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Edited by A.V. Balakrishnan and M. Thoma

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Control Theory for Distributed Parameter Systems and Applications

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F. Kappel, K. Kunisch, W. Schappacher

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PREFACE

This volume comprises the proceedings of the "Conference on Control Theory for Distributed Parameter Systems" held at the Chorherrenstift Vorau (Styria), July 11 - 17, 1982.

Control theory for distributed parameter systems presently is a very thriving part of applied mathematics with problems equally challenging for theoretically and practically minded researchers. The aim of the conference was to stimulate the exchange of ideas and to provide information on recent advances in various directions of research. It was a great pleasure for us to welcome 30 participants coming from 8 different countries. The program of the meeting included 19 lectures. Our thanks go to the lecturers, to all participants and especially to the authors of the contributions contained in this volume.

The conference was made possible by grants from the European Research Office of the US Army (under Grant No. DAJA 45-82-M-0282), from the Amt der Steiermärkischen Landesregierung and from the Bundesministerium für Wissenschaft und Forschung. We greatly appreciate the financial support rendered by these institutions.

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March 1983

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CONTENTS

THE MATHEMATICAL STRUCTURE OF THE FEEDBACK CONTROL PROBLEM FOR LINEAR DISTRIBUTED PARAMETER SYSTEMS WITH FINITE-DIMENSIONAL CONTROLLERS

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1. INTRODUCTION

In previous work (summarized in [1]), we have emphasized finite-dimensional feedback control of (usually) linear infinite-dimensional distributed parameter systems (DPS). This is the only situation of practical interest in engineering applications because the controllers must be implemented by on-line digital computers with finite wordlength and finite memory-access-time. Since our work on DPS control has been motivated by engineering systems, e.g. large aerospace structures [2], Tokomak fusion reactors, and other process control applications, we have been inclined to develop new DPS control theory with some practical constraints. This has been done in the hope that our results would help engineers to see the limitations of what can be accomplished with implementable DPS controllers and would make use of their experience and intuition in the design and operation of complex systems. In other words, we would like to understand the theoretical structure of the problem to see what can be accomplished with finite-dimensional control.

We do not mean to suggest that the above is the only important issue in DPS control; there are, of course, many mathematical problems of interest such as controllability, observability, and stabilizability of linear and nonlinear DPS by both interior and boundary control (e.g.[22,19,10]). However, not much attention has been paid to the finite-dimensional control of DPS; notable exceptions are [11,23,20].

In the past, we have concentrated on model reduction of DPS, i.e. obtaining finite-dimensional approximations of an infinite-dimensional system, and the synthesis of controllers based on these reduced-order models. This has meant that stability analysis must be an intrinsic part of the design because the stability of the closed-loop system, consisting of the actual DPS and a reduced-order controller, is not theoretically guaranteed. In finite-dimensions, when the controller and the plant have the same dimension, the (deterministic) separation principle saves the day (e.g.[17]); however, for DPS, the plant dimension must always be (substantially) larger than the controller dimension.

Of course, model reduction and reduced-order controller design are not new in the engineering community; they are the most natural approach to large-scale system control problems and have been used in various forms (and occasionally disguises) for DPS in mechanical, chemical, aerospace, and electrical engineering applications.

Often the stability analysis has been based entirely on computer simulation (i.e. a few initial situations appear stable; therefore, the system is stable) or has been entirely disregarded. Although the former is at least a step in the right direction, the latter is unconscionable. We have obtained various stability bounds for DPS via singular and regular perturbation techniques (e.g. [1],[3] - [6]).

The real problem is to apply stable and effective control to a complex DPS whose parameters and structure are usually not very well known. Put simply: controlling the heat equation in one space dimension is no big deal; in fact, engineers have been doing much more complicated things for a long time without the help of mathematical control theory (e.g. [21]). However, when the application is, for example, a large flexible structure which is to be constructed and operated in space (where no such things have been done before) where data like the damping and stiffness are poorly known and the vibration modes can only be approximated for a given configuration, then control theory may have something useful (and even comforting) to say. Perturbation methods seem to us to be especially well suited to this type of problem and may be able to give indications of stability and performance that can be used in the design (and redesign) of finite-dimensional controllers for DPS.

