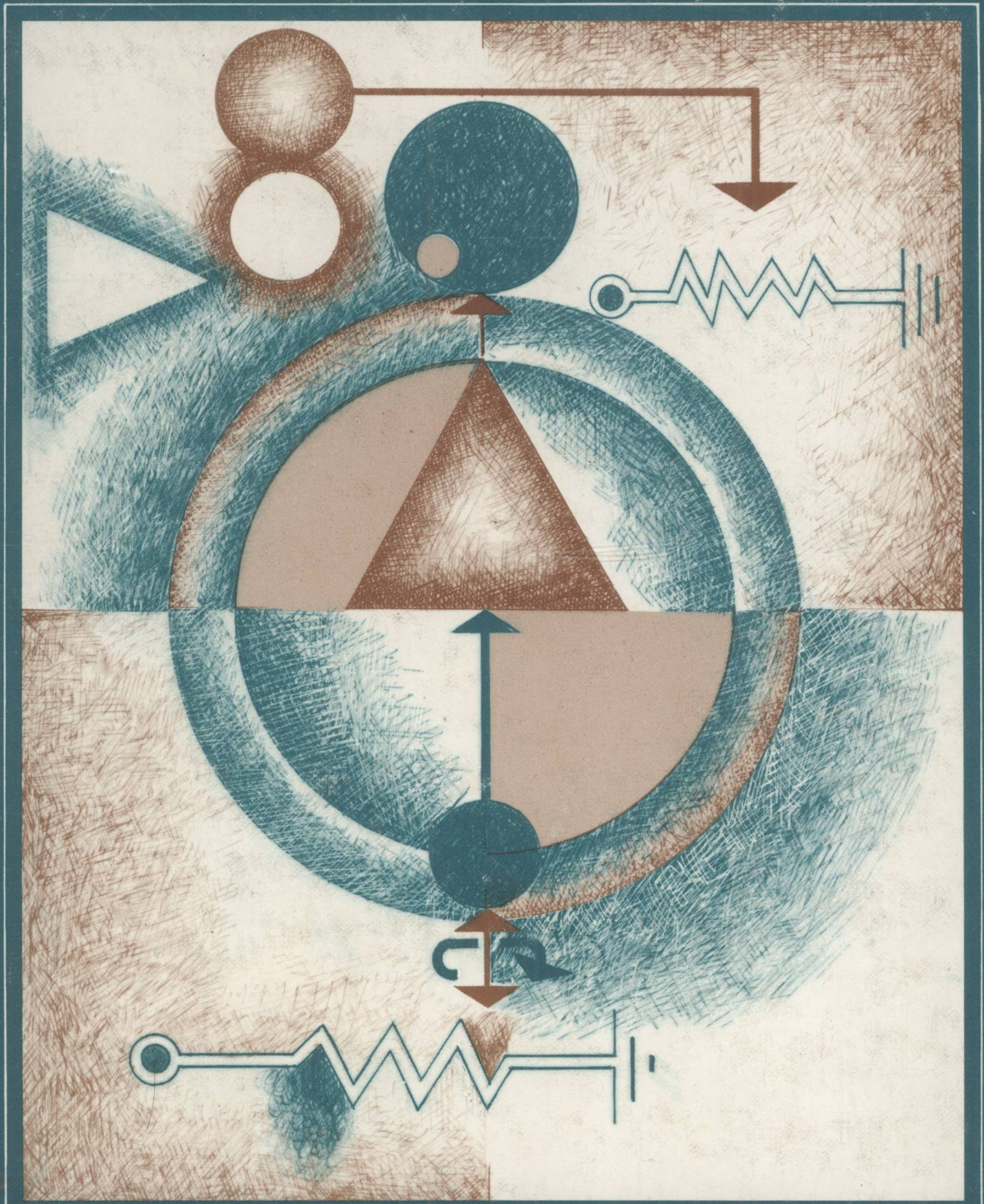
Theodore F. Bogart, Jr.

