Robust Control System Techniques and Applications

Part 1 of 2

CONTROL AND DYNAMIC SYSTEMS

ADVANCES IN THEORY AND APPLICATIONS

Volume Editor

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CONTROL AND DYNAMIC SYSTEMS

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ROBUST CONTROL SYSTEM TECHNIQUES AND APPLICATIONS

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PREFACE

In the early days of modern control theory, the techniques developed were relatively simple but, nevertheless, quite effective for the relatively simple systems applications of those times. Basically, the techniques were frequency domain analysis and synthesis techniques. Then, toward the latter part of the 1950s, system state space techniques began to emerge. In parallel with these developments, computer technology was evolving.These two parallel developments (i.e., increasingly effective system analysis and synthesis techniques and increasingly powerful computer technology) have resulted in a requisite powerful capability to deal with the increasingly complex systems of today's world.

In these modern day systems of various levels of complexity, the need to deal with a wider variety of situations, including significant parameter variations, modeling large scale systems with models of lower dimension, fault tolerance, and a rather wide variety of other problems, has resulted in a need for increasingly powerful techniques, that is, system robustness techniques, for dealing with these issues. As a result, this is a particularly appropriate time to treat the issue of robust system techniques in this international series. Thus, this volume is the first volume of a two-volume sequence devoted to the most timely theme of "Robust Control Systems Techniques."

The first contribution to this volume is "Trade-offs Among Conflicting Objectives in Robust Control Design," by Brian D. O. Anderson, Wei-Yong Yan, and Robert Bitmead. This contribution presents important techniques for dealing with conflicting design objectives in control systems.

The next contribution is "Aspects of Robust Control Systems Design," by Rafael T. Yanushevsky. It provides an in-depth treatment of robustness techniques of systems described by differential-difference equations, and it also presents methods for the design of a wide class of robust nonlinear systems.

The next contribution is "System Observer Techniques in Robust Control Systems Design Synthesis," by Tsuyoski Okada, Masahiko Kihara, Masakazu Ikeda, and Toshihiro Honma. This contribution discusses techniques for dealing with the problems resulting from the use of observers in robust systems design, and it offers three distinct design techniques for treating these problems.

The next contribution is "Robust Tracking Control of Non-Linear Systems with Uncertain Dynamics," by Dauchang Wang and Cornelius T. Leondes. It presents rather effective techniques for the robust control on non-linear time varying of tracking control systems with uncertainties.

The next contribution is "Adaptive Robust Control of Uncertain Systems," by Y. H. Chen and J. S. Chen. This article sets forth techniques for incorporating adaptive control techniques into a (non-adaptive) robust control design.

The next contribution is "Robustness Techniques in Nonlinear Systems with Applications to Manipulator Adaptor Control," by Nader Sadegh. It presents techniques for achieving exponential and robust stability for a rather general class of nonlinear systems.

The next contribution is "Techniques in Modeling Uncertain Dynamics for Robust Control Systems Design," by Altuğ İftar and Ümit Özgüner. This contribution discusses a unified framework for robust control systems design for systems with both parameter uncertainties and uncertain dynamics due to the difficulties in modeling complex systems.

The next contribution is "Neoclassical Control Theory: A Functional Analysis Approach to Optimal Frequency Domain Controller Synthesis," by A. M. Holohan and M. G. Safonov. It offers techniques for the optimal synthesis (optimal in the sense defined in this contribution) of robust control systems.

The next contribution is "A Generalized Eigenproblem Solution for Singular H^2 and H^∞ Problems," by B. R. Copeland and M. G. Safonov. It presents techniques for the design of H^2 and H^∞ which apply equally well to both singular and nonsingular system cases.

The final contribution to this first volume of this two-volume sequence on the theme of "Robust Control System Techniques" is "Techniques in Stability Robustness Bounds for Linear Discrete-Time Systems," by James B. Farison and Sri R. Kollaq. This contribution provides a unified treatment of stability robustness design for discrete-time systems.

This volume is a particularly appropriate one as the first of a companion set of two volumes on robust control system analysis and synthesis techniques. The authors are all to be commended for their superb contributions, which will provide a significant reference source for workers on the international scene for years to come.

CONTENTS

Trade-offs among Conflicting Objectives in Robust Control Design

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I. Introduction

Consider the feedback control system depicted in Figure 1.

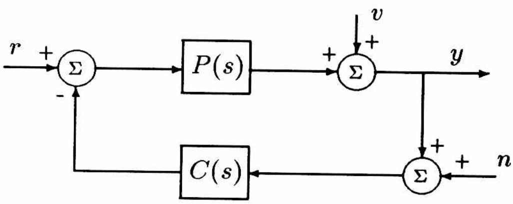

Figure 1. Feedback control system.

Here P is the plant system concerned, C is the controller and the signal r, y, u, v and n are the reference, system output, control input, output disturbance and sensor noise respectively. Classical control systems design emphasizes the securing of a number of conflicting objectives for this feedback system such as;

- securing closed loop stability from each external input to the loop signals u and y. (We refer to a controller $C(s)$ achieving this as [internally] *stabilizing* the plant $P(s)$.)

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- securing the rejection of disturbance signals v and n from y. This is reflected by the properties of the closed loop sensitivity and complementary sensitivity functions.

- securing robustness to plant modeling errors and plant variations. Depending upon the nature of plant uncertainty, this objective may appear in terms of gain margins, phase margins or sensitivity functions.

A classical controller synthesis aims to achieve all these ends but usually proceeds by focusing on a single objective and then de-tunes or compromises to effect the trade-off between these sometimes conflicting desires. Alternatively, one may optimize with respect to one design issue and then select, from the class of available solutions, that controller which best meets one of the other objectives [1]. Of critical importance in such approaches is to know to what extent each of these individual design criteria are conflicting in their demands of the controller. Further, it is advantageous to know whether successive optimization of objective functions is feasible and, if so, in what order. Lastly, one may pose the question of the ability to ameliorate these conflicts through the choice of a time-varying or nonlinear controller.

In this chapter, we study the trade-offs between a number of these design objectives firstly for the class of linear, time invariant controller. We use a tool introduced in [2] for this purpose. This tool deals with the construction of functions of a complex variable which fulfill certain interpolation and analyticity conditions arising from stability requirements and design objectives. The specific problems treated are:

- the combined sensitivity-gain margin problem

- the combined sensitivity-phase margin problem

- the combined sensitivity-complementary sensitivity problem

Having made apparent some of the compromises available to the designer with time invariant linear controllers, we then move on to study the benefits achievable in reducing some of this conflict by using a periodically time varying controller. Our analysis treats the single-input/single-output case but, where extension to the multi-input/multi-output case is direct, this is noted.

II. The Optimal Sensitivity and Gain Margin Problem as Separate Problems

The material of this section is largely drawn from [2], and serves as a tutorial introduction to the main ideas of the chapter. Let $P(s)$ be a scalar linear time-invariant plant with poles $p_1, \ldots, p_n \in \mathrm{Re}[s] \geq 0$ and zeros $z_1, \ldots, z_n$ (including possibly infinity) $\in \mathrm{Re}[s] \geq 0$. Consider a stable closed-loop as depicted in Figure 1.

A. Optimal Sensitivity and Gain Margin

The sensitivity function $S(s)$ is defined by

$$S(s) = [1 + P(s)C(s)]^{-1} \tag{1}$$

and the *sensitivity* is defined by

$$R[C(s)] \triangleq \|S(s)\|_\infty = \sup_{s \in \mathrm{Re}[s] \geq 0} |S(s)| = \sup_{\omega} |S(j\omega)| \tag{2}$$

The last equality in Eq. (2) follows from the maximum modulus principle; closed-loop stability ensures that $S(s)$ is analytic in $\mathrm{Re}[s] \geq 0$.

The sensitivity $R[C(s)]$ of course depends on $C(s)$. Its minimization through choice of $C(s)$ serves to secure a design which minimizes the maximum (over ω) of the gain from a disturbance entering at the plant output to the actual output.

A natural question is: what is

$$r_{\min} = \inf_{C(s)} \{R[C(s)] : \quad C(s) \text{ stabilizes } P(s)\} \tag{3}$$

(and what is the associated $C(s)$; and how may it be found)?

With a fixed controller $C(s)$, the upper and lower gain margins $b_{\max}$ and $a_{\min}$ are defined by

$$b_{\max} = \sup\{b : \quad C(s) \text{ stabilizes } kP(s)\ \forall k \in [1, b]\} \tag{4}$$

$$a_{\min} = \inf\{a : \quad C(s) \text{ stabilizes } kP(s)\ \forall k \in [a, 1]\} \tag{5}$$

Of course, it is possible to have $b_{\max} = \infty$ or $a_{\min} = 0$ (or both), but not if the sets $\{z_i\}$ or $\{p_j\}$ are nonempty. We shall define the *gain margin* as

$$K[C(s)] \triangleq \sup\{b/a : \quad 0 < a < 1 < b \quad \text{and} \quad C(s) \text{ stabilizes } kP(s)\ \forall k \in [a,b]\} \tag{6a}$$

$$= \frac{b_{\max}}{a_{\min}} \tag{6b}$$

Evidently, K can be infinite for certain plants. Note that the gain margin K is the same for $P(s)$ and $\alpha P(s)$, for any $\alpha > 0$, so long as $C(s)$ stabilizes $\alpha P(s)$.

