Circular and Hyperbolic Sines and Cosines

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NATIONAL BUREAU OF STANDARDS • A. V. Astin, *Director*

Tables of Circular and Hyperbolic Sines and Cosines

for Radian Arguments

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(A reissue of Mathematical Table 3, with corrections)

Preface

The present volume, a third edition of Tables of Circular and Hyperbolic Sines and Cosines, was prepared by the New York Mathematical Tables Project, the predecessor of the present Computation Laboratory of the National Bureau of Standards.

The Mathematical Tables Project was organized in January 1938 under NBS scientific sponsorship, and later, in March 1943, the actual operation of the project was taken over by the Bureau. When the NBS National Applied Mathematics Laboratories were established in July 1947, the Mathematical Tables Project became identified with the unit of these laboratories known as the Computation Laboratory. The main offices of the Computation Laboratory have always been in Washington, D. C., but the work was continued in New York for some time after July 1947.

The computations of the present volume were carried out in New York City during 1938–9 under the scientific sponsorship of the National Bureau of Standards, and were published in 1939 as a report of the Federal Works Agency, Work Projects Administration for the City of New York. This publication became known as Mathematical Table MT3, and was reissued in a second edition in 1949 by the National Bureau of Standards through the U. S. Government Printing Office. To meet a continuing demand, the tables are herewith issued for the third time. To unify the mathematical table publications of the National Bureau of Standards, the volume is being incorporated in the Applied Mathematics Series.

The technical staff at the time of preparation of these tables included Arnold N. Lowan, technical director, Milton Abramowitz, Gertrude Blanch, William Kaufman, Frederick G. King, Jack Laderman, and Matilda Persily.

Errors reported in former editions have been corrected in the present volume. The following is a list of all changes in the tabular material since the tables first appeared.

Page	Argument	Function	Correction
59	0. 2905	cos x	For . 9581 . . . read 0. 9581 . . .
121	0. 6010	cos x	For . 8247 . . . read 0. 8247 . . .
223	1. 1115	sinh x	*For . 3549 . . . read 1. 3549 . . .
223	1. 1115	cosh x	*For . 6839 . . . read 1. 6839 . . .
235	1. 1740	cosh x	For . 7720 . . . read 1. 7720 . . .
337	1. 6848	Argument	*For 1. 6868 read 1. 6848
358	1. 7850	Argument	*For . 7850 read 1. 7850
364	1. 8190	sinh x	*For 2. 0017 . . . read 3. 0017 . . .

The corrections indicated by asterisks represent errors (or inconsistencies in format) that were corrected in the second edition; the other three items represent inconsistencies in format in both the first and second editions.

Supplementary Table III has been extended in this edition. This table expresses degrees, minutes, and seconds in terms of radians to 10 decimal places. It also gives radians from 10^{-10} to 10^{2} in terms of degrees, minutes, and seconds, to an accuracy of 0.000005 second.

A. V. Astin, *Director.*

May 20, 1953.

Foreword[1]

The trigonometries based on the circular and hyperbolic functions are a fundamental part of pure and applied mathematics.

Even in their simplest forms, the circular and hyperbolic functions oftentimes represent natural phenomena as, for example, the periodic flow of solar heat at a given depth in the earth is represented approximately by a single sine term; and the curve assumed by a cable suspended between two points, the catenary, the well-known curve of the suspension bridge, is represented by hyperbolic sines and cosines. The more complicated relations are represented by functions of functions in which functions like $\sin x$ and $\sinh x$ appear as factors, exponents, or individual terms. A simple illustration is the product of an exponential and a sine which represents the temperatures resulting from the annual flow of solar heat into and out of the earth. The preceding illustration is merely the simplest application of the Fourier series by means of which natural phenomena of the greatest complexity in many branches of science, especially in the mathematical theory of heat conduction, can be represented to almost any desired degree of precision by the use of circular and hyperbolic functions. These functions are used very extensively, also, in the mathematical theories of elasticity, electricity, hydrodynamics, mathematical statistics, and other subjects.

Considered from the standpoint of economy of human energy, the advantage of having reliable and efficient tables at one's disposal can hardly be overestimated. In fact, the amount of energy lost in using inadequate tables exceeds by many times the total quantity of energy required to construct the necessary tables by an organization of mathematicians and computers.

Fortunately, once a function is accurately computed and properly tabulated, it belongs to the present and future ages; and, furthermore, inasmuch as tables of the required accuracy and convenience frequently lead to new applications, no one can foresee the benefits that will gradually accrue as the result of constructing a mathematical table which is fundamental in character.

C. E. VAN ORSTRAND.

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[1] Approved for publication by the Director of the U. S. Geological Survey. Reprinted from the first edition in 1930.

Introduction

The circular and hyperbolic functions tabulated in this volume may be defined by the infinite series

$$\sin x = U_1(x) - U_3(x) + U_5(x) - U_7(x) + \ldots$$

$$\cos x = U_0(x) - U_2(x) + U_4(x) - U_6(x) + \ldots$$

$$\sinh x = U_1(x) + U_3(x) + U_5(x) + U_7(x) + \ldots$$

$$\cosh x = U_0(x) + U_2(x) + U_4(x) + U_6(x) + \ldots,$$

where

$$U_n(x) = x^n/n!.$$

The starting point in the computation of the functions under consideration was the computation of $U_n(x)$. The values [1] of this function for the key arguments $x_0 = 0.01, 0.02, \ldots$ were computed to fifteen decimal places for $n = 0, 1, 2, \ldots, r$, where r is such that $U_r(x_0)$ is equal to or smaller than one unit in the fifteenth place. The values of $U_n(x)$ for all other arguments were computed with the aid of the recurrence formula

$$U_n(x \pm h) = U_n(x) \pm U_1(h)U_{n-1}(x) + U_2(h)U_{n-2}(x) \pm \ldots + (\pm 1)^n U_n(h),$$

where $h = 0.0001$.