LAPLACE TRANSFORMS AND CONTROL SYSTEMS THEORY FOR TECHNOLOGY

Including Microprocessor-Based Control Systems



LAPLACE TRANSFORMS AND CONTROL SYSTEMS THEORY FOR TECHNOLOGY

INCLUDING MICROPROCESSOR-BASED CONTROL SYSTEMS

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TO MY PARENTS
ON THEIR GOLDEN WEDDING ANNIVERSARY

PREFACE

LaPlace Transforms and Control Systems Theory for Technology is intended for one- or two-semester courses in network analysis and control-systems theory. The material covered in Chapters 1 through 5 and the Appendix form the basis for a one-semester course in LaPlace transformation and its application to network analysis. The remainder of the book can be used for a course that covers the basics of continuous control-systems theory, analog- and digital-analysis techniques, and microprocessor-based control systems. Also, an accelerated single-semester course, which would touch on a majority of topics, could be designed around the material in Chapters 3, 4, 5, and 6 (through Section 6.9), 7 (through Section 7.5), and 8 (through Section 8.5).

The material covered and its level of treatment are appropriate for any two- or four-year program that includes technical calculus. Students beginning the two-course sequence at the University of Southern Mississippi have completed two semesters of calculus, although conscientious students with only one semester of calculus, including an introduction to integration, have successfully completed the sequence. Students should also have completed the traditional courses in dc- and ac-circuit analysis and electronic devices and circuits. For the material in Chapter 10, students should have completed at least one course in digital electronics, but no prior knowledge of microprocessors is assumed.

The intent of this book is to expose students to as broad a range of topics as possible, from traditional continuous-systems analysis, to the most recent developments in digital and microprocessor control techniques. The breadth of coverage precludes an exhaustive treatment of any one topic, though it is hoped that the depth is sufficient to stimulate the interest of readers and to make them feel comfortable with new concepts in *every* area covered. In any event, the level of mathematical abstraction that would be required to treat many topics in greater detail is not considered appropriate for the technology student. Thus, for example, the mathematical procedure for finding an inverse LaPlace transform and the rigorous justification of the Nyquist stability criterion, as well as its application to generalized systems, are omitted, as they might not be in a traditional engineering curriculum. A knowledge of calculus of the complex plane is not an expected background for this course, nor is it discussed in the text.

The profusion of topics that today's electronics technologist should know something about (even if only the jargon) is so great that a single course in, say, LaPlace transforms, or continous-type control-systems theory, may not be justified in a technology program simply because such specialty courses must necessarily displace instruction in other equally vital areas of the curriculum. Nevertheless, the application of LaPlace transforms to both circuit and systems analysis receives thorough coverage in this text. Topics that are traditionally relevant to this area are also covered: control-systems stability analysis, compensation, operational amplifiers, and analog computers, though not in the same depth. A substantial part of the text is devoted to digital techniques, in recognition of the incursion that the digital computer has made in recent years both as a tool for systems analysis and as an integral part of control systems. Therefore, one entire chapter deals with digital-computer analysis of continuous systems, and another chapter introduces the microprocessor-based control system. Appendix A also deals with the application of LaPlace transforms to the analysis of circuits driven by pulse-type waveforms.

Wherever possible and appropriate, the theory is illustrated by examples from mechanical as well as electrical systems. The analogy between mechanical and electrical networks, and the equations that describe their behavior, is stressed throughout. Portions of this book are appropriate for use in some mechanical or manufacturing curriculums as well as electronics and electrical-technology programs, particularly in view of the increasingly important role of the microprocessor in automation.

A large number of worked-out examples and student exercises reinforce the theory; this is especially true in those chapters that introduce new ideas or make greater demands on the students' ability to reason quantitatively with abstract concepts. Answers to exercises are provided, except to those for which a large number of correct solutions is possible or whose solutions are quite long or require extensive diagrams.

Many illustrative examples are based on manufacturers' product literature and specifications. Specification sheets and illustrations of commercially available servo-system components, both analog and digital, are reproduced in Appendix B. Data taken from this product literature are used in examples to give students a feeling for the orders of magnitude of variables used in the theory and to motivate and reinforce learning by confirming the existence of real hardware that behaves according to theory.

The SI system of units is used throughout the book, except in illustrations where actual product specifications are used. Lamentably, most American manufacturers' specifications are still given in English or CGS units; but in the majority of the examples that use these units, the conversions to SI are shown and used.

I wish to thank Professor C. Howard Heiden, Chairman of the Department of Industrial Technology at the University of Southern Mississippi, for the encouragement and departmental support given me during the many months required to prepare this book. Thanks also to Professor Jack Lipscomb, University of Southern Mississippi, for reviewing portions of the manuscript and for his assistance in the section dealing with CSMP. I am grateful to those instructors from other institutions, who reviewed the manuscript for John Wiley & Sons, and made many constructive suggestions. A large measure of thanks is due my students, who suffered through the first drafts of the book in class, and who were very diligent in seeking out errors.

