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LINEAR and NONLINEAR MULTIVARIABLE FEEDBACK CONTROL

A Classical Approach

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Linear and Nonlinear Multivariable Feedback Control: A Classical Approach

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Linear and Nonlinear Multivariable Feedback Control

To my beloved family: Lilit, Yulia, and Nikolay

Preface

This textbook provides a unified control theory of linear and nonlinear multivariable feedback systems, also called multi-input multi-output (MIMO) systems, as a straightforward extension of the classical control theory. The central idea of the book is to show how the *classical* (frequency- and root-domain) engineering methods look in the multidimensional case, and how a practising engineer or researcher can apply them to the analysis and design of linear and nonlinear MIMO systems.

At present, there is a great number of fundamental textbooks on classical feedback control as applied to single-input single-output (SISO) systems, such as the books by Dorf and Bishop (1992), K. Ogata (1970), Franklin, Powell and Emami-Naeini (1991), Atherton (1975) and E. Popov (1973), the last two being devoted to nonlinear SISO systems, and many others. A general quality of all these books is a united conceptual approach to introducing the classical control theory, as well as clearly indicated branches of that theory; in fact, a lecturer can successfully use any of these books in teaching his course on related subjects. On the other hand, there are many remarkable textbooks and monographs on multivariable feedback control, but the situation here is not so plain. Historically, at the outset, the development of multivariable control theory was conducted in different ways and manners, varying from massive efforts to extend directly the basic classical methods and techniques, to no less massive attempts to reformulate radically and even 'abolish' the classical heritage of control theory. Besides, the initial stages of formation of multivariable control essentially coincided with the advent of state-space methods and approaches, and with rapid development of optimal control theory, equally dealing with SISO and MIMO systems. At last, at around that time, there the robust control theory also applicable to both SISO and MIMO systems emerged. As a result, the notion of 'modern' multivariable control is so manifold and embraces so many directions and aspects of feedback control that it is difficult to list them all without running the risk of missing something significant. Nevertheless, it is obvious that optimal, adaptive and robust methods (and their variations) are predominant in the scientific and technical literature, and advances in these methods considerably exceed the achievements of the 'classical' branch in multivariable control. At the same time, it should be acknowledged that modern MIMO control theory just 'jumped over' many important problems of the classical theory and now there is an evident gap between the topics presented in most textbooks on SISO control and those in many books on multivariable control (Skogestad and Postlethwaite 2005; Safonov 1980; Maciejowski 1989, etc.).

The goal of this book is to bridge that gap and to provide a holistic multivariable control theory as a direct and natural extension of the classical control theory, for both linear and nonlinear MIMO systems. The need for such a book is particularly evident now that modern computer aids and specialized programming languages (first of all, MATLAB®[1]) allow control specialists to restore and successfully use in practice many powerful classical approaches which in fact have been disregarded recently as useless and non-effective, especially for multivariable control. That is why the author hopes that a text in which many key problems of multivariable control are introduced and explained in common terms and notions of the classical control would be helpful for practitioners and researchers engaged in control engineering, as well as for lecturers on both classical and modern control.

The textbook can be used for an advanced undergraduate (fourth-year) course or for an introductory graduate course in multivariable feedback control. The necessary prerequisites for understanding the book contents are a typical introductory course in classical control and some elementary knowledge of the theory of matrices and linear spaces. The presented material has partially been used in an undergraduate multivariable control course given by the author in the Cybernetics Department at State Engineering University of Armenia (SEUA) since 2002.

The restrictions on the book's length forced the author to exclude some material that had been regarded as very useful and appropriate for the textbook. The matter concerns problems, exercises, appendices on the theory of matrices and functional analysis, etc. All these materials are available over the internet from the author's home page (www.seua.am/ogasp).

All worked examples in the book were solved with the help of graphical user interface (GUI) *ControlSysCAD*, working in the MATLAB environment, which was developed by the author. A very simplified version of that GUI destined for solving simple exercises on two-dimensional linear MIMO systems of different structural classes is also available over the internet from the author's home page.

The author will be grateful for any comments, remarks, discovered errors, etc. concerning the book. Please send them to the author's email address (ogasparyan@seua.am).

The companion website for the book is http://www.wiley.com/go/gasparyan

[1] MATLAB® is a registered trademark of The MathWorks, Inc.