Starting with the given key value $U_n(x_0)$, a first application of the recurrence formula yielded the values of $U_n(x_0+h)$ and $U_n(x_0-h)$. Successive application of the recurrence formula generated the values of U_n for the successive pairs of arguments x_0+2h and x_0; x_0+3h and x_0+h, and so forth. The method was therefore stepwise and self-checking.

A simple analysis leads to the conclusion that the one-hundredth value computed by the recurrence formula must agree with the precomputed value for the next higher key argument $x_0+0.01$ to within three units in the thirteenth place. This served as an additional check on the accuracy of the values of $U_n(x)$.

From the basic terms $U_n(x)$, the following sums were computed

$$S_0 = U_0 + U_4 + U_8 + \ldots \qquad S_1 = U_1 + U_5 + U_9 + \ldots$$

$$S_2 = U_2 + U_6 + U_{10} + \ldots \qquad S_3 = U_3 + U_7 + U_{11} + \ldots$$

$$e^x = S_0 + S_1 + S_2 + S_3 \qquad e^{-x} = S_0 - S_1 + S_2 - S_3.$$

The functions to be tabulated were obtained from the relations:

$$\sin x = S_1 - S_3, \qquad \cos x = S_0 - S_2, \qquad \sinh x = S_1 + S_3, \qquad \cosh x = S_0 + S_2.$$

The values of e^x and e^{-x} were compared with those given in our comprehensive tables of these functions,[2] computed by a different method. The agreement of the values served as a check on the accuracy of the functions sinh x and cosh x. These functions were further tested by comparing the sums of groups of ten entries with the precomputed sums obtained in the process of checking the table of exponential functions.

[1] These values are available in "Table of $x^n/n!$", contained in National Bureau of Standards *Tables of functions and of zeros of functions*, Applied Mathematics Series 37 (in press, U. S. Government Printing Office, Washington, D. C.). The table is to 13D, and the range is $x = 0(.01)2$ with n taken up to the point where $x^n/n!$ vanishes in the last place.

[2] National Bureau of Standards, Tables of the exponential function e^x, (Applied Mathematics Series 14, 3d ed., 1951, U. S. Government Printing Office, Washington, D. C.).

The values of sin x and cos x at intervals of 0.001 were compared with the values given in Van Orstrand's table [3] and in our own tables of circular functions, computed by an independent method.[4]

The values of the circular and hyperbolic functions which had undergone the tests just described were rounded to nine decimal places and typed from the worksheets onto a form from which this volume was reproduced by a photo-offset process.

The typewritten manuscript was carefully proofread against the worksheets and then subjected to a differencing test designed to detect errors that may have been overlooked in previous tests. Finally, the manuscript was subjected to an additional proofreading against the worksheets with the primary object of checking the roundings. It is believed that the rounding error does not exceed 0.51 units in the ninth decimal place.

Direct Interpolation

The following formulas based on Taylor's expansion will yield values correct to within one unit in the ninth decimal place.

$$\sin(x \pm pw) = \sin x \pm pw \cos x - \tfrac{1}{2}p^2w^2 \sin x$$

$$\cos(x \pm pw) = \cos x \mp pw \sin x - \tfrac{1}{2}p^2w^2 \cos x$$

$$\sinh(x \pm pw) = \sinh x \pm pw \cosh x + \tfrac{1}{2}p^2w^2 \sinh x$$

$$\cosh(x \pm pw) = \cosh x \pm pw \sinh x + \tfrac{1}{2}p^2w^2 \cosh x.$$

In the above formulas $w=0.0001$. If $f(x \pm pw)$ denotes any of the above functions and $f(x)$ the tabulated value corresponding to the argument x which is nearest to $x \pm pw$, the value of p will be no greater than $\frac{1}{2}$, and the last term in the above formulas, involving p^2w^2, will be less than 1.3×10^{-9} and 4.8×10^{-9} for the circular and hyperbolic functions, respectively. Allowing one unit for rounding, the error of linear interpolation will therefore be less than 2.3×10^{-9} in the circular functions and less than 5.8×10^{-9} in the hyperbolic functions.

EXAMPLE: Let it be required to find sinh 1.7576 60789.
SOLUTION:

$x=1.7577$	$\cosh x=$	2.9857 63
$p=$ .39211	$\frac{1}{2}p^2w^2=$	.77 × 10^{-9}
	$\sinh x=$	2.8133 21564
	$-pw \cosh x=-$	.0001 170748
		2.8132 044892
	$\frac{1}{2}p^2w^2 \sinh x=$	22
	sinh 1.75766 0789=	2.8132 04491

Inverse Interpolation

Let $f(x)=X$ denote any of the circular or hyperbolic functions tabulated, and let it be required to find the value of the inverse function $x=F(X)$ when X is given. Since X is periodic, x will be a multiple-valued function; it will suffice to confine ourselves to the principal solution and to consider the case when X is positive.

Generally X will not be a tabulated value, but will lie between two tabulated values X_1 and X_1+d, corresponding to two consecutive arguments x_1 and $x_1+0.0001$. Hence the value of x may usually be

[3] Tables of the exponential function and of the circular sine and cosine to radian argument, vol. XIV, Fifth Memoir, National Academy of Sciences (1921).
[4] National Bureau of Standards, Table of sines and cosines for radian arguments (Applied Mathematics Series, 2d ed. in press. U. S. Government Printing Office, Washington, D. C.)

given by inspection to four decimal places, with an error less than $\frac{1}{2}\cdot 10^{-4}$. By interpolation, it is possible to express $x=F(X)$ more accurately; the maximum attainable accuracy varies with X.

