

TRANSPORT PHENOMENA IN POROUS MEDIA II

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TRANSPORT PHENOMENA IN POROUS MEDIA II

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Preface

Transport phenomena in porous media continues to be an area of intensive research activity and this is primarily due to the fact that it plays an important role in a large variety of engineering and technological applications which span from the transport processes in biomechanical systems, such as blood flow in the pulmonary alveolar sheet, to the large scale circulation of brine in a geothermal reservoir. The acceleration in the progress in science and in the improvement in the design, efficiency and reliability of heat transfer equipment in power engineering, chemical, oil and gas industries are directly associated with the effective use of the modern tools of heat transfer analysis and measurement, predictive correlation equations, and with the sharing of the practical experience on the operation of all types of thermal equipment. This has caused a rapid expansion of research in diversified areas of heat transfer, including also porous media, and this has produced a huge amount of theoretical and experimental work.

The first volume of this book series, published in 1998, met with a very favourable reception within the porous media community, showing that there was a large demand for a series of books that emphasizes both the fundamentals and the applications of research in porous media. This has encouraged us to prepare the present second volume. In doing so, we have maintained the original concept of including a wide and diverse range of topics. In choosing the material we have also been influenced by the needs of the practical applications of porous media. Also, the book provides an up-to-date review of the current state-of-the-art in the various topics of heat transfer in porous media, presenting both the fundamental and experimental results of very active and internationally well-recognized authors. Thus, the book is primarily aimed at advanced researchers in porous media, and these may be applied mathematicians, physicists, geologists, chemists and practicing engineers.

All of the chapters in this book are very much interrelated. Further, some of the views expressed by some of the authors are contradictory and controversial, and therefore it was not easy to decide how best to order the chapters.

In Chapter 1, Nield has produced a very provoking and wide-ranging review of non-Darcy models, including comments on inertial effects, boundary friction, non-Newtonian fluids, viscous dissipation, rotation, magnetic field, radiation, and porous-medium/clear-fluid interface conditions. The solution of these model equations may be performed using many of the standard techniques, e.g., finite differences, finite elements, etc., but in

Chapter 2 Škerget and Jecl describe an alternative method which is becoming ever more popular, namely the boundary element method.

The work described in Chapters 1 and 2 assumes that the flows are stable and therefore in the next three chapters the stability of various fluid flows are investigated. In Chapter 3, Rees presents an overview of recent work on the onset of instabilities in thermal boundary-layer flows. In Chapter 4, Tyvand investigates the linear stability in three dimensions for the onset of Rayleigh–Bénard convection in finite porous bodies. In Chapter 5, Mamou looks at the stability of double-diffusive convection and finite amplitude flows in a tilted porous enclosure.

In Chapter 6, Howle investigates buoyancy-driven convection and notes the discrepancy that can exist between theoretical and laboratory results. This can be as a result of an instability or that the medium is not spatially homogeneous. Wang, in Chapter 7, further investigates the effects of the micromechanics of ordered, unidirectional heterogeneous materials.

In Chapters 8 and 9, the effects of turbulence are discussed. Lage, de Lemos and Nield, in Chapter 8, review four available methodologies for developing microscopic turbulence models for single-phase flow in rigid, fully-saturated porous media. Masuoka and Takatsu, in Chapter 9, experimentally and theoretically investigate the turbulence characteristics and discuss the mechanism for the production and thermal dissipation.

In Chapters 10 to 13, the effects of phase change, bubble growth, solidification and surface reactions are considered. In Chapter 10, Chang and Weng review coupled heat and moisture transfer in porous material with applications in the drying and storage of grain. In Chapter 11, Bories and Prat investigate the nucleation and bubble growth which are important in processes where there is pressure depletion and boiling. In Chapter 12, Riahi discusses the effects of rotation on convection adjacent to the solid–liquid interface during the solidification of a binary alloy. In Chapter 13, Pop, Merkin and Ingham investigate the effects of exothermic catalytic reactions which take place on a surface next to a porous medium.

In Chapters 14 to 16, problems involving porosity, either within or on the surface of the Earth, are considered. In Chapter 14, Bejan, Rocha and Cherry review the work that has been performed on the generation and flow of methane gas through a porous medium impregnated with solid clathrate hydrates. In Chapter 15, Woods develops a hierarchy of models to describe gravity driven flows in porous rocks, including the effects of layering, reaction, boiling and double advection. Finally, in Chapter 16, Lane and Hardy highlight the basic limitations of traditional treatments of complex bed geometries and vegetation in traditional models of river flows and the need for a more sophisticated porosity approach.

In the preparation of this volume, we have collaborated with a large number of researchers. Thus, our thanks go first to all the twenty-six authors, who have kindly accepted to contribute a chapter to the realization of the present volume, for their patience and dedicated effort. We would also like to express our sincere thanks to Professors P. J. Heggs and A. Nakayama who have throughout the preparation of the book given their support generously.

The formatting of this book and the preparation of the figures were performed by Dr Julie M. Harris and Dr Simon D. Harris and we are deeply indebted to them for all of their care and attention, and the patience that they have shown in both the preparation and the proof reading. Finally, but most sincerely, our grateful appreciation is extended to Mr Keith Lambert, Senior Publishing Editor, not only for his thoughtfulness, but also for his constant encouragement throughout the preparation of this book.

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