Contents

Part I

Linear Multivariable Control Systems

1

Canonical representations and stability analysis of linear MIMO systems

1.1 INTRODUCTION

In the first section of this chapter, we consider in general the key ideas and concepts concerning canonical representations of linear multi-input multi-output (MIMO) control systems (also called *multivariable* control systems) with the help of the *characteristic transfer functions* (or *characteristic gain functions*) method (MacFarlane and Belletrutti 1970; MacFarlane *et al.* 1977; MacFarlane and Postlethwaite 1977; Postlethwaite and MacFarlane 1979). We shall see how, using simple mathematical tools of the theory of matrices and linear algebraic operators, one can associate a set of N so-called *one-dimensional characteristic systems* acting in the complex space of input and output vector-valued signals along N linearly independent directions (axes of the *canonical basis*) with an N-dimensional (i.e. having N inputs and N outputs) MIMO system. This enables us to reduce the stability analysis of an interconnected MIMO system to the stability analysis of N *independent* characteristic systems, and to formulate the generalized Nyquist criterion. We also consider some notions concerning the *singular value decomposition* (SVD) used in the next chapter for the performance analysis of MIMO systems. In the subsequent sections, we focus on the structural and geometrical features of important classes of MIMO systems – *uniform* and *normal* systems – and derive canonical representations for their transfer function matrices. In the last section, we discuss multivariable root loci. That topic, being immediately related to the stability analysis, is also very significant for the MIMO system design.

1.2 GENERAL LINEAR SQUARE MIMO SYSTEMS

1.2.1 Transfer matrices of general MIMO systems

Consider an N-dimensional controllable and observable *square* (that is having the same number of inputs and outputs) MIMO system, as shown in Figure 1.1. Here, $\varphi(s)$, $f(s)$ *and* $\varepsilon(s)$

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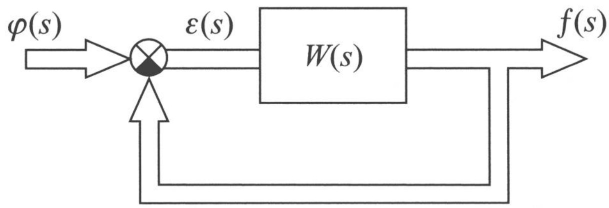

Figure 1.1 Block diagram of a general-type linear MIMO feedback system.

stand for the Laplace transforms of the N-dimensional input, output and error vector signals $\varphi(t)$, $f(t)$ and $\varepsilon(t)$, respectively (we shall regard them as elements of some N-dimensional complex space $\mathbb{C}^N$); $W(s) = \{w_{kr}(s)\}$ denotes the square transfer function matrix of the open-loop system of order $N \times N$ (for simplicity, we shall call this matrix the *open-loop transfer matrix*) with entries $w_{kr}(s)$ $(k, r = 1, 2, \ldots, N)$, which are scalar proper rational functions in complex variable s. The elements $w_{kk}(s)$ on the principal diagonal of $W(s)$ are the transfer functions of the *separate* channels, and the nondiagonal elements $w_{kr}(s)$ $(k \neq r)$ are the transfer functions of *cross-connections* from the rth channel to the kth.

Henceforth, we shall not impose any restrictions on the number N of separate channels, i.e. on the dimension of the MIMO system, and on the *structure* (type) of the matrix $W(s)$. At the same time, so as not to encumber the presentation and to concentrate on the primary ideas, later on, we shall assume that the scalar transfer functions $w_{kr}(s)$ do not have multiple poles (we mean each individual transfer function). Also, we shall refer to the general-type MIMO system of Figure 1.1 as simply the *general* MIMO system (so as not to introduce any ambiguity concerning the *type* of system, which is conventionally defined in the classical control theory as the number of pure integrators in the open-loop system transfer function).

The output $f(s)$ and error $\varepsilon(s)$ vectors, where

$$\varepsilon(s) = \varphi(s) - f(s), \tag{1.1}$$

are related to the input vector $\varphi(s)$ by the following operator equations:

$$f(s) = \Phi(s)\varphi(s), \qquad \varepsilon(s) = \Phi_\varepsilon(s)\varphi(s), \tag{1.2}$$

where

$$\Phi(s) = [I + W(s)]^{-1}\, W(s) = W(s)\, [I + W(s)]^{-1} \text{ and} \tag{1.3}$$

$$\Phi_\varepsilon(s) = [I + W(s)]^{-1} \tag{1.4}$$

are the transfer function matrices of the closed-loop MIMO system (further, for short, referred to as the *closed-loop transfer matrices*) with respect to output and error signals, and I is the unit matrix. The transfer matrices $\Phi_\varepsilon(s)$ and $\Phi(s)$ are usually called the *sensitivity function matrix* and *complementary sensitivity function matrix*.[1]

By straightforward calculation, it is easy to check that $\Phi_\varepsilon(s)$ and $\Phi(s)$ satisfy the relationship:

$$\Phi(s) + \Phi_\varepsilon(s) = I. \tag{1.5}$$

[1] The terms *sensitivity function* and *complementary sensitivity function* were introduced by Bode (1945).