In the table, the value of x is assumed exact and the tabulated value of $f(x)$ is given to nine decimal places. In inverse interpolation, it is convenient to regard $f(x)=X$ as exact, in which case x is inexact. The relation $\Delta F(X)=F'(X)\Delta X$ affords a close approximation to the error in x if X is regarded as exact. If we replace ΔX by the upper bound of error in the tabulated values, namely $\frac{1}{2}\cdot 10^{-9}$, we shall have a close estimate of the upper bound of the error in $x=F(X)$.

The derivatives of the inverse functions are

$$\frac{d(\sin^{-1}X)}{dX}=\frac{1}{\sqrt{1-X^2}}, \qquad \frac{d(\cos^{-1}X)}{dX}=\frac{-1}{\sqrt{1-X^2}},$$

$$\frac{d(\sinh^{-1}X)}{dX}=\frac{1}{\sqrt{1+X^2}}, \qquad \frac{d(\cosh^{-1}X)}{dX}=\frac{1}{\sqrt{X^2-1}}.$$

An examination of these expressions shows that $x=\sinh^{-1}X$ can be considered correct to $\frac{1}{2}\cdot 10^{-9}$ for all tabulated arguments. In the case of the remaining three functions, $F(X)$ is considerably less accurate when X is close to unity. If we regard the entries x as exact, the following table gives the number of decimal places n which may be given in the inverse function $x=F(X)$ with error less than one unit in the nth place:

$\lvert 1-X\rvert$	n
5.0×10^{-9} to 1.2×10^{-7}	5
1.2×10^{-7} to 1.2×10^{-5}	6
1.2×10^{-5} to 1.2×10^{-3}	7
1.2×10^{-3} to 0.12	8
0.12 to 1.0	9

($X=\sin x$, $\cos x$, or $\cosh x$)

For example, the tabulated value of cos 0.01 is $1-0.00005$. But it may be readily verified that $\cos^{-1}(1-.00005)=0.01000\ 00417$; thus, in this case only seven decimal places may be regarded as correct in the value of x. This is in agreement with the value of n given in the above table.

Since an interpolated value is in general no more accurate than the tabulated values between which interpolation is performed, the values of n given above indicate the maximum number of decimal places in the values of $\sin^{-1}X$, $\cos^{-1}X$, $\cosh^{-1}X$, which may be regarded as correct.

The interpolation formulas given below will yield the maximum attainable accuracy if inverse interpolation is performed for any of the functions given in this volume.

Let X be a value lying between two tabulated entries A and $A+d$, corresponding to two consecutive arguments x_0 and $x_0\pm w$. If $X-A=a$, and $a/d=p$, the formulas are, respectively

$$\sinh^{-1}X=x_0+pw+K \qquad \cosh^{-1}X=x_0+pw+K$$
$$\sin^{-1}X=x_0+pw-K \qquad \cos^{-1}X=x_0-pw+K,$$

where $K=\frac{1}{2}p(1-p)w^2X/d$.

It may be verified that the value of K in the above formulas has an upper bound of 2.6×10^{-n} (n as previously defined) in the case of the functions $\cosh^{-1}X$, $\sin^{-1}X$, or $\cos^{-1}X$, and is always less than 1.3×10^{-9} in the case of $\sinh^{-1}X$. For all the functions under consideration, linear interpolation

will be in error by less than 4×10^{-8} after rounding. The formulas given above will therefore yield values only slightly more accurate than those obtained by linear interpolation.

EXAMPLE: Let it be required to find $\sinh^{-1}$ 2.8132 04491

SOLUTION:

$$A+d=\sinh(x_0+w)=2.8133\ 21564 \qquad d=298562$$

$$A=\sinh x_0=2.8130\ 23002 \qquad a=181489$$

$$X=2.8132\ 04491$$

$$p=a/d=0.6078\ 77 \qquad w^3X/d=9.4\times10^{-9}$$

$$x_0=1.7576$$

$$pw=0.0000\ 607877$$

$$K=\tfrac{1}{2}p(1-p)w^3X/d= \qquad 11$$

$$\sinh^{-1} 2.8132\ 04491=1.7576\ 60789$$

As a check on the calculations, the value of sinh 1.7576 60789 may be computed with the aid of the formulas for direct interpolation. This yields $X=2.8132\ 04491$ and verifies the calculations in the example in the section on direct interpolation.

When $|1-X|$ is less than 0.12 and $x=F(X)$ is any of the functions $\sin^{-1}X$, $\cos^{-1}X$, or $\cosh^{-1}X$ whose value is required, it is, of course, possible to find $\sqrt{|1-X^2|}$, and to interpolate in the corresponding range of $\cos x$, $\sin x$, or $\sinh x$, where inverse interpolation is more accurate.

