Signal Analysis and Estimation

An Introduction

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A WILEY-INTERSCIENCE PUBLICATION

JOHN WILEY & SONS

New York · Chichester · Brisbane · Toronto · Singapore

Library of Congress Cataloging in Publication Data:

Fante, Ronald L. (Ronald Louis), 1936–
Signal analysis and estimation: an introduction/Ronald L. Fante.

p. cm.
"A Wiley-Interscience publication."
Bibliography: p.
Includes index.

ISBN 0-471-62425-X
1. Signal theory (Telecommunication) 2. Fourier transformations.
I. Title

TK5102.5.F36 1988
621.38′043--dc19

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

To the honor and glory of God
and the memory of my brother
Mel Fante

Signal Analysis and Estimation

PREFACE

This text is intended as an introduction to the analysis (using Fourier techniques) of continuous and discrete deterministic signals, along with both the estimation and spectral analysis of random signals. Because the treatment is applicable to optical, acoustical, electrical, etc., signals, it should be useful to students of both physics and engineering (especially electrical).

In order to be appropriate for a course given at any time in the curriculum, the presentation has been designed so that no detailed mathematical knowledge beyond the completion of sophomore calculus is required. No familiarity with Fourier transforms is assumed, nor is any extensive knowledge of probability and random processes. Furthermore, vector calculus and complex variables have been avoided wherever possible. Also, matrix notation is used sparingly, although a limited familiarity with matrix operations is assumed.

The text is designed for a 12-week course and divided into two portions: deterministic signals and random signals. Chapters 1–5 are devoted to the analysis of continuous and discrete deterministic signals, and it is expected that this material could be covered in approximately 5 weeks. The properties, spectral analysis and estimation of random signals are covered in Chapters 6–9, and should occupy the remainder of the course.

In order to assist the reader in understanding the (sometimes complex) concepts presented, examples have been liberally included in every chapter. Usually, those examples chosen are related to engineering applications, such as radar, antennas, optical processing, tomography, analog and digital filters, and target tracking, so that the student can understand how the techniques studied are applied in practice. Also, the problems are designed not only to test the readers understanding, but also to illustrate applications and extensions that could not be included in the main text.

The question of notation is always a vexing one. It is, of course, impossible to find a system of notation that is consistent with that used by all other authors. I have tried to use conventional notation wherever possible, but there were several instances where I chose to depart from usual electrical engineering notation. In particular, I use a prime on a symbol to indicate that it represents an estimate, whereas many authors use a caret over the symbol for this purpose. Instead, I have used a caret over a symbol to indicate that it is the Fourier transform of a discrete signal (to differentiate it from the Fourier transform of a continuous signal). The other departure from the usual notation of electrical engineering, but not physics, is my use of an overbar (i.e., $\bar{x}$ = expectation of x) to denote an expectation (or ensemble average) of a random variable, whereas many authors use $E(x)$ for this purpose. I was forced to abandon $E(x)$ in favor of $\bar{x}$ because many of the results in Chapters 7–9 became too cumbersome when the $E(\cdots)$ notation was used.

Finally, I wish to express my thanks to Avco/Textron for providing an environment conducive to the preparation of this manuscript, and especially to Clara for her continuous support and encouragement.

Wilmington, Massachusetts
September 1987

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INTRODUCTION

Fourier methods will be used here to study the properties of both deterministic and random signals. Although other techniques are sometimes used, Fourier analysis is by far the most common and useful, having important applications in virtually every field of engineering and physics. Some examples are

1. *Signal Analysis*. The Fourier transform allows one to deduce the frequency content (i.e., how fast the signal varies) of an arbitrary signal.
2. *Quantum Mechanics*. The position and momentum of a particle are a Fourier transform pair; the Heisenberg uncertainty principle expresses a property common to all Fourier transform pairs.
3. *Linear Systems*. The analysis and design of both continuous and discrete linear systems is considerably simplified through use of Fourier transforms. In particular, it is found that the Fourier transform of the output of an arbitrary linear system is simply the product of the Fourier transform of the input to the system and a function (called the system function) that describes the linear system.
4. *Antennas*. The signal radiated by either an electromagnetic or acoustic antenna is equal to the Fourier transform of the source distribution.
5. *Optical Processors*. The electric field distribution in an appropriate plane behind a thin lens is the Fourier transform of the field in an appropriate plane in front of the lens. This property forms the basis of coherent optical processing and is a cornerstone of the computer of the future.
6. *Spectroscopy*. There is a technique known as "Fourier Transform Spectroscopy" where Fourier transforms are used to determine the spectral content (brightness versus wavelength) of an arbitrary light source.
7. *Tomography*. A special property of two-dimensional Fourier transforms forms the basis of tomographic image reconstruction.
8. *Partial Differential Equations*. Fourier transforms are routinely used in the solution of partial differential equations and other areas of mathematical physics.

Thus, Fourier transforms are quite important and the discovery of fast algorithms for computing these transforms has made them an even more valuable and useful tool.

Part I of this text will be devoted to the study of deterministic signals. In Chapter 1 it is shown how Fourier series can be used to represent and analyze periodic functions. Then after a digression (Chapter 2) on the properties of the singularity functions (Dirac delta function, etc.), Chapter 3 covers the application of the Fourier transform to the analysis of aperiodic continuous (analog) signals. Chapter 4 shows how one obtains discrete signals from continuous ones, and the role of the Fourier transform in the analysis of digital signals. Chapter 5 discusses the discrete Fourier transform and special algorithms that are appropriate for their rapid computer-computation.