T. F. Bogart, Jr.

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CHAPTER 1

INTRODUCTION

1.1 WHAT IS A LAPLACE TRANSFORM?

Since a major portion of this text deals with the rather formidable sounding term *LaPlace transform*, we wish to reassure the reader at the outset that it is a concept designed to *simplify* rather than complicate the analysis of circuits and systems. Although differential and integral calculus are the mathematical prerequisites for the material in this book, the student who has weathered these subjects will be pleased to learn that LaPlace transforms effectively convert many calculus problems into ordinary algebra problems. Indeed, a good command of algebra, particularly the ability to handle algebraic fractions, may be a more important skill in using LaPlace transforms than is a thorough knowledge of calculus.

Calculus, as we all know, is the mathematical tool that we must have in order to analyze, predict, and understand *dynamic* systems: systems whose variables change with time. A derivative, after all, represents a *rate of change* and is, therefore, the fundamental means we have for describing the behavior of time-varying quantities. For example, the familiar relation between the charge and voltage on a capacitor $Q = CV$ is useful when the charge and voltage are constant with respect to time, that is, in the dc case, but it is more useful when we differentiate both sides of the equation to obtain

$$\frac{dQ}{dt} = C\,\frac{dv}{dt}$$

or equivalently,

$$i = C\,\frac{dv}{dt}$$

This allows us to relate the current through and voltage across a capacitor when these two quantities are changing with time. When we attempt to relate voltage and charge, or current, in a circuit containing capacitors, inductors, and resistors, we find that we suddenly have a number of derivatives to deal with and that we, in fact, have a *differential equation* to solve. We intend to cover the techniques for setting up the differential equations of electrical as well as mechanical networks in a later chapter, but suffice it for now to say that LaPlace transformations of these equations will allow us to solve them by simply solving ordinary algebra equations.

A *transformation* is a new name for an already familiar procedure. In simplest terms, we say that we transform a function whenever we change or modify what that function does. We usually think of a function as a rule we follow to change the value of a number; similarly, a transformation is a rule we follow to change what a function does. For example, the transformation "multiply by 2" would transform the function $y = x^2$ into $y = 2x^2$, the function $y = \sin x$ into $y = 2 \sin x$, and so forth. Thus, a transformation operates on functions to change *them*, just as a function operates on numbers to produce new numbers. For this reason, transformations are sometimes called *operators*. When the operator "multiply by 2" is applied to the function $y = x^2$, we obtain the transform $y = 2x^2$.

The LaPlace transformation is an *integral operator*, so called because it changes a given function into a new function by the process of integration. In practice, we rarely need to perform that integration when we wish to obtain the LaPlace transform of a function. We rely instead on *tables* of LaPlace transforms and upon our ability to manipulate algebraic expressions so that they fit the format of the tables. The major advantage of the LaPlace transformation is that it transforms the mathematical operations of differentiation and integration into multiplication and division. Thus, a differential equation that has been transformed in this manner has no derivatives present, and its solution may therefore be obtained using purely algebraic methods (and a table of transforms). Those who have struggled with the classical methods for solving differential equations will appreciate what this means and will reap considerable dividends from investing a little more time and effort to learn a little more mathematics.

So rather than feeling that vaguely threatening and familiar apprehension often experienced on a first encounter with a new mathematical concept, the student will, we trust, welcome the transform method and embrace it with confidence as simply a useful tool for simplifying complicated problems.

1.2 WHAT IS A CONTROL SYSTEM?

The subject we call control systems pervades all of technology. In fact, we may go so far as to say that at least one aspect or another of control-systems theory is appropriate to every human activity. In its most general sense, a control system is simply an aggregate of components that, working together, generate a response to some stimulus.

The human body, which responds to internal and extenal stimuli, is certainly a control system. Internal stimuli (thoughts, ideas, physiological changes) stimulate human responses, as do such external stimuli as temperature changes, threatening events, social interactions, and a host of others. The very act of reaching out to pick up a pencil is an example of a human control system in operation. It is an example of a closed-loop control system where a visual stimulus (the relative position of the hand with respect to the pencil as perceived by the eye) is continually fed back to the brain and where the response, that is, the movement of the hand, is continually adjusted until the hand successfully reaches the pencil.