A. N. LOWAN.

TABLE I

Contents

x	sin x	cos x	sinh x	cosh x
0.0000	0.0000 00000	1.0000 00000	0.0000 00000	1.0000 00000
.0001	.0001 00000	0.9999 99995	.0001 00000	.0000 00005
.0002	.0002 00000	.9999 99980	.0002 00000	.0000 00020
.0003	.0003 00000	.9999 99955	.0003 00000	.0000 00045
.0004	.0004 00000	.9999 99920	.0004 00000	.0000 00080
0.0005	0.0005 00000	0.9999 99875	0.0005 00000	1.0000 00125
.0006	.0006 00000	.9999 99820	.0006 00000	.0000 00180
.0007	.0007 00000	.9999 99755	.0007 00000	.0000 00245
.0008	.0008 00000	.9999 99680	.0008 00000	.0000 00320
.0009	.0009 00000	.9999 99595	.0009 00000	.0000 00405
0.0010	0.0010 00000	0.9999 99500	0.0010 00000	1.0000 00500
.0011	.0011 00000	.9999 99395	.0011 00000	.0000 00605
.0012	.0012 00000	.9999 99280	.0012 00000	.0000 00720
.0013	.0013 00000	.9999 99155	.0013 00000	.0000 00845
.0014	.0014 00000	.9999 99020	.0014 00000	.0000 00980
0.0015	0.0014 99999	0.9999 98875	0.0015 00001	1.0000 01125
.0016	.0015 99999	.9999 98720	.0016 00001	.0000 01280
.0017	.0016 99999	.9999 98555	.0017 00001	.0000 01445
.0018	.0017 99999	.9999 98380	.0018 00001	.0000 01620
.0019	.0018 99999	.9999 98195	.0019 00001	.0000 01805
0.0020	0.0019 99999	0.9999 98000	0.0020 00001	1.0000 02000
.0021	.0020 99998	.9999 97795	.0021 00002	.0000 02205
.0022	.0021 99998	.9999 97580	.0022 00002	.0000 02420
.0023	.0022 99998	.9999 97355	.0023 00002	.0000 02645
.0024	.0023 99998	.9999 97120	.0024 00002	.0000 02880
0.0025	0.0024 99997	0.9999 96875	0.0025 00003	1.0000 03125
.0026	.0025 99997	.9999 96620	.0026 00003	.0000 03380
.0027	.0026 99997	.9999 96355	.0027 00003	.0000 03645
.0028	.0027 99996	.9999 96080	.0028 00004	.0000 03920
.0029	.0028 99996	.9999 95795	.0029 00004	.0000 04205
0.0030	0.0029 99996	0.9999 95500	0.0030 00005	1.0000 04500
.0031	.0030 99995	.9999 95195	.0031 00005	.0000 04805
.0032	.0031 99995	.9999 94880	.0032 00005	.0000 05120
.0033	.0032 99994	.9999 94555	.0033 00006	.0000 05445
.0034	.0033 99993	.9999 94220	.0034 00007	.0000 05780
0.0035	0.0034 99993	0.9999 93875	0.0035 00007	1.0000 06125
.0036	.0035 99992	.9999 93520	.0036 00008	.0000 06480
.0037	.0036 99992	.9999 93155	.0037 00008	.0000 06845
.0038	.0037 99991	.9999 92780	.0038 00009	.0000 07220
.0039	.0038 99990	.9999 92395	.0039 00010	.0000 07605
0.0040	0.0039 99989	0.9999 92000	0.0040 00011	1.0000 08000
.0041	.0040 99989	.9999 91595	.0041 00011	.0000 08405
.0042	.0041 99988	.9999 91180	.0042 00012	.0000 08820
.0043	.0042 99987	.9999 90755	.0043 00013	.0000 09245
.0044	.0043 99986	.9999 90320	.0044 00014	.0000 09680
0.0045	0.0044 99985	0.9999 89875	0.0045 00015	1.0000 10125
.0046	.0045 99984	.9999 89420	.0046 00016	.0000 10580
.0047	.0046 99983	.9999 88955	.0047 00017	.0000 11045
.0048	.0047 99982	.9999 88480	.0048 00018	.0000 11520
.0049	.0048 99980	.9999 87995	.0049 00020	.0000 12005

x	sin x	cos x	sinh x	cosh x
0.0050	0.0049 99979	0.9999 87500	0.0050 00021	1.0000 12500
.0051	.0050 99978	.9999 86995	.0051 00022	.0000 13005
.0052	.0051 99977	.9999 86480	.0052 00023	.0000 13520
.0053	.0052 99975	.9999 85955	.0053 00025	.0000 14045
.0054	.0053 99974	.9999 85420	.0054 00026	.0000 14580
0.0055	0.0054 99972	0.9999 84875	0.0055 00028	1.0000 15125
.0056	.0055 99971	.9999 84320	.0056 00029	.0000 15680
.0057	.0056 99969	.9999 83755	.0057 00031	.0000 16245
.0058	.0057 99967	.9999 83180	.0058 00033	.0000 16820
.0059	.0058 99966	.9999 82595	.0059 00034	.0000 17405
0.0060	0.0059 99964	0.9999 82000	0.0060 00036	1.0000 18000
.0061	.0060 99962	.9999 81395	.0061 00038	.0000 18605
.0062	.0061 99960	.9999 80780	.0062 00040	.0000 19220
.0063	.0062 99958	.9999 80155	.0063 00042	.0000 19845
.0064	.0063 99956	.9999 79520	.0064 00044	.0000 20480
0.0065	0.0064 99954	0.9999 78875	0.0065 00046	1.0000 21125
.0066	.0065 99952	.9999 78220	.0066 00048	.0000 21780
.0067	.0066 99950	.9999 77555	.0067 00050	.0000 22445
.0068	.0067 99948	.9999 76880	.0068 00052	.0000 23120
.0069	.0068 99945	.9999 76195	.0069 00055	.0000 23805
0.0070	0.0069 99943	0.9999 75500	0.0070 00057	1.0000 24500
.0071	.0070 99940	.9999 74795	.0071 00060	.0000 25205
.0072	.0071 99938	.9999 74080	.0072 00062	.0000 25920
.0073	.0072 99935	.9999 73355	.0073 00065	.0000 26645
.0074	.0073 99932	.9999 72620	.0074 00068	.0000 27380
0.0075	0.0074 99930	0.9999 71875	0.0075 00070	1.0000 28125
.0076	.0075 99927	.9999 71120	.0076 00073	.0000 28880
.0077	.0076 99924	.9999 70355	.0077 00076	.0000 29645
.0078	.0077 99921	.9999 69580	.0078 00079	.0000 30420
.0079	.0078 99918	.9999 68795	.0079 00082	.0000 31205
0.0080	0.0079 99915	0.9999 68000	0.0080 00085	1.0000 32000
.0081	.0080 99911	.9999 67195	.0081 00089	.0000 32805
.0082	.0081 99908	.9999 66380	.0082 00092	.0000 33620
.0083	.0082 99905	.9999 65555	.0083 00095	.0000 34445
.0084	.0083 99901	.9999 64720	.0084 00099	.0000 35280
0.0085	0.0084 99898	0.9999 63875	0.0085 00102	1.0000 36125
.0086	.0085 99894	.9999 63020	.0086 00106	.0000 36980
.0087	.0086 99890	.9999 62155	.0087 00110	.0000 37845
.0088	.0087 99886	.9999 61280	.0088 00114	.0000 38720
.0089	.0088 99883	.9999 60395	.0089 00117	.0000 39605
0.0090	0.0089 99879	0.9999 59500	0.0090 00122	1.0000 40500
.0091	.0090 99874	.9999 58595	.0091 00126	.0000 41405
.0092	.0091 99870	.9999 57680	.0092 00130	.0000 42320
.0093	.0092 99866	.9999 56755	.0093 00134	.0000 43245
.0094	.0093 99862	.9999 55820	.0094 00138	.0000 44180
0.0095	0.0094 99857	0.9999 54875	0.0095 00143	1.0000 45125
.0096	.0095 99853	.9999 53920	.0096 00147	.0000 46080
.0097	.0096 99848	.9999 52955	.0097 00152	.0000 47045
.0098	.0097 99843	.9999 51980	.0098 00157	.0000 48020
.0099	.0098 99838	.9999 50995	.0099 00162	.0000 49005

