

Most of the work on the theory of systems of stochastic differential equations is centered around problems (i) and (ii). This is because the theory of deterministic systems of differential equations provides many mathematical tools and ideas. It is problem (iii) that deserves more attention. Since 1970, some serious efforts have been made to address this issue in the context of stochastic modeling of dynamic processes by means of systems of stochastic differential equations. In the light of this interest, now is an appropriate time to present an account of stochastic versus deterministic issues in a systematic and unified way.

Two of the most powerful methods for studying systems of nonlinear differential equations are nonlinear variation of parameters and Lyapunov's second method. About a quarter century ago a hybrid of these two methods evolved. This hybrid method is called variational comparison method. In addition, a generalized variation of constants method has also developed in the same period of time. These new techniques are very suitable and effective tools to investigate problems concerning stochastic systems of differential equations, in particular, stochastic versus deterministic issues.

This book offers a systematic and unified treatment for systems of stochastic differential equations in the framework of three methods: a) variational comparison method, b) generalized variation of constants method, and c) probability distribution method. The book is divided into five chapters. The first chapter deals with random algebraic polynomials. Chapter 2 is devoted to the initial value problem (IVP) for ordinary differential systems with random parameters. Stochastic boundary value problems (SBVP) with random parameters are treated in Chapter 3. Chapters 4 and 5 cover IVP and SBVP for systems of stochastic differential equations of Itô type, respectively.

A few important features of the monograph are as follows:

(i) This is the first book that offers a systematic study of the well-known problem of stochastic mathematical modeling in the context of systems of stochastic differential equations, namely, "stochastic versus deterministic;"

(ii) It complements the existing books in stochastic differential equations;

(iii) It provides a unified treatment of stability, relative stability and error estimate analysis;

(iv) It exhibits the role of randomness as well as rate functions in explicit form;

(v) It provides several illustrative analytic examples to demonstrate the scope of methods in stochastic analysis;

(vi) The methods developed in the book are applied to the existing stochastic mathematical models described by stochastic differential equations in population dynamics, hydrodynamics, and physics;

(vii) Last but not least, it provides several numerical examples and figures to illustrate and compare the analytic techniques that are outlined in the book.

The monograph can be used as a textbook for graduate students. It can also be used as a reference book for both experimental and applied scientists working in the mathematical modeling of dynamic processes.

G. S. Ladde
M. Sambandham

NOTATION AND ABBREVIATIONS

For the convenience of readers we list below the various notations and abbreviations employed in the monograph.

Vectors (column vectors) of dimension n are basically treated as $n \times 1$ matrices. All relations such as equations, inequalities, belonging to, and limits involving random variables or functions are valid with probability one. Sometimes the symbols $x(t)$ and $x(t, \omega)$ are used interchangeably as a random function.

R^n	As n-dimensional Euclidean space with a convenient norm $\|\| \bullet \|\|$
$\|\| \bullet \|\|$	The norm of a vector or matrix
R	The set of all deterministic real numbers or a real line
R_+	The set of all $t \in R$ such that $t \geq 0$
I	An arbitrary index set, in particular, a finite, countable set, or any interval in R
$I(1,n)$	$\{1, 2, \ldots, n\}$, that is, the set of first n positive integers
J	$[t_0, t_0 + a]$, where $t_0 \in R$ and a is a positive real number
$B(z, \rho)$	The set of all $x \in R^n$ such that $\|\|x - z\|\| < \rho$ for given $z \in R^n$ and positive real number ρ
$\overline{B}(z, \rho)$	The closure of $B(z, \rho)$
$B(\rho)$	The set $B(z, \rho)$ with $z = 0 \in R^n$
$\mathcal{F}^n$	The σ-algebra of Borel sets in R^n
$\mathcal{B}$	The σ-algebra of Borel sets in a metric space (X, d), where d is a metric induced by the norm $\|\| \bullet \|\|$ and X is a separable Banach space

$(\Omega, \mathcal{F}, P) \equiv \Omega$ — A complete probability space, where Ω is a sample space, $\mathcal{F}$ is a σ-algebra of Ω, and P is a probability measure defined on $\mathcal{F}$