We are surrounded by man-made control systems, both open-loop and closed-loop. All manner of electrical appliances and mechanical devices perform useful functions, that is, *respond* to all manner of stimuli. In control-systems parlance, we refer to a stimulus as an input, while the response is called an output. Thus, turning a knob on an oven represents an input, and the oven responds by producing an output that causes an increase or decrease in temperature. More sophisticated examples include aircraft autopilots, guided missiles, automobile-speed controllers, nuclear-reactor safety systems, scanning radio receivers, dc power supplies, oil refineries and chemical-processing plants, robots, and many, many others.

Perhaps the very broadness of scope has led us to characterize the subject as a "system"—a word nearly as vague and as often misused as "thing." However, in the evolution of this subject as a specialty area of mechanical and electrical engineering and technology, control-systems theory has come to mean the study of certain resonably well-defined techniques for predicting, analyzing, and improving the behavior of devices (traditionally electromechanical devices) that perform some useful function in response to electrical or mechanical inputs. These techniques have been successfully applied in other disciplines including the human control system. Control-systems theory is sufficiently general, so that its concepts and results, though frequently expressed in electrical or mechanical terms, have their counterparts in a wide variety of scientific studies.

1.3 ANALOG VERSUS DIGITAL SYSTEMS

An analog system is one whose variables, for example, input and output voltages, can assume *any* values within their permissible range. Such variables are said to be *continuous*. For example, a sinusoidal voltage with 10-volt (V) peak value is a continuous or analog variable, since it can have *any* value between -10 V and $+10$ V. A motor-driven shaft is an analog device if we regard its output as some angular rotation, since the shaft can assume any angular position we wish.

A digital system, on the other hand, is one whose variables can assume only certain *discrete* values. Typically, the magnitudes of the variables are represented by binary numbers, which, in turn, are represented by a sequence of "on" or "off" levels; these levels correspond to distinct voltages. For example, a system may produce only the four binary outputs 00, 01, 10, and 11, where a 0 is represented by, say, -5 V and a 1 is represented by $+5$ V. Processing these signals is done by digital-logic circuitry or by a digital computer. A stepping motor whose shaft can only be rotated in fixed increments of, say, ten degrees is an example of a digital device.

A control system that employs analog devices and processes analog signals is an analog, or continuous, control system, while one that utilizes digital devices and discrete-voltage levels is a digital-control system. Frequently, a control system uses both types of devices and performs both analog- and digital-signal processing; such a system is said to be *hybrid*. For example, while the input signal to a system may be analog in nature, it is often converted to digital form by an analog-to-digital (A/D) converter. The digital signals are processed as necessary, and the result is then converted to an analog output by a digital-to-analog (D/A) converter.

Because the development of analog systems preceded the widespread use of digital systems, much of the classical theory of control systems is based on mathematical relationships between continuous variables. Control-systems theory is, for example, very much concerned with the solution of differential equations. These equations represent continuous, analog-type relations between variables and their rates of change, about which we will have more to say in subsequent chapters.

In recent years, the trend has been to replace analog-signal processing with its digital counterpart. Digital-signal processing has some significant advantages, most notably a greater immunity to error caused by noise and circuit simplicity. Furthermore, the advent of the microprocessor, a compact, inexpensive means of handling large quantities of digital data rapidly and efficiently, has resulted in its appearance in many control systems that have been traditionally analog in nature. Automobile ignition systems, microwave ovens, automated production lines, and navigation systems are but a few examples.

It is nonetheless true that our world is essentially analog in nature, since we deal with continuous variables (our position in a coordinate system, a shaft rotation, light intensity, and the like). The mathematical relations between these continuous variables and their rates of change are still of interest to us; analyzing analog systems in terms of continuous variables is, therefore, still an appropriate endeavor. In a sense, we can think of a digital system as one that operates by approximating the actual analog variables that we experience and observe. Thus, analog solutions represent the true state of affairs with which we must certainly be acquainted in order to implement reasonable and efficient approximations. This is not to imply that digital systems and digital-control signals are any less valid, useful, or efficient than their analog counterparts. Indeed, as we have already suggested, digital-signal processing enjoys significant practical advantages. Accuracy is limited only by the resolution (number of binary digits) that the user is willing to employ. A case in point: A differential equation may be solved using an analog computer, which yields a continuously time-varying function, or by a digital computer, which yields a finite set of values of the function at specific instants of time. The accuracy of the digital solution may be as good or better than a practical analog solution, since the user may choose as many instants of time as he wishes and use numbers with as many significant digits as he wishes.

In view of these considerations, this text treats the subject of control-systems theory in the classic manner, that is, by analog methods, after developing the mathematical tools necessary for such a treatment. Then attention focuses on