x	sin x	cos x	sinh x	cosh x
0.0100	0.0099 99833	0.9999 50000	0.0100 00167	1.0000 50000
.0101	.0100 99828	.9999 48995	.0101 00172	.0000 51005
.0102	.0101 99823	.9999 47980	.0102 00177	.0000 52020
.0103	.0102 99818	.9999 46955	.0103 00182	.0000 53045
.0104	.0103 99813	.9999 45920	.0104 00187	.0000 54080
0.0105	0.0104 99807	0.9999 44876	0.0105 00193	1.0000 55126
.0106	.0105 99801	.9999 43821	.0106 00199	.0000 56181
.0107	.0106 99796	.9999 42756	.0107 00204	.0000 57246
.0108	.0107 99790	.9999 41681	.0108 00210	.0000 58321
.0109	.0108 99784	.9999 40596	.0109 00216	.0000 59406
0.0110	0.0109 99778	0.9999 39501	0.0110 00222	1.0000 60501
.0111	.0110 99772	.9999 38396	.0111 00228	.0000 61606
.0112	.0111 99766	.9999 37281	.0112 00234	.0000 62721
.0113	.0112 99760	.9999 36156	.0113 00240	.0000 63846
.0114	.0113 99753	.9999 35021	.0114 00247	.0000 64981
0.0115	0.0114 99747	0.9999 33876	0.0115 00253	1.0000 66126
.0116	.0115 99740	.9999 32721	.0116 00260	.0000 67281
.0117	.0116 99733	.9999 31556	.0117 00267	.0000 68446
.0118	.0117 99726	.9999 30381	.0118 00274	.0000 69621
.0119	.0118 99719	.9999 29196	.0119 00281	.0000 70806
0.0120	0.0119 99712	0.9999 28001	0.0120 00288	1.0000 72001
.0121	.0120 99705	.9999 26796	.0121 00295	.0000 73206
.0122	.0121 99697	.9999 25581	.0122 00303	.0000 74421
.0123	.0122 99690	.9999 24356	.0123 00310	.0000 75646
.0124	.0123 99682	.9999 23121	.0124 00318	.0000 76881
0.0125	0.0124 99674	0.9999 21876	0.0125 00326	1.0000 78126
.0126	.0125 99667	.9999 20621	.0126 00333	.0000 79381
.0127	.0126 99659	.9999 19356	.0127 00341	.0000 80646
.0128	.0127 99650	.9999 18081	.0128 00350	.0000 81921
.0129	.0128 99642	.9999 16796	.0129 00358	.0000 83206
0.0130	0.0129 99634	0.9999 15501	0.0130 00366	1.0000 84501
.0131	.0130 99625	.9999 14196	.0131 00375	.0000 85806
.0132	.0131 99617	.9999 12881	.0132 00383	.0000 87121
.0133	.0132 99608	.9999 11556	.0133 00392	.0000 88446
.0134	.0133 99599	.9999 10221	.0134 00401	.0000 89781
0.0135	0.0134 99590	0.9999 08876	0.0135 00410	1.0000 91126
.0136	.0135 99581	.9999 07521	.0136 00419	.0000 92481
.0137	.0136 99571	.9999 06156	.0137 00429	.0000 93846
.0138	.0137 99562	.9999 04782	.0138 00438	.0000 95222
.0139	.0138 99552	.9999 03397	.0139 00448	.0000 96607
0.0140	0.0139 99543	0.9999 02002	0.0140 00457	1.0000 98002
.0141	.0140 99533	.9999 00597	.0141 00467	.0000 99407
.0142	.0141 99523	.9998 99182	.0142 00477	.0001 00822
.0143	.0142 99513	.9998 97757	.0143 00487	.0001 02247
.0144	.0143 99502	.9998 96322	.0144 00498	.0001 03682
0.0145	0.0144 99492	0.9998 94877	0.0145 00508	1.0001 05127
.0146	.0145 99481	.9998 93422	.0146 00519	.0001 06582
.0147	.0146 99471	.9998 91957	.0147 00529	.0001 08047
.0148	.0147 99460	.9998 90482	.0148 00540	.0001 09522
.0149	.0148 99449	.9998 88997	.0149 00551	.0001 11007