These approaches are applicable to deterministic signals. For random signals the approach must be generalized, and Part II is devoted to this end. Chapter 6 is a relatively extensive discussion of the properties and representation of random signals, and in Chapter 7 we show how Fourier techniques are used to determine their spectral (frequency) content. Chapter 8 is devoted to the principles of signal estimation, and in Chapter 9 these techniques are used to estimate the power spectral density of random signals.

CONTENTS

PART I

DETERMINISTIC SIGNALS

1

Fourier Series

1.1 THE FOURIER SINE/COSINE SERIES

We shall begin our discussion of signal analysis by considering some simple periodic signals such as shown in Figure 1.1 (these are typical of the pulse trains produced by an electronic signal generator for use as a clock in a digital system). A signal $s(t)$ is called "periodic" in time t if it repeats itself every T seconds, so that $s(t+T)=s(t)$. The quantity T is called the period of the signal, and $F=1/T$ is known as its frequency. A quantity $\omega = 2\pi \cdot$ frequency is also often used and is known as the "radian frequency." The units for the period are "seconds," those for the frequency are usually "hertz" (abbreviated Hz) or sec^{-1}, and those for the radian frequency are "radians per second."

Upon observing the signal $s_1(t)$ in Figure 1.1a, one might guess that, without too great an error, it could be approximated by $\cos(2\pi t/T)$, which is shown as a dashed line. It might also be expected that the fit would be ever better if more cosines are added and $s_1(t)$ approximated by

$$s_1(t) = A_1 \cos\left(\frac{2\pi t}{T}\right) + A_2 \cos 2\left(\frac{2\pi t}{T}\right) + \cdots + A_n \cos n\left(\frac{2\pi t}{T}\right), \qquad (1)$$

where the coefficients are chosen (in a way to be discussed later) to provide the best possible fit to the actual signal. Finally, we could speculate that if we kept enough terms in Eq. (1) the fit would become perfect. This can be shown to be precisely what happens (except for very near any discontinuities in $s_1(t)$, where there is a mismatch known as the Gibbs phenomenon; this will be discussed later).

The series in Eq. (1) contains only cosines because the function $s_1(t)$ is symmetric about the time origin (a symmetric function is one for which $s(-t)=s(t)$). Now suppose we displace $s_1(t)$ to the right by a time Δ to produce the function $s_2(t)$ shown in Figure 1.1b. Then in all the terms on the right-hand side of Eq. (1) t is replaced by $t-\Delta$ and the nth term becomes

$$\cos\frac{2\pi n}{T}(t-\Delta) = \cos\frac{2\pi nt}{T}\cos\frac{2\pi n\Delta}{T} + \sin\frac{2\pi nt}{T}\sin\frac{2\pi n\Delta}{T}, \qquad (2)$$

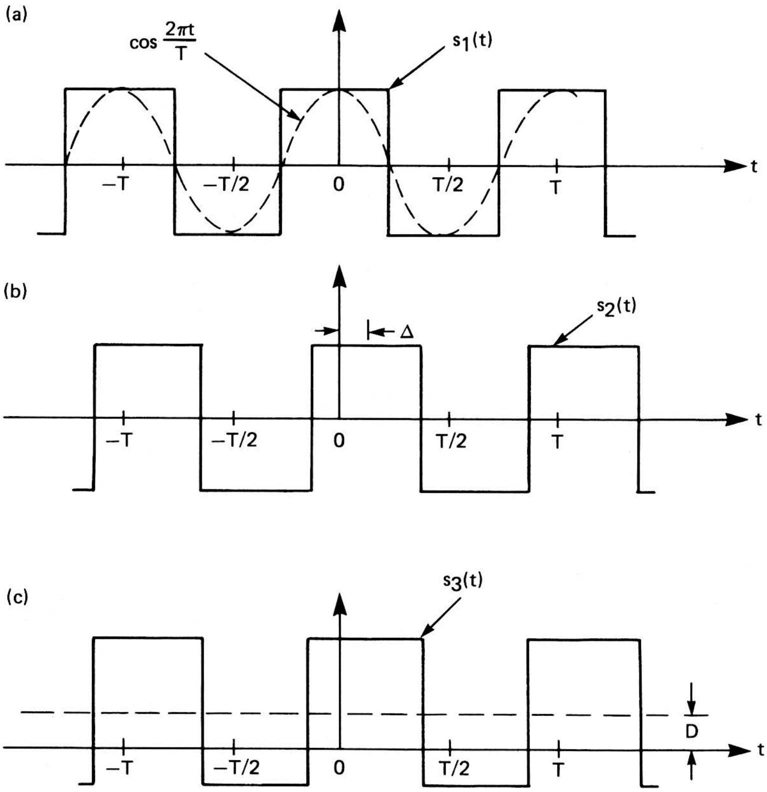

Figure 1.1 Periodic pulse trains.

so that the result in Eq. (1) now is

$$s_2(t) = \sum_{n=1}^{\infty} \left[A_n \cos\frac{2\pi n\Delta}{T} \right] \cos\frac{2\pi nt}{T} + \sum_{n=1}^{\infty} \left[A_n \sin\frac{2\pi n\Delta}{T} \right] \sin\frac{2\pi nt}{T} \tag{3}$$

$$= \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nt}{T} + \sum_{n=1}^{\infty} b_n \sin\frac{2\pi nt}{T}\ , \tag{4}$$

where $a_n = A_n \cos(2\pi n\Delta/T)$ and $b_n = A_n \sin(2\pi n\Delta/T)$.

Finally, suppose that the function $s_2(t)$ is displaced upward by an amount D to produce $s_3(t)$ as shown in Figure 1.1c. Then a constant term $a_0/2 = D$ would need to be added to the series in Eq. (4), and this gives

$$s_3(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nt}{T} + \sum_{n=1}^{\infty} b_n \sin\frac{2\pi nt}{T}\ . \tag{5}$$