$\mathcal{F}_t$ — A sub-σ-algebra of $\mathcal{F}$ for $t \in R_+$

$\mathcal{L}_t$ — The smallest sub-σ-algebra of $\mathcal{F}$ generated by a k-dimensional normalized Wiener process $z(t)$ for $t \in R_+$

$R[\Omega, R^n]$ — The collection of all random vectors defined on complete probability space $(\Omega, \mathcal{F}, P)$ into R^n

$R[\Omega, R^{nm}] \equiv [\Omega, \mathcal{M}_{n\times m}]$ A collection of all $n \times m$ random matrices $A(\omega) = (a_{ij}(\omega))$ such that $a_{ij} \in R[\Omega, R]$

$\|x\|_p$ — $\|x\|_p = \left(E[\|x(\omega)\|^p]\right)^{1/p} = \left(\int_\Omega \|x(\omega)\|^p P(d\omega)\right)^{1/p}$ for $p \geq 1$

$\mathcal{L}^p$ — The collection of all n-dimensional random vectors x such that $E[\|x(\omega)\|^p] < \infty$ for $p \geq 1$

$L^p[\Omega, R^n]$ — A collection of all equivalence classes of n-dimensional random vectors such that an element of an equivalence class belongs to $\mathcal{L}^p$

$R[[a,b], R[\Omega, R^n]] \equiv R[[a,b] \times \Omega, R^n]$ A collection of all R^n-valued separable random functions defined on $[a,b]$ with a state space $(R^n, \mathcal{F}^n)$, $a, b \in R$

$M[[a,b], R[\Omega, R^n]] \equiv M[[a,b] \times \Omega, R^n]$ A collection of all random functions in $R[[a,b], R[\Omega, R^n]]$ which are product-measurable on $([a,b] \times \Omega, \mathcal{F}^1 \times \mathcal{F}, m \times P)$, where $(\Omega, \mathcal{F}, P) \equiv \Omega$ and $([a,b], \mathcal{F}^1, m)$ are complete probability and Lebesgue-measurable spaces, respectively

$M[R_+ \times R^n, R[\Omega, R^n]] \equiv M[R_+ \times R^n \times \Omega, R^n]$ A class of R^n-valued random functions $F(t, x, \omega)$ such that $F(t, x(t, \omega), \omega)$ is product measurable whenever $x(t, \omega)$ product is measurable

$M[[0,1] \times R^n \times R^n, R[\Omega, R^n]] \equiv M[[0,1] \times R^n \times R^n \times \Omega, R^n]$ A class of R^n-valued random functions $F(t, x, y, \omega)$ such that $F(t, x(t, \omega), y(t, \omega), \omega)$ is product measurable whenever $x(t, \omega)$ and $y(t, \omega)$ are product measurables

$C[D, R^n]$ The class of all deterministic continuous functions defined on an open (t, x) subset D of R^{n+1} into R^n

$C[R_+ \times R^n, R^m]$ The class of all deterministic continuous functions defined on $R_+ \times R^n$ into R^m

$C[[0,1] \times R^n \times R^n, R^m]$ The collection of all deterministic continuous functions $[0,1] \times R^n \times R^n$ into R^m

$C[[a,b], R[\Omega, R^n]] \equiv C[R_+ \times R^n, R[\Omega, R^n]]$ A collection of all sample continuous R^n-valued random functions $x(t, \omega)$

$C[R_+ \times R^n, R[\Omega, R^n]] \equiv C[R_+ \times R^n \times \Omega, R^n]$ A class of sample continuous R^n-valued random functions $F(t, x, \omega)$ defined on $R_+ \times R^n \times \Omega$ into R^n

$C[[0,1] \times R^n \times R^n, R[\Omega, R^n]] \equiv C[[0,1] \times R^n \times R^n \times \Omega, \mathcal{R}^n]$ A class of sample continuous R^n-valued random functions $F(t, x, y, \omega)$ defined on $[0,1] \times R^n \times R^n \times \Omega$ into R^n

A^T The transpose of a vector or matrix A

A^{-1} The inverse of a square matrix A

$tr(A)$ The trace of a square matrix A

$\det(A)$ The determinant of a square matrix A