A natural question is: what is

$$k_{\max} = \sup\{K[C(s)] : \quad C(s) \text{ stabilizes } P(s)\}? \tag{7}$$

We shall now review how these questions can be answered. There are two key relevant ideas, one tied to interpolation properties of $S(s)$ and the other tied to mapping properties. The overall thrust is to work with $S(s)$ rather than $C(s)$; once $S(s)$ is known, $C(s)$ of course follows easily.

B. Interpolation Properties of $S(s)$

Recall that p_i is a pole in Re$[s] \geq 0$ of $P(s)$. Because $C(s)$ is stabilizing, $C(p_i) = 0$ is impossible. Hence $S(p_i) = 0$. Recall also that z_j is a zero in Re$[s] \geq 0$ of $P(s)$. Again because $C(s)$ is stabilizing, z_j cannot be a pole of $C(s)$. Hence $S(z_j) = 1$. Thus we have

$$S(p_i) = 0 \quad \forall p_i \in \text{Re}(s) \geq 0,\ p_i \text{ a pole of } P(s) \tag{8}$$

$$S(z_j) = 1 \quad \forall z_j \in \text{Re}(s) \geq 0,\ z_j \text{ a zero of } P(s) \tag{9}$$

For convenience, we shall assume poles and zeros in Re$(s) \geq 0$ of $P(s)$ are simple. The theory can be extended to cope with multiple poles and zeros, but is more complex.

C. Mapping Properties of $S(s)$

Suppose $C(s)$ achieves a sensitivity of r. Let $\bar{H}$ denote Re$[s] \geq 0$. Then, clearly

$$S(s) : \quad \bar{H} \to G_1 \triangleq \{s \in \mathbf{C} : \ |s| < r\} \tag{10}$$

Also, suppose $C(s)$ achieves a gain margin pair of a, b. Then for all $k \in [a,b]$, we have $\forall s \in \bar{H}$,

$$1 + kP(s)C(s) \neq 0$$

or

$$P(s)C(s) \neq -1/k, \qquad \forall s \in \bar{H}$$

or

$$S(s) = [1 + P(s)C(s)]^{-1} \neq \frac{k}{k-1}, \quad \forall s \in \bar{H}$$

or

$$S(s): \ \bar{H} \to G_2 \triangleq \mathbf{C} \setminus \left\{ \left(-\infty, -\frac{a}{1-a} \right] \cup \left[\frac{b}{b-1}, \infty \right) \right\} \tag{11}$$

Evidently, if a sensitivity of r is achieved, $S(s)$ satisfies the interpolation conditions (8)-(9), and the mapping condition (10), while if a gain margin pair a, b is achieved, it again satisfies the interpolation conditions (8)-(9), but now the mapping condition (11).

Importantly, and conversely, if we can find an $S(s)$ satisfying the interpolation conditions and a mapping condition, we can then construct $C(s)$ from $S(s)$ to achieve a controller of the desired properties, i.e. one which yields a sensitivity of r or a gain margin pair of a, b. The quantities $r_{\min}$ and $k_{\max}$ are characterized by finding the infimum of the r and supremum of the ratio b/a such that $S(s)$ exists.

To examine the question of simultaneous satisfaction of mapping and interpolation conditions, we shall first look at a special case.

D. Nevanlinna-Pick Theory

The Nevanlinna-Pick theory is concerned with the existence and construction of a function $F(z)$ mapping the closed unit disk $\bar{D} = \{|z| \leq 1\}$ into the open unit disk $D = \{|z| < 1\}$. Let $\beta_1, \ldots, \beta_p$ satisfy $|\beta_i| \leq 1$ and let $\gamma_1, \ldots, \gamma_p$ satisfy $|\gamma_i| < 1$. If $\beta_i = \beta_j^*$, then $\gamma_i = \gamma_j^*$. Ask the question: does there exist

$$F: \quad \bar{D} \to D \tag{12}$$

such that

$$F(\beta_i) = \gamma_i, \qquad i = 1, \ldots, p? \tag{13}$$

Suppose first of all that all β_i are in D, so that $|\beta_i| < 1$. Then the simple answer is that F exists if and only if the following $p \times p$ matrix is positive definite:

$$\Gamma = (\Gamma_{ij})_{p\times p}, \quad \Gamma_{ij} = \frac{1 - \gamma_i \bar{\gamma}_j}{1 - \beta_i \bar{\beta}_j} \tag{14}$$

A variant on the Nevanlinna-Pick problem is to seek

$$\alpha_{\max} \triangleq \sup\{\gamma > 0 : \exists\, F : \bar{D} \to D \text{ for which } F(\beta_i) = \gamma\gamma_i\}$$