From here, we come to the important conclusion that it is impossible to bring to zero the system error if the input signal is a sum (mixture) of a reference signal and disturbances, where the latter may be, for example, the measurement or other noises. Indeed, if the system ideally tracks the input reference signal, that is if the matrix $\Phi_{\varepsilon}(s)$ identically equals the zero matrix, then, due to the superposition principle (Ogata 1970; Kuo 1995), that system also ideally reproduces at the output the input noise [since, if $\Phi_{\varepsilon}(s) = 0$, then the matrix $\Phi(s)$ in Equation (1.5) is equal to the unit matrix I]. A certain trade-off may only be achieved provided the input reference signal and the measurement noise have nonoverlapping (at least, partially) frequency ranges.[2]

1.2.2 MIMO system zeros and poles

1.2.2.1 Open-loop MIMO systems

A single-input single-output (SISO) feedback control system with the open-loop transfer function $W(s)$ is depicted in Figure 1.2. That system may be regarded, if $N = 1$, as a specific case of

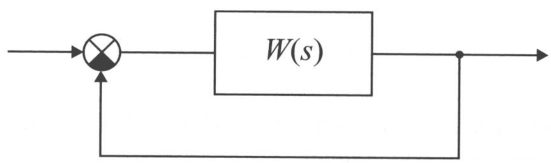

Figure 1.2 Block diagram of a SISO control system ($N = 1$).

the MIMO system of Figure 1.1. The transfer function $W(s)$ is a rational function in complex variable s and can be expressed as a quotient of two polynomials $M(s)$ and $D(s)$ with real coefficients:

$$W(s) = \frac{M(s)}{D(s)}, \tag{1.6}$$

where the order m of $M(s)$ is equal to or less than the order n of $D(s)$, that is we consider only physically feasible systems.

From the classical control theory, we know that the *poles* p_i of $W(s)$ are the roots of the denominator polynomial $D(s)$, and *zeros* z_i are the roots of the numerator polynomial $M(s)$ (Ogata 1970; Kuo 1995). In the case of usual SISO systems with *real* parameters, complex poles and zeros always occur in complex conjugate pairs. Obviously, at the zeros z_i, the transfer function $W(s)$ vanishes and, at the poles p_i, it tends to infinity (or $1/W(s)$ vanishes).

In the multivariable case, the situation is not so simple, and this refers to the MIMO system zeros in particular. This indeed explains the large number of papers in which there are given different definitions and explanations of the MIMO system zeros: from the state-space positions, by means of polynomial matrices and the Smith-McMillan form, etc. (Sain and Schrader 1990; Wonham 1979; Rosenbrock 1970, 1973; Postlethwaite and MacFarlane 1979; Vardulakis 1991).

First, let us consider the *open-loop* MIMO system *poles*. We call any complex number p_i the *pole* of the open-loop transfer matrix $W(s)$ if p_i is the pole of at least one of the entries

[2] The MIMO system accuracy is discussed in Chapter 2.

$w_{kr}(s)$ of the matrix $W(s)$. In fact, if at least one of the entries $w_{kr}(s)$ of $W(s)$ tends to infinity as $s \to p_i$, then $W(s)$ tends (strictly speaking, by norm) to infinity. Therefore, p_i may be regarded as the pole of $W(s)$. As a result, we count the set of the poles of all $w_{kr}(s)$ as the poles of $W(s)$. Such a *prima facie* formal definition of the MIMO system pole seems evident but it leads, as we shall see later, to rather interesting results.

Let the transfer matrix $W(s)$ be expanded, taking into account the above assumption that $w_{kr}(s)$ have no multiple roots, into partial fractions as:

$$W(s) = \sum_{i=1}^{n} \frac{K_i}{s - p_i} + D, \tag{1.7}$$

where n is the total number of simple poles of $W(s)$;

$$K_i = \lim_{s \to p_i} (s - p_i) W(s) \tag{1.8}$$

are the *residue matrices* of $W(s)$ at the finite poles p_i; and the constant matrix D is

$$D = \lim_{s \to \infty} W(s). \tag{1.9}$$

Note that the matrix D differs from the zero matrix if any of $w_{kr}(s)$ have the same degree of the numerator and denominator polynomials.

