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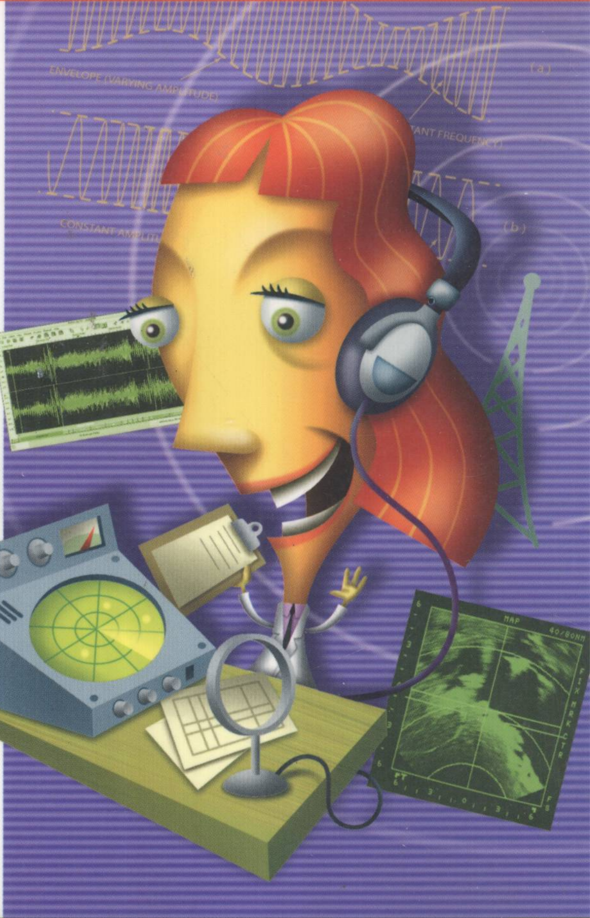

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Signals and Systems Demystified

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PREFACE

Signals and Systems is a core subject in electrical engineering, and unfortunately it's one of the most difficult. Laden with heavy mathematics, many, if not all, students find courses in the areas of signal processing and systems to be very difficult. This book is aimed primarily at those students. It can serve as a supplemental text for students studying signals, systems, and communications courses in electrical engineering. The topics covered in this book are suitable for both undergraduate and graduate students.

This book is also very useful for electrical engineers who have been out of school for a long time and would like a refresher. We assume the reader has had calculus and some exposure to differential equations; however, this book is well suited for self-study. If you are an intelligent person simply looking to learn electrical engineering on your own, this is the book for you. Once you go through it you will be well prepared to tackle full-blown textbooks written on this topic.

Our approach in this text is to briefly describe concepts, theorems, and formulas and to focus on problem-solving. We explicitly demonstrate the *how-to* aspect of problem-solving. As a result each chapter is built around a core of explicitly solved problems. We try to demonstrate as many steps as possible so that the student does not have to guess how to get from Point A to Point B in a problem solution. Theorems and formulas are stated briefly. Curious students who are interested in derivations of formulas and theorems or detailed explanations of concepts can seek the references at the end of the book or their own textbook if they are interested.

Each chapter includes a chapter quiz with problems similar to those solved in the text. The answers to every question are provided at the end of the book, so that the student can try the problems and determine whether or not they have really grasped the material. A final exam and answer key at the end of the book provide the reader with a means to review the concepts laid out in the chapters.

We try to cover all major areas of signals and systems. The book begins by covering methods used to calculate energy and power in signals. Next, we spend time studying signals in the frequency domain using Fourier analysis. Other topics covered include amplitude, frequency, phase modulation, spectral analysis, convolution, the Laplace transform, and the z-transform. The primary aim of the book is to cover basic topics a student should master on a first exposure to the subject. Therefore, topics such as probability and digital signal processing are not covered in this edition.

Unfortunately, in a book with this size and scope it is not possible to cover every aspect of the field or to cover all topics in great detail. We have tried to put the best sampling of basic concepts together which is representative of most courses and texts. In any case, you should be able to get through this book relatively quickly and it will give you the confidence to solve problems and prepare you to go on to further study in the field.