In this paper, we will take a somewhat different viewpoint: assuming that a finite-dimensional linear controller is available, what is the most we can expect to accomplish with it on a linear DPS? In [15], Gibson showed that compact perturbations can never produce exponential stability in a contractive, strongly stable system. Therefore, since most practical systems can only introduce feedback through a finite number of actuators, such finite-rank perturbations, being compact, can never produce a margin of stability (i.e. rate of exponential decay) in a DPS which does not already have such a margin initially. This type of result shows, for example, that a flexible structure without inherent damping can never be stabilized with an exponential rate of decay by feedback through a finite number of actuators. Luckily, real structures have some inherent damping; however, that is not the important point. The result of Gibson is exactly the sort of thing that is needed from DPS control theory, it tells us that we must be careful of the way we idealize (model) DPS for the purpose of control: no damping, no hope! Of course, the Gibson result assumes perfect state feedback into the actuators and this would never be available in practice. At best, observations can be made from a finite number of sensors and this data passed through a filter of finite-order to produce the control commands for a finite number of actuators. In the spirit (it not the same mathematical direction) of Gibson's result, we will present results that show what a given finite-dimensional controller is doing: it is asymptotically recreating the projection of the infinite-dimensional DPS state onto a finite-dimensional subspace and this finite-dimensional control is all that is available to modify the DPS by feedback. The finite-dimensional projection created by the controller is not necessarily the one the designer has chosen by model reduction. Hence, our results give a better insight to the structure of the control problem but do not necessarily indicate how to improve the design.

In Section 2, the preliminaries are presented for the class of linear DPS considered here. In Sections 3 and 4, our main results on the structure of the finite-dimensional feedback control problem for DPS are given. Some connections between the structural results of Sections 3 and 4 and our previous analysis of the controller design via model reduction are presented in Section 5. Although boundary control is usually treated as a separate problem from interior control of DPS, many boundary control problems can be converted to equivalent interior control problems; this is developed in Section 6 and it extends the results of the previous sections to a large class of practical boundary control problems for DPS. Our conclusions and recommendations form Section 7.

2. PRELIMINARIES FOR LINEAR DPS

The class of linear distributed parameter systems (DPS) considered here will have the following state space form:

$$\left.\begin{aligned} \frac{\partial v(t)}{\partial t} &= Av(t) + Bf(t); \qquad v(0) = v_o \\ y(t) &= Cv(t) \end{aligned}\right\} \tag{2.1}$$

where the state v(t) is in an infinite-dimensional Hilbert space H with inner product denoted by $(.,.)$ and corresponding norm $||.||$. The operator A is a closed, linear, unbounded differential operator with domain D(A) dense in H, and A generates a C_o-semigroup of bounded operators U(t) on H. The operators B & C have finite ranks M & P, respectively, and f(t), y(t) represent the inputs from M actuators and the outputs from P sensors, respectively. Thus,

$$Bf(t) = \sum_{i=1}^{M} b_i f_i(t) \tag{2.2}$$

and

$$y(t) = [y_1(t),\ldots,y_P(t)]^T \qquad \text{where}$$

$$y_j(t) = (c_j, v(t)); \quad 1 \leq j \leq P \tag{2.3}$$

with b_i and c_j in H.

This is the form of most interior control problems and, as we shall point out in Section 6, it also represents many boundary control problems. When (2.1) - (2.3) is a model of an actual engineering system, the choice of Hilbert space H and the norm $||\cdot||$ are usually dictated by the practical problem (e.g. $||\cdot||$ is the energy norm). However, some care must be used in this choice because, unlike the finite-dimensional case, the state space forms for (2.1) need not be equivalent (even when (A,B,C) is controllable and observable).

From the Hille-Yosida Theorem [12] or [25], the operator A generates a C_o-semigroup U(t) satisfying:

$$||U(t)|| \leq Ke^{-\sigma t} \; ; \quad t \geq 0 \tag{2.4}$$

where $K \geq 1$ and σ is real, when

$$||R(\lambda,A)^n|| \leq \frac{K}{(\lambda+\sigma)^n} \; ; \quad n = 1,2,... \tag{2.5}$$

for all real $\lambda > -\sigma$ in the resolvent set of A. The operator $R(\lambda,A) = (\lambda I-A)^{-1}$ is called the resolvent operator for A, and it is a bounded linear operator for each λ in the resolvent set $\rho(A)$; the spectrum $\sigma(A)$ of A is the set $\sigma(A) = \rho^c(A)$.