x	sin x	cos x	sinh x	cosh x
0.0150	0.0149 99438	0.9998 87502	0.0150 00563	1.0001 12502
.0151	.0150 99426	.9998 85997	.0151 00574	.0001 14007
.0152	.0151 99415	.9998 84482	.0152 00585	.0001 15522
.0153	.0152 99403	.9998 82957	.0153 00597	.0001 17047
.0154	.0153 99391	.9998 81422	.0154 00609	.0001 18582
0.0155	0.0154 99379	0.9998 79877	0.0155 00621	1.0001 20127
.0156	.0155 99367	.9998 78322	.0156 00633	.0001 21682
.0157	.0156 99355	.9998 76758	.0157 00645	.0001 23248
.0158	.0157 99343	.9998 75183	.0158 00657	.0001 24823
.0159	.0158 99330	.9998 73598	.0159 00670	.0001 26408
0.0160	0.0159 99317	0.9998 72003	0.0160 00683	1.0001 28003
.0161	.0160 99304	.9998 70398	.0161 00696	.0001 29608
.0162	.0161 99291	.9998 68783	.0162 00709	.0001 31223
.0163	.0162 99278	.9998 67158	.0163 00722	.0001 32848
.0164	.0163 99265	.9998 65523	.0164 00735	.0001 34483
0.0165	0.0164 99251	0.9998 63878	0.0165 00749	1.0001 36128
.0166	.0165 99238	.9998 62223	.0166 00762	.0001 37783
.0167	.0166 99224	.9998 60558	.0167 00776	.0001 39448
.0168	.0167 99210	.9998 58883	.0168 00790	.0001 41123
.0169	.0168 99196	.9998 57198	.0169 00804	.0001 42808
0.0170	0.0169 99181	0.9998 55503	0.0170 00819	1.0001 44503
.0171	.0170 99167	.9998 53799	.0171 00833	.0001 46209
.0172	.0171 99152	.9998 52084	.0172 00848	.0001 47924
.0173	.0172 99137	.9998 50359	.0173 00863	.0001 49649
.0174	.0173 99122	.9998 48624	.0174 00878	.0001 51384
0.0175	0.0174 99107	0.9998 46879	0.0175 00893	1.0001 53129
.0176	.0175 99091	.9998 45124	.0176 00909	.0001 54884
.0177	.0176 99076	.9998 43359	.0177 00924	.0001 56649
.0178	.0177 99060	.9998 41584	.0178 00940	.0001 58424
.0179	.0178 99044	.9998 39799	.0179 00956	.0001 60209
0.0180	0.0179 99028	0.9998 38004	0.0180 00972	1.0001 62004
.0181	.0180 99012	.9998 36199	.0181 00988	.0001 63809
.0182	.0181 98995	.9998 34385	.0182 01005	.0001 65625
.0183	.0182 98979	.9998 32560	.0183 01021	.0001 67450
.0184	.0183 98962	.9998 30725	.0184 01038	.0001 69285
0.0185	0.0184 98945	0.9998 28880	0.0185 01055	1.0001 71130
.0186	.0185 98928	.9998 27025	.0186 01072	.0001 72985
.0187	.0186 98910	.9998 25160	.0187 01090	.0001 74850
.0188	.0187 98893	.9998 23285	.0188 01107	.0001 76725
.0189	.0188 98875	.9998 21400	.0189 01125	.0001 78610
0.0190	0.0189 98857	0.9998 19505	0.0190 01143	1.0001 80505
.0191	.0190 98839	.9998 17601	.0191 01161	.0001 82411
.0192	.0191 98820	.9998 15686	.0192 01180	.0001 84326
.0193	.0192 98802	.9998 13761	.0193 01198	.0001 86251
.0194	.0193 98783	.9998 11826	.0194 01217	.0001 88186
0.0195	0.0194 98764	0.9998 09881	0.0195 01236	1.0001 90131
.0196	.0195 98745	.9998 07926	.0196 01255	.0001 92086
.0197	.0196 98726	.9998 05961	.0197 01274	.0001 94051
.0198	.0197 98706	.9998 03986	.0198 01294	.0001 96026
.0199	.0198 98687	.9998 02002	.0199 01313	.0001 98012