The *rank* r_i of the ith pole p_i is defined as the rank of the residue matrix K_i, and it is called the *geometric multiplicity* of that pole. Among all poles of the open-loop MIMO system, of special interest are those of rank N, which are also the poles of *all* the nonzero elements $w_{kr}(s)$. In what follows, we shall call such poles the *absolute* poles of the open-loop MIMO system. It is easy to see that if a complex number p_i is an absolute pole of the transfer matrix $W(s)$, then the latter can be represented as

$$W(s) = \frac{1}{s - p_i} W_1(s), \tag{1.10}$$

where the matrix $W_1(p_i)$ is nonsingular [that matrix cannot have entries with poles at the same point p_i owing to the assumption that $w_{kr}(s)$ have no multiple poles].

In a certain sense, it is more complicated to introduce the notion of *zero* of the transfer matrix $W(s)$, as an arbitrary complex number s that brings any of the transfer functions $w_{kr}(s)$ to vanishing, cannot always be regarded as the zero of $W(s)$. We introduce the following two definitions:

1. A complex number z_i is said to be an *absolute* zero of the transfer matrix $W(s)$ if it reduces the latter to the zero matrix.

2. A complex number z_i is said to be a *local* zero of rank k of $W(s)$, if substituting it into $W(s)$ makes the latter singular and of rank $N - k$. The local zero of rank N is, evidently, the absolute zero of $W(s)$.[3]

[3] The notion of MIMO system zero as a complex number z that reduces at $s = z$, the *local rank* of the matrix $W(s)$, is given, for example, in MacFarlane (1975).

Let us discuss these statements. It is clear that if a number z_i is an absolute zero of $W(s)$, then we can express that matrix as

$$W(s) = (s - z_i)W_1(s), \tag{1.11}$$

where $W_1(z_i)$ differs from the zero matrix and has rank N. In other words, the absolute zero must also be the *common zero* of all the nonzero elements $w_{kr}(s)$ of $W(s)$.

We are not quite ready yet for detailed discussion of the notion of the open-loop MIMO system local zero, but, as a simple example, consider the following situation. Let z_i be the common zero of all elements $w_{kr}(s)$ of the kth row or the rth column of $W(s)$, i.e. $w_{kr}(z_i) = 0$ when $k = const, r = 1, 2, \ldots, N$, or when $r = const,\ k = 1, 2, \ldots, N$. Then, obviously, if the rank of $W(s)$ is N for almost all values of s [i.e. the *normal* rank of $W(s)$ is N], then the matrix $W(z_i)$ will have at least rank $N - 1$, since, for $s = z_i$, the elements of the kth row or the rth column of $W(z_i)$ are zero. Structurally, the equality to zero of all elements of the kth row of $W(s)$ means that for $s = z_i$, both the direct transfer function $w_{kk}(s)$ of the kth channel and the transfer functions of all cross-connections leading to the kth channel from all the remaining channels become zeros. Analogously, the equality to zero of all elements of the rth column of $W(s)$ means that for $s = z_i$, both the direct transfer function $w_{rr}(s)$ of the rth channel and the transfer functions of all cross-connections leading from the rth channel to all the remaining channels become zeros. This situation may readily be expanded to the case of local zero of rank k. Thus, if, for $s = z_i$, the elements of any k rows or any k columns of $W(s)$ become zeros, then z_i is the local zero of rank k. At this point, however, a natural question arises of whether local zeros of the matrix $W(s)$ exist which reduce its normal rank but do not have the above simple explanation and, if such zeros, then what is their number?

A sufficiently definite answer to that question is obtained in the following subsections, and here we shall try to establish a link between the introduced notions of the open-loop MIMO system poles and zeros, and the determinant of $W(s)$. It is easy to see that both the absolute and local zeros of $W(s)$ make $\det W(s)$ vanishing, since the determinants of the zero matrices as well as of the singular matrices identically equal zero. Besides, from the standard rules of calculating the determinants of matrices (Gantmacher 1964; Bellman 1970), we have that if some elements of $W(s)$ tend to infinity, then the determinant $\det W(s)$ also tends to infinity. In other words, the poles of $W(s)$ are the poles of $\det W(s)$. Based on this, we can represent $\det W(s)$ as a quotient of two polynomials in s:

$$\det W(s) = \frac{Z(s)}{P(s)} \tag{1.12}$$

and call the *zeros* of $W(s)$ the roots of the equation

$$Z(s) = 0 \tag{1.13}$$

and the *poles* of $W(s)$ the roots of the equation

$$P(s) = 0. \tag{1.14}$$

Let us denote the degrees of polynomials $Z(s)$ and $P(s)$ as m and n, respectively, where, in practice, $m \leq n$. We shall call $Z(s)$ the *zeros polynomial* and $P(s)$ the *poles polynomial*, or the *characteristic polynomial*, of the open-loop MIMO system.