When $\sigma > 0$ in (2.4), the semigroup U(t) and the system (2.1) are exponentially stable with stability margin σ; for simplicity, we will say that the operator A is exponentially stable in (2.1), when $\sigma > 0$.

In some cases, A can be shown to satisfy dissipative conditions:

$$\left.\begin{aligned} &(Av,v) \leq -\sigma(v,v) \qquad \sigma > 0 \\ &(A^*v,v) \leq -\sigma(v,v) \end{aligned}\right\} \tag{2.6}$$

for all v in D(A) or D(A*) where A* is the adjoint operator for A. When (2.6) is true and A generates a C_o-semigroup U(t), then U(t) satisfies (2.4) with K = 1 and $\sigma > 0$ ([19] Theo. 2.4 or [25] Theo. 3.2). However, not every exponentially stable system operator A satisfies a dissipativity condition in the original norm; see [25] Theo. 3.2, p. 92.

The generation of a semigroup for (2.1) is the mathematical way of saying that the model (2.1) is well-posed and, hence, represents a physical system. The physical system modeled by (2.1) is the weak (or mild) formulation of the DPS:

$$\left.\begin{aligned} &v(t) = U(t)v_o + \int_0^t U(t-\tau)Bf(\tau)d\tau \\ &y(t) = Cv(t) \end{aligned}\right\} \tag{2.7}$$

There are other types of stability besides exponential stability (in fact, these are all related to the types of convergence of solutions of (2.7) to zero); however, for engineering systems, a margin of stability is essential in order that the system be able to tolerate small parameter variations, noise, and nonlinearities which are ignored in the model (2.1). Of course, a more detailed model, including all these factors, could be developed, in theory, but in practice such detail is poorly known. Consequently, this is one of the trade-offs in controller design: either make a simplified model of the DPS and design a controller which yields exponential stability with as satisfactory a stability-margin as possible or make an extremely detailed DPS model containing all possible factors affecting performance and design a corresponding controller to deal with this system, e.g. make it strongly stable. The latter can lead ultimately to madness since the more closely you look at a system the more detail is revealed. Therefore, even a detailed model of the DPS may not incorporate all the possible factors, hence, such an approach is very likely to lead to an unstable closed-loop system if weaker stability than exponential stability is used in the design criterion. Furthermore, the level of detail of the model can quickly exhaust the available possibilities for controller design to handle such systems. Enough detail must be included so that the controller can be designed to yield a reasonable level of performance from the closed-loop system. Most control engineers would agree with this imprecise statement of what they do; however, it takes quite a bit of experience with specific engineering systems to decide what the words "enough" and "reasonable" mean (and it is not our intention to presume to do this here).

Feedback control for such a DPS as (2.1) should be accomplished with finite-dimensional, discrete - time controllers of the form:

$$\left.\begin{aligned} f(k) &= L_{11}\, y(k) + L_{12}\, z(k) \\ z(k+1) &= L_{21}\, y(k) + L_{22}\, z(k) \end{aligned}\right\} \qquad (2.8)$$

where $z(k)$ belongs to R^{α}. Such controllers can be implemented with on-line digital computers whose memory-access-time and memory capacity is related to the controller dimension α. Although the discrete-time acpect of the controller is not a trivial issue (e.g. [18]), for convenience here, we shall deal only with the continuous-time version of (2.8); therefore, the finite-dimensional linear controller will have the form:

$$f(t) = L_{11}\, y(t) + L_{12}\, z(t) \qquad (2.9a)$$

$$\dot{z}(t) = L_{21}\, y(t) + L_{22}\, z(t) = Fz(t) + Ky(t) + Ef(t) \qquad (2.9b)$$

where $z(t)$ belongs to R^{α}.