x	sin x	cos x	sinh x	cosh x
0.0200	0.0199 98667	0.9998 00007	0.0200 01333	1.0002 00007
.0201	.0200 98647	.9997 98002	.0201 01353	.0002 02012
.0202	.0201 98626	.9997 95987	.0202 01374	.0002 04027
.0203	.0202 98606	.9997 93962	.0203 01394	.0002 06052
.0204	.0203 98585	.9997 91927	.0204 01415	.0002 08087
0.0205	0.0204 98564	0.9997 89882	0.0205 01436	1.0002 10132
.0206	.0205 98543	.9997 87828	.0206 01457	.0002 12188
.0207	.0206 98522	.9997 85763	.0207 01478	.0002 14253
.0208	.0207 98500	.9997 83688	.0208 01500	.0002 16328
.0209	.0208 98478	.9997 81603	.0209 01522	.0002 18413
0.0210	0.0209 98457	0.9997 79508	0.0210 01544	1.0002 20508
.0211	.0210 98434	.9997 77403	.0211 01566	.0002 22613
.0212	.0211 98412	.9997 75288	.0212 01588	.0002 24728
.0213	.0212 98389	.9997 73164	.0213 01611	.0002 26854
.0214	.0213 98367	.9997 71029	.0214 01633	.0002 28989
0.0215	0.0214 98344	0.9997 68884	0.0215 01656	1.0002 31134
.0216	.0215 98320	.9997 66729	.0216 01680	.0002 33289
.0217	.0216 98297	.9997 64564	.0217 01703	.0002 35454
.0218	.0217 98273	.9997 62389	.0218 01727	.0002 37629
.0219	.0218 98249	.9997 60205	.0219 01751	.0002 39815
0.0220	0.0219 98225	0.9997 58010	0.0220 01775	1.0002 42010
.0221	.0220 98201	.9997 55805	.0221 01799	.0002 44215
.0222	.0221 98177	.9997 53590	.0222 01824	.0002 46430
.0223	.0222 98152	.9997 51365	.0223 01848	.0002 48655
.0224	.0223 98127	.9997 49130	.0224 01873	.0002 50890
0.0225	0.0224 98102	0.9997 46886	0.0225 01898	1.0002 53136
.0226	.0225 98076	.9997 44631	.0226 01924	.0002 55391
.0227	.0226 98051	.9997 42366	.0227 01950	.0002 57656
.0228	.0227 98025	.9997 40091	.0228 01975	.0002 59931
.0229	.0228 97999	.9997 37806	.0229 02002	.0002 62216
0.0230	0.0229 97972	0.9997 35512	0.0230 02028	1.0002 64512
.0231	.0230 97946	.9997 33207	.0231 02054	.0002 66817
.0232	.0231 97919	.9997 30892	.0232 02081	.0002 69132
.0233	.0232 97892	.9997 28567	.0233 02108	.0002 71457
.0234	.0233 97865	.9997 26232	.0234 02136	.0002 73792
0.0235	0.0234 97837	0.9997 23888	0.0235 02163	1.0002 76138
.0236	.0235 97809	.9997 21533	.0236 02191	.0002 78493
.0237	.0236 97781	.9997 19168	.0237 02219	.0002 80858
.0238	.0237 97753	.9997 16793	.0238 02247	.0002 83233
.0239	.0238 97725	.9997 14409	.0239 02275	.0002 85619
0.0240	0.0239 97696	0.9997 12014	0.0240 02304	1.0002 88014
.0241	.0240 97667	.9997 09609	.0241 02333	.0002 90419
.0242	.0241 97638	.9997 07194	.0242 02362	.0002 92834
.0243	.0242 97609	.9997 04770	.0243 02392	.0002 95260
.0244	.0243 97579	.9997 02335	.0244 02421	.0002 97695
0.0245	0.0244 97549	0.9996 99890	0.0245 02451	1.0003 00140
.0246	.0245 97519	.9996 97435	.0246 02481	.0003 02595
.0247	.0246 97489	.9996 94971	.0247 02512	.0003 05061
.0248	.0247 97458	.9996 92496	.0248 02542	.0003 07536
.0249	.0248 97427	.9996 90011	.0249 02573	.0003 10021

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x	sin x	cos x	sinh x	cosh x
0.0250	0.0249 97396	0.9996 87516	0.0250 02604	1.0003 12516
.0251	.0250 97365	.9996 85012	.0251 02636	.0003 15022
.0252	.0251 97333	.9996 82497	.0252 02667	.0003 17537
.0253	.0252 97301	.9996 79972	.0253 02699	.0003 20062
.0254	.0253 97269	.9996 77437	.0254 02731	.0003 22597
0.0255	0.0254 97237	0.9996 74893	0.0255 02764	1.0003 25143
.0256	.0255 97204	.9996 72338	.0256 02796	.0003 27698
.0257	.0256 97171	.9996 69773	.0257 02829	.0003 30263
.0258	.0257 97138	.9996 67198	.0258 02862	.0003 32838
.0259	.0258 97104	.9996 64614	.0259 02896	.0003 35424
0.0260	0.0259 97071	0.9996 62019	0.0260 02929	1.0003 38019
.0261	.0260 97037	.9996 59414	.0261 02963	.0003 40624
.0262	.0261 97003	.9996 56800	.0262 02998	.0003 43240
.0263	.0262 96968	.9996 54175	.0263 03032	.0003 45865
.0264	.0263 96933	.9996 51540	.0264 03067	.0003 48500
0.0265	0.0264 96899	0.9996 48896	0.0265 03102	1.0003 51146
.0266	.0265 96863	.9996 46241	.0266 03137	.0003 53801
.0267	.0266 96828	.9996 43576	.0267 03172	.0003 56466
.0268	.0267 96792	.9996 40901	.0268 03208	.0003 59141
.0269	.0268 96756	.9996 38217	.0269 03244	.0003 61827
0.0270	0.0269 96720	0.9996 35522	0.0270 03281	1.0003 64522
.0271	.0270 96683	.9996 32817	.0271 03317	.0003 67227
.0272	.0271 96646	.9996 30103	.0272 03354	.0003 69943
.0273	.0272 96609	.9996 27378	.0273 03391	.0003 72668
.0274	.0273 96572	.9996 24643	.0274 03429	.0003 75403
0.0275	0.0274 96534	0.9996 21899	0.0275 03466	1.0003 78149
.0276	.0275 96496	.9996 19144	.0276 03504	.0003 80904
.0277	.0276 96458	.9996 16380	.0277 03542	.0003 83670
.0278	.0277 96419	.9996 13605	.0278 03581	.0003 86445
.0279	.0278 96381	.9996 10820	.0279 03620	.0003 89230
0.0280	0.0279 96341	0.9996 08026	0.0280 03659	1.0003 92026
.0281	.0280 96302	.9996 05221	.0281 03698	.0003 94831
.0282	.0281 96263	.9996 02406	.0282 03738	.0003 97646
.0283	.0282 96223	.9995 99582	.0283 03778	.0004 00472
.0284	.0283 96182	.9995 96747	.0284 03818	.0004 03307
0.0285	0.0284 96142	0.9995 93902	0.0285 03858	1.0004 06152
.0286	.0285 96101	.9995 91048	.0286 03899	.0004 09008
.0287	.0286 96060	.9995 88183	.0287 03940	.0004 11873
.0288	.0287 96019	.9995 85309	.0288 03981	.0004 14749
.0289	.0288 95977	.9995 82424	.0289 04023	.0004 17634
0.0290	0.0289 95935	0.9995 79529	0.0290 04065	1.0004 20529
.0291	.0290 95893	.9995 76625	.0291 04107	.0004 23435
.0292	.0291 95851	.9995 73710	.0292 04150	.0004 26350
.0293	.0292 95808	.9995 70786	.0293 04192	.0004 29276
.0294	.0293 95765	.9995 67851	.0294 04236	.0004 32211
0.0295	0.0294 95721	0.9995 64907	0.0295 04279	1.0004 35157
.0296	.0295 95678	.9995 61952	.0296 04323	.0004 38112
.0297	.0296 95634	.9995 58987	.0297 04367	.0004 41077
.0298	.0297 95590	.9995 56013	.0298 04411	.0004 44053
.0299	.0298 95545	.9995 53028	.0299 04455	.0004 47038