David McMahon

ABOUT THE AUTHOR

David McMahon works as a researcher in the national laboratories on nuclear energy. He has advanced degrees in physics and applied mathematics, and has written several titles of McGraw-Hill.

CONTENTS

CHAPTER 1

Introduction

A *signal* is a function of time that carries information. Physically, it may be a voltage across a capacitor or a current through a resistor for example, but in this book we will primarily be interested in the *mathematical* properties of signals. As such in general we will ignore the specific realization of a signal, beyond the understanding that it has some electrical form. It is typical to denote a signal by $x(t)$.

Often signals are real functions of time. However, it is also possible to have a signal that is a complex function. In this book we will follow the convention used in electrical engineering and denote $j = \sqrt{-1}$. Therefore a complex signal can be written in the form $x(t) = x_1(t) + jx_2(t)$, where $x_1(t)$ and $x_2(t)$ are real functions.

Continuous and Discrete Signals

A signal can be *continuous* or *discrete*. Basically, in this case the signal is a plain old function you are familiar with from calculus. In short, a continuous signal can assume any value in some continuous interval (a, b). In fact, a and b can

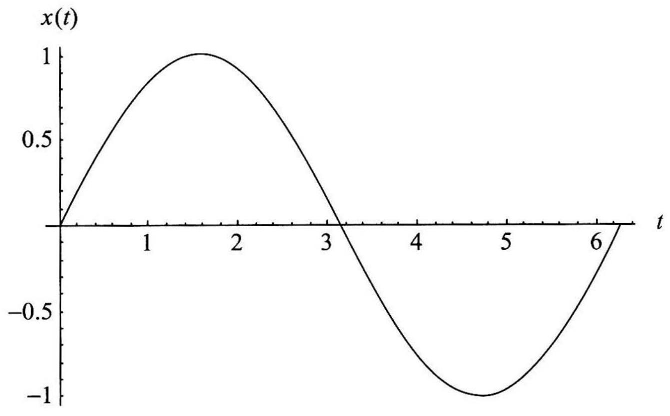

Fig. 1-1. An example of a continuous signal, a sine wave.

range over the entire real numbers, such that the signal is defined for $(-\infty, \infty)$. A continuous signal is shown in Fig. 1-1.

A *discrete time signal* is defined at discrete times we can label with an integer n. Therefore we often define a discrete time signal by $x[n]$. A discrete time signal can be created from a continuous signal by sampling $x(t)$ at regular intervals. Mathematically, we can think of a discrete signal as a *sequence* of numbers. If we denote the sampling interval by T_s, then $x[n] = x(nT_s)$; that is, we compute the discrete values of $x[n]$ by passing the argument $t = nT_s$ to the function $x(t)$. In Fig. 1-2, we show a discrete time signal formed by sampling the sin function at regular intervals.

In this chapter we will examine some basic properties of continuous signals; in the next chapter we'll do some examples with discrete signals.

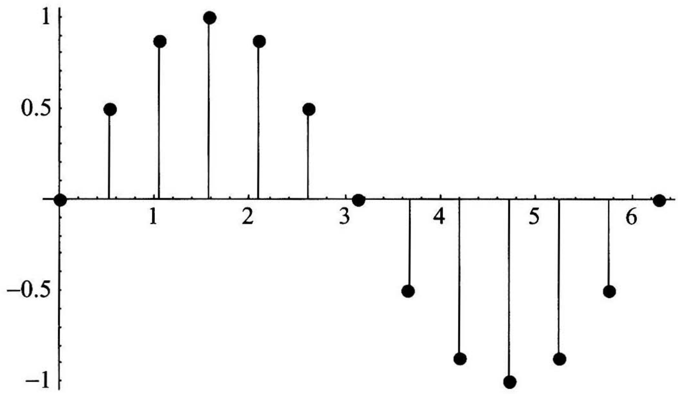

Fig. 1-2. A discrete time signal formed by sampling the sine function.