The matrices F, K, and E are related to L_{21} and L_{22} by:

$$L_{21} = K + EL_{11} \quad (2.10a)$$

$$L_{22} = F + EL_{12} \quad . \quad (2.10b)$$

The controller dynamics (2.9b) provide a filtering effect on the sensor data; these dynamics can be very helpful but, as we shall point out in Secs. 3 and 4, they cannot perform miracles (such as reconstructing the full DPS state). Special cases of (2.9) are static (or output) feedback:

$$L_{12} = 0, \quad L_{21} = 0, \quad L_{22} = 0 \quad (2.11)$$

where no dynamics are present in the controller, and full dynamic (or α-dimensional) feedback:

$$L_{11} = 0 \quad (2.12)$$

where no direct feedthrough is present and all sensor measurements are passed through the controller dynamics.

3. FINITE-DIMENSIONAL OBSERVERS FOR DPS

In this section we will examine what can be accomplished with a finite-dimensional observer of the form:

$$q(t) = Q_{11}\, y(t) + Q_{12}\, z(t) \quad (3.1a)$$

$$\dot{z}(t) = Fz(t) + Ky(t) + Ef(t) \quad (3.1b)$$

where $z(t)$ belongs to R^{α} with $\alpha < \infty$. If this observer is used to estimate the state of the infinite-dimensional DPS (2.1), then at best it can asymptotically reconstruct only the projection of the DPS state onto a finite-dimensional subspace. This is made precise by the following result:

Theorem 1. Assume $f(t)$ in (2.1) is continuously differentiable.

If (a) F is stable (i.e. all eigenvalues of F are in the open left-half of the complex plane),

(b) there exists a bounded linear operator $T: H \to R^{\alpha}$ such that

$$(FT - TA + DC)v = 0 \quad (3.2)$$

for all v in $D(A)$, and

(c) E is chosen so that $E = TB$ (3.3)

then $z(t)$ in (3.1b) is given by

$$z(t) = Tv(t) + e(t) \quad (3.4)$$

where

$$\left.\begin{aligned} \dot{e}(t) &= Fe(t) \\ e(0) &= z_o - Tv_o . \end{aligned}\right\} \qquad (3.5)$$

Furthermore, there exists a pair of nontrivial subspaces $\tilde{H}_N$ and $\tilde{H}_R$ in H such that:

$$H = \tilde{H}_N \oplus \tilde{H}_R \qquad (3.6)$$

$$\dim \tilde{H}_N \equiv N \leq P + \alpha \qquad (3.7)$$

$$\lim_{t\to\infty} [q(t) - \tilde{P}_N v(t)] = 0 \qquad (3.8a)$$

$$\lim_{t\to\infty} [q(t) - v(t)] = - \lim_{t\to\infty} \tilde{P}_R v(t) \qquad (3.8b)$$

where $\tilde{P}_N$ and $\tilde{P}_R$ are the projections onto $\tilde{H}_N$ and $\tilde{H}_R$ defined by (3.6). In fact, these subspaces are given by

$$\tilde{H}_N = \tilde{N}(T)^{\perp}$$

$$\tilde{H}_R = \tilde{N}(T) \equiv \{v \in D(\tilde{T}) \mid \tilde{T}v = 0\}$$

where $\tilde{T} \equiv \begin{bmatrix} C \\ T \end{bmatrix}: H \to R^{P+\alpha}$.

In order to prove Theo. 1, we will need the following result about pseudo-inverses of operators:

<u>Theorem 2.</u> Given a bounded linear operator $T: H_1 \to H_2$ with H_i Hilbert spaces. If T is onto (surjective), then the pseudo-inverse $T^{\#}$ of T defined by

$$T^{\#}: H_2 \to H_1 \text{ with}$$

$$T^{\#}T = \tilde{P}_N \qquad (3.9)$$

where $\tilde{P}_N$ is orthogonal projection onto N(T) has the following properties:

(a) $T^{\#}$ is well defined and linear on H_2

(b) $T\,T^{\#}T = T$ (3.10)

(c) $T^{\#}$ is a bounded operator

(d) If $\dim H_2 < \infty$, then $\dim N(T)^{\perp} = \dim H_2$.

The proofs of Theos. 1 and 2 appear in Appendix I. Although properties (a) and (c) of Theo. 1 are easy to guarantee by the choice of the observer parameters F and E, property (b) may seem to be more formidable. However, the following result suggests otherwise: