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# Studies in Advanced Mathematics

S.-T. Yau, Series Editor

## Wavelet Analysis and Applications

Proceedings of an International Conference  
on Wavelet Analysis and Its Applications

November 15-20, 1999

Zhongshan University

Guangzhou, China

Donggao Deng, Daren Huang,  
Rong-Qing Jia, Wei Lin, and  
Jianzhong Wang, Editors

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Volume 25

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# **Wavelet Analysis and Applications**

## Preface

Wavelet analysis has been one of the major research directions in science in the last decade. More and more, mathematicians and scientists are joining this exciting research area. Certainly, wavelet analysis has had a great impact on areas such as approximation theory, harmonic analysis, and scientific computation. More importantly, wavelet analysis has shown great potential in applications to information technology such as signal processing, image processing, and computer graphics.

China has played a significant role in this development of wavelet analysis as evidenced by many fruitful theoretical results and practical applied projects. In order to exchange ideas and results with international research groups, we organized an international conference on wavelet analysis and its applications during the period of November 15–November 20, 1999, at Zhongshan University, Guangzhou, China. The conference brought together more than 80 researchers in wavelet analysis and related areas from all over the world, including Australia, Canada, China, Germany, Hong Kong, Italy, Japan, Korea, Russia, Singapore, and the United States.

The present proceedings contain twenty-six selective papers from the lectures given in the conference. These papers cover a wide range of research topics of current interest and include many significant results in the study of refinement equations and refinable functions, properties and construction of wavelets, spline wavelets, multi-wavelets, wavelet packets, shift-invariant spaces, approximation schemes and subdivision algorithms, and tilings. Several papers also focus on applications of wavelets to numerical solutions of partial differential equations and integral equations, image processing and facial recognition, computer vision, and feature extraction from data. We thank all the authors for their contributions and believe that these papers will stimulate further interest in wavelet analysis and its applications.

This conference was sponsored by the Ministry of Education of China, the National Science Foundation of China, Nankai University, Peking University, Zhejiang University, and Zhongshan University. We would like to express our sincere thanks to all of the above organizations for their generous financial support to the conference. Special thanks go to our host: the School of Mathematics and Computing Science of Zhongshan University. In particular, we appreciate the efficient work done by Dr. Jianhuang Lai and Ms. Shouzhuang Liang for the conference.

During the process of editing these proceedings, we have received much needed help from Drs. Bin Han and Qingtang Jiang—we are grateful for their assistance. We also thank the publisher, the International Press in Boston, for their cooperation in preparing the manuscript.

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## Non-uniform Sampling in Multiply Generated Shift-invariant Subspaces of $L^p(\mathbb{R}^d)$

Akram Aldroubi, Qiyu Sun and Wai-Shing Tang

ABSTRACT. Given the samples  $\{f(x_j) : j \in J\}$  of a function  $f$  belonging to a shift invariant subspace of  $L^p(\mathbb{R}^d)$ , we give a fast reconstruction algorithm that allows the exact reconstruction of  $f$ , as long as the sampling set  $X = \{x_j : j \in J\}$  is sufficiently dense.

### 1. Introduction

In the sampling problem, the objective is to recover a function  $f$  on  $\mathbb{R}^d$  from its samples  $\{f(x_j) : j \in J\}$ , where  $J$  is a countable indexing set, and where  $f$  satisfies some apriori constraints. Typically, the function is assumed to belong to the space of band-limited functions  $B_\Omega$ , i.e.,  $\text{supp } \hat{f} \subset [-\Omega, \Omega]$  (see for example [5, 6, 10, 11]). For dimension  $d = 1$ , another characterization states that the space of band-limited functions  $B_{[-1/2, 1/2]}$  is identical to the space

$$(1.1) \quad V^2(\text{sinc}) = \left\{ f = \sum_{k \in \mathbb{Z}} c(k) \text{sinc}(x - k) : (c(k)) \in \ell^2 \right\}.$$

Recently, the problem of regular and irregular sampling and reconstruction have been extended to more general shift-invariant spaces of the form

$$(1.2) \quad V^2(\phi) = \left\{ f = \sum_{k \in \mathbb{Z}^d} c(k) \phi(x - k) : c \in \ell^2 \right\} \subset L^2(\mathbb{R}^d)$$

(see for instance [1, 2, 3, 7, 16, 17]), and weighted shift-invariant spaces of the form

$$(1.3) \quad V_\nu^p(\phi) = \left\{ f = \sum_{k \in \mathbb{Z}^d} c(k) \phi(x - k) : c \in \ell_\nu^p \right\} \subset L_\nu^p(\mathbb{R}^d)$$

[3]. The results on irregular sampling and reconstruction as well as their connection to Reproducing Kernel Hilbert Spaces (RKHS) and frames, can be found in [2],

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1991 *Mathematics Subject Classification*. Primary 41A27, 41A63, 42C15, 42C40, 46S30, 46T99, 46A35, 46E15, 46N99, 47B37, 94A12.