As we will see throughout much of the book, it is possible to analyze signals by studying their frequency content. Later, we will see that by using a mathematical tool known as the *Fourier transform* we can transform a function of time into a function of frequency ω, which is denoted by $X(\omega)$. For now, we will begin by looking at some important properties that can be studied as functions of time. We often say that we are working in the *time domain* when looking at signals this way. When studying $X(\omega)$, we say that we are working in the *frequency domain.* We begin by computing the energy and power content of a signal in the time domain.

Energy and Power in Signals

Consider a voltage $v(t)$ across a resistor R. Recalling Ohm's law, $v(t) = Ri(t)$, where $i(t)$ is the current, the instantaneous power is given by

$$p(t) = \frac{v(t)^2}{R}$$

Equivalently, we can instead consider the current $i(t)$ through the resistor, in which case the power is given by

$$p(t) = Ri(t)^2$$

Now if we calculate the power on a per-ohm basis, then we have $p(t) = v^2(t) = i^2(t)$. Then the total energy in joules is found by integrating

$$E = \int_{-\infty}^{\infty} p(t)\,\mathrm{d}t = \int_{-\infty}^{\infty} i^2(t)\,\mathrm{d}t$$

The average power is given by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} i^2(t)\,\mathrm{d}t$$

We can generalize these notions to find the energy and power content in an arbitrary signal $x(t)$. In general a signal can be complex, so we consider the squared modulus given by $|x(t)|^2 = x(t)\overline{x}(t)$, where $\overline{x}(t)$ is the complex conjugate given by $\overline{x}(t) = x_1(t) - jx_2(t)$. If the signal is real then $|x(t)|^2 = x^2(t)$.

Therefore given a signal $x(t)$, the normalized energy content E is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,\mathrm{d}t \tag{1.1}$$

The normalized average power of a signal is given by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \qquad (1.2)$$

If a signal is discrete, then the normalized energy content is found by calculating

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad (1.3)$$

And the normalized average power becomes

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \qquad (1.4)$$

Classification of Energy Signals and Power Signals

Signals can be classified as *energy signals* or *power signals.* An energy signal is one for which the energy E is finite, that is $0 < E < \infty$, while the average power vanishes ($P = 0$). On the other hand, if P is nonzero but finite (i.e., $0 < P < \infty$) and the energy is infinite, then the signal is a power signal. It is possible for a signal to be neither an energy signal nor a power signal. In the next few sections, we summarize some common signal types.

DC SIGNALS

A *DC signal* is simply a signal that has a constant value. In Fig. 1-3, we show a signal that maintains the constant value of unity for all times.

PERIODIC SIGNALS

In many cases of interest a signal will be periodic. This means that there exists some positive number T_0 which we denote the *period* such that

$$x(t) = x(t + T_0) \qquad (1.5)$$

The *fundamental frequency* f_0 is given by the inverse of the period:

$$f_0 = \frac{1}{T_0} \text{ Hz} \qquad (1.6)$$

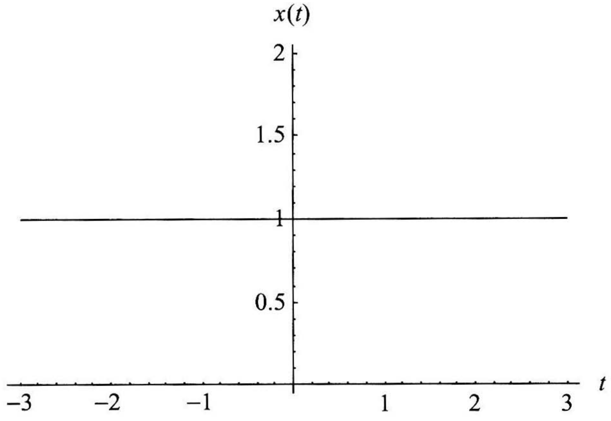

Fig. 1-3. A DC signal.

When considering periodic signals, we will examine the energy content over one period. If the energy for one period E_0 is finite, then the signal is a power signal, and the power is given by

$$P = \frac{E_0}{T_0} \quad \text{(for periodic signals)} \tag{1.7}$$

Many types of periodic functions are possible. For example, in Fig. 1-4 we have a "sawtooth" wave. To find the period, we look for the smallest value of t at which a feature of the function repeats (recall (1.5)). For example, in the sawtooth

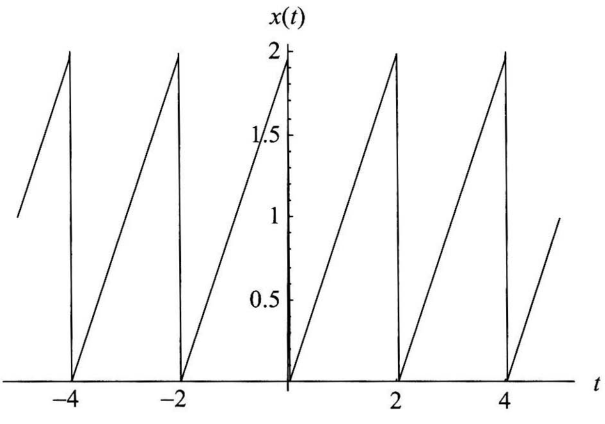

Fig. 1-4. A sawtooth wave. The period is 2 s, and the frequency is 0.5 Hz.

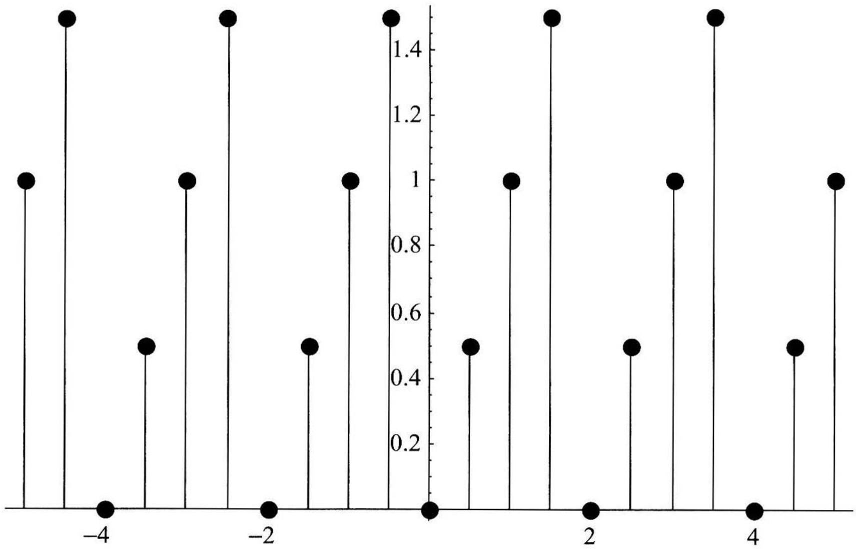

Fig. 1-5. A periodic discrete time signal.

wave we can look at the location of each peak in the wave. By inspection we see that the peaks repeat every 2 s, and so the fundamental period is $T_0 = 2$ s. The fundamental frequency is then found to be

$$f_0 = \frac{1}{T_0} = \frac{1}{2}\,\text{Hz}$$

It is also possible to have periodic discrete time signals. A discrete time signal $x[n]$ is periodic with period N, where N is a positive integer if

$$x[n] = x[n + N] \tag{1.8}$$

A simple example of a periodic discrete time signal is found by sampling the sawtooth wave at regular intervals. This is shown in Fig. 1-5.

SINUSOIDAL SIGNALS

The most familiar periodic functions are the trigonometric functions. In particular, we are interested in *sinusoidal* signals. A sinusoidal signal can be written as

$$x(t) = A\cos(\omega t + \theta) \tag{1.9}$$