x	sin x	cos x	sinh x	cosh x
0.0300	0.0299 95500	0.9995 50034	0.0300 04500	1.0004 50034
.0301	.0300 95455	.9995 47029	.0301 04545	.0004 53039
.0302	.0301 95410	.9995 44015	.0302 04591	.0004 56055
.0303	.0302 95364	.9995 40990	.0303 04637	.0004 59080
.0304	.0303 95318	.9995 37956	.0304 04683	.0004 62116
0.0305	0.0304 95271	0.9995 34911	0.0305 04729	1.0004 65161
.0306	.0305 95225	.9995 31857	.0306 04776	.0004 68217
.0307	.0306 95178	.9995 28792	.0307 04823	.0004 71282
.0308	.0307 95131	.9995 25717	.0308 04870	.0004 74357
.0309	.0308 95083	.9995 22633	.0309 04918	.0004 77443
0.0310	0.0309 95035	0.9995 19538	0.0310 04965	1.0004 80538
.0311	.0310 94987	.9995 16434	.0311 05014	.0004 83644
.0312	.0311 94938	.9995 13319	.0312 05062	.0004 86759
.0313	.0312 94890	.9995 10195	.0313 05111	.0004 89885
.0314	.0313 94840	.9995 07061	.0314 05160	.0004 93021
0.0315	0.0314 94791	0.9995 03916	0.0315 05210	1.0004 96166
.0316	.0315 94741	.9995 00762	.0316 05259	.0004 99322
.0317	.0316 94691	.9994 97597	.0317 05309	.0005 02487
.0318	.0317 94641	.9994 94423	.0318 05360	.0005 05663
.0319	.0318 94590	.9994 91238	.0319 05411	.0005 08848
0.0320	0.0319 94539	0.9994 88044	0.0320 05462	1.0005 12044
.0321	.0320 94488	.9994 84839	.0321 05513	.0005 15249
.0322	.0321 94436	.9994 81625	.0322 05565	.0005 18465
.0323	.0322 94384	.9994 78400	.0323 05617	.0005 21690
.0324	.0323 94332	.9994 75166	.0324 05669	.0005 24926
0.0325	0.0324 94279	0.9994 71921	0.0325 05722	1.0005 28171
.0326	.0325 94226	.9994 68667	.0326 05775	.0005 31427
.0327	.0326 94173	.9994 65403	.0327 05828	.0005 34693
.0328	.0327 94119	.9994 62128	.0328 05882	.0005 37968
.0329	.0328 94065	.9994 58844	.0329 05936	.0005 41254
0.0330	0.0329 94011	0.9994 55549	0.0330 05990	1.0005 44549
.0331	.0330 93956	.9994 52245	.0331 06044	.0005 47855
.0332	.0331 93901	.9994 48931	.0332 06099	.0005 51171
.0333	.0332 93846	.9994 45606	.0333 06155	.0005 54496
.0334	.0333 93790	.9994 42272	.0334 06210	.0005 57832
0.0335	0.0334 93734	0.9994 38927	0.0335 06266	1.0005 61177
.0336	.0335 93678	.9994 35573	.0336 06323	.0005 64533
.0337	.0336 93622	.9994 32209	.0337 06379	.0005 67899
.0338	.0337 93565	.9994 28834	.0338 06436	.0005 71274
.0339	.0338 95507	.9994 25450	.0339 06493	.0005 74660
0.0340	0.0339 93450	0.9994 22056	0.0340 06551	1.0005 78056
.0341	.0340 93392	.9994 18651	.0341 06609	.0005 81461
.0342	.0341 93333	.9994 15237	.0342 06667	.0005 84877
.0343	.0342 93275	.9994 11813	.0343 06726	.0005 88303
.0344	.0343 93216	.9994 08378	.0344 06785	.0005 91738
0.0345	0.0344 93156	0.9994 04934	0.0345 06844	1.0005 95184
.0346	.0345 93097	.9994 01480	.0346 06904	.0005 98640
.0347	.0346 93037	.9993 98015	.0347 06964	.0006 02105
.0348	.0347 92976	.9993 94541	.0348 07024	.0006 05581
.0349	.0348 92916	.9993 91057	.0349 07085	.0006 09067