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and a complete discussion can be found in [3]. Let  $\Phi = (\phi_1, \dots, \phi_r)^T$  and  $C = (c_1, \dots, c_r)^T$ . The goal of this article is to discuss the problem of sampling and to extend the results in [1] to the case of multiply generated shift-invariant spaces of the form

$$(1.4) \quad V^p(\Phi) = \left\{ f = \sum_{k \in \mathbb{Z}^d} C^T(k) \Phi(x - k) : C \in (\ell^p)^{(r)} \right\} \subset L^p(\mathbb{R}^d).$$

## 2. Notation and preliminaries

Let  $\mathcal{B}$  be a Banach space,  $\mathcal{B}^{(r)}$  will denote  $r$  copies  $\mathcal{B} \times \dots \times \mathcal{B}$  of  $\mathcal{B}$ , together with an appropriate norm  $\|\cdot\|_{\mathcal{B}^{(r)}}$  induced by the norm  $\|\cdot\|_{\mathcal{B}}$  of  $\mathcal{B}$ . In particular  $L^p(\mathbb{R}^d)$ ,  $1 \leq p \leq \infty$ , will denote the space of all measurable vector-valued functions  $F = (f_1, \dots, f_r)^T$  on  $\mathbb{R}^d$  with finite norm  $\|F\|_{L^p(\mathbb{R}^d)}$  given by

$$\begin{aligned} \|F\|_{L^p(\mathbb{R}^d)} &= \left( \sum_{i=1}^r \|f_i\|_{L^p}^p \right)^{1/p} \quad \text{if } 1 \leq p < \infty, \\ \|F\|_{L^\infty(\mathbb{R}^d)} &= \max_{1 \leq i \leq r} \|f_i\|_{L^\infty} \quad \text{if } p = \infty. \end{aligned}$$

For  $1 \leq p \leq \infty$ , let  $(\ell^p)^{(r)}$  be the space of all sequences  $D = (d_1, \dots, d_r)^T$  with finite norms  $\|D\|_{(\ell^p)^{(r)}}$ , where

$$\begin{aligned} \|D\|_{(\ell^p)^{(r)}} &= \left( \sum_{i=1}^r \|d_i\|_{\ell^p}^p \right)^{1/p} \quad \text{if } 1 \leq p < \infty, \\ \|D\|_{(\ell^\infty)^{(r)}} &= \max_{1 \leq i \leq r} \|d_i\|_{\ell^\infty} \quad \text{if } p = \infty. \end{aligned}$$

Let  $W(L^p) = W(L^\infty, L^p)$  denote the space of measurable functions  $f$  which are “locally in  $L^\infty$  and globally in  $L^p$ ” with finite norm

$$\begin{aligned} \|f\|_{W(L^p)} &= \left( \sum_{k \in \mathbb{Z}^d} \text{ess sup}_{x \in [0,1]^d} |f(x+k)|^p \right)^{1/p} < \infty \quad \text{if } 1 \leq p < \infty, \\ \|f\|_{W(L^\infty)} &= \sup_{k \in \mathbb{Z}^d} \text{ess sup}_{x \in [0,1]^d} |f(x+k)| < \infty \quad \text{if } p = \infty. \end{aligned}$$

Endowed with this norm,  $W(L^p)$  becomes a Banach space [8, 9]. Let  $W_0(L^p) = W(C, L^p)$  be the space of all continuous functions in  $W(L^p)$ . Then it is a closed subspace of  $W(L^p)$  and thus also a Banach space [8, 9]. We have the following inclusions between the various spaces [9]:

$$(2.1) \quad W_0(L^p) \subset W_0(L^q) \subset L^q \quad \text{for any } 1 \leq p \leq q \leq \infty.$$

For  $g \in W(L^1)$  and  $f \in L^p$ , it is not difficult to check that there exists a constant  $C$  such that

$$\|f * g\|_{W(L^p)} \leq C \|f\|_{L^p} \|g\|_{W(L^1)}.$$

For any vector sequence  $D = (d_1, \dots, d_r)^T \in (\ell^p)^{(r)}$  and  $F \in W(L^1)^{(r)}$ , define their *semi-convolution*  $D *' F$  by

$$D *' F = \sum_{j \in \mathbb{Z}^d} D^T(j) F(\cdot - j).$$