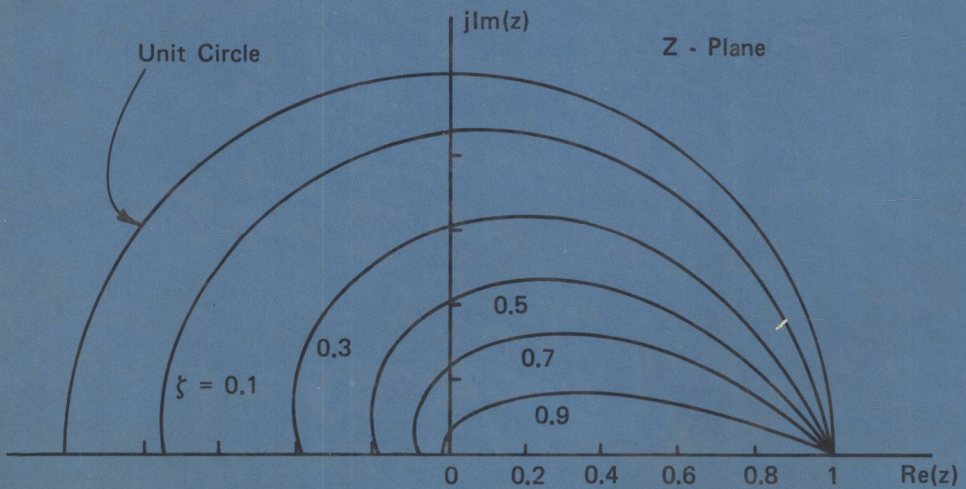


Modern Digital Control Systems



Raymond G. Jacquot

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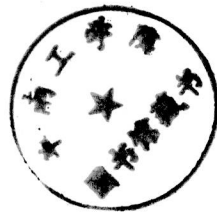
MODERN DIGITAL CONTROL SYSTEMS

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To Nancy and Byron, who gave so many evenings

Preface

This book is written for students who have had one course in conventional feedback control systems with or without an introduction to the modern concepts of state variables. Sufficient material is provided for students lacking background in modern concepts. It is assumed, however, that students will be well grounded in the use of the Laplace transform and s -domain concepts and will also possess a background in elementary matrix theory and linear algebra. For study of the final chapters, it is also assumed that the student is versed in the theory of probability and possesses introductory knowledge in the area of continuous-time stochastic processes. This material should also be useful to practicing engineers who have studied the concepts of continuous-time control of dynamic systems.

In the author's experience, it seems that learning discrete data concepts was unnecessarily complicated by introduction of the artifice of impulse sampling. After considerable study of the subject, the author decided that those subjects necessary for discrete-time control systems and discrete-time signal processing could be introduced without the complication of modulated impulse functions. A chapter on impulse sampling is provided for completeness but is not a prerequisite to any other material in the text.

Several example problems with a physical basis provide interesting applications of the theory developed. These examples present a recurring theme in the text and tie the theoretical developments together. These examples are introduced in Chapter 3 and are continuously used through the final chapter.

The author has taught a course from preliminary versions of this material and most recently found it possible to cover all the material in Chapters 1 through 7 with fundamental points taken from Chapters 8 and 9 in a semester. The material in the latter portions of the text has been used as supplementary material in an estimation theory course. The author's course meets for three lectures weekly with one laboratory exercise every two weeks. The importance of these laboratory exercises cannot be overemphasized in the understanding of the theoretical developments they provide.

The material contained herein is largely the product of ideas developed by the author during a sabbatical leave to the Department of Aeronautics and Astronautics at Stanford University. The author is particularly indebted to those valuable discussions with Professors Arthur E. Bryson, Jr. and J. David Powell that stimulated the effort in this writing.

The author is particularly grateful to his colleague Dr. John Steadman, who continually provided encouragement and advice in the development of this work. He was also responsible for the development of the minicomputer system which inspired the laboratory exercises that are so valuable in the teaching of this material.

A course built around this material is no substitute for knowledge of digital hardware and software design. Knowledge of the hardware and software of mini- and microcomputers is highly desirable because these are the devices which are the heart of any digital control system.

The author was considerably aided in the development of the manuscript by the careful attention to detail by students Ken Jensen, Kim Sturm, Mizan Rahman, Mike Petrea, Mark Hepworth, Barry Eklund, Paul Nelson, and Mike Mundt. Without these diligent students, no effort to write text material would have been successful.

Raymond G. Jacquot



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1

Introduction to Digital Control

1.1 The Basic Idea of System Control

Consider a dynamic plant which is to be controlled to do a given task. Generally this plant has continuous-in-time inputs and outputs. In general, the plant has r input variables (manipulable variables) and m output variables. There may also be internal variables which are not outputs. Which variables are outputs is a matter for the system designer to decide. The control problem is one of manipulating the input variables in an attempt to influence the output variables in a desired fashion, for example, to achieve certain values or certain rates.

If we know the system model and the initial conditions with reasonable accuracy, we can manipulate the input variables so as to drive the outputs in the desired manner. This is what we would call "open-loop control" since it is accomplished without knowledge of the current outputs. This situation is depicted in Fig. 1.1. This type of control also assumes that the system operates in the absence of disturbances that would cause the system outputs to vary from those predicted by a model of the plant.

Because of uncertainties in the system model and initial conditions and because of disturbances there is a better way to accomplish the control task. This technique involves measuring the behavior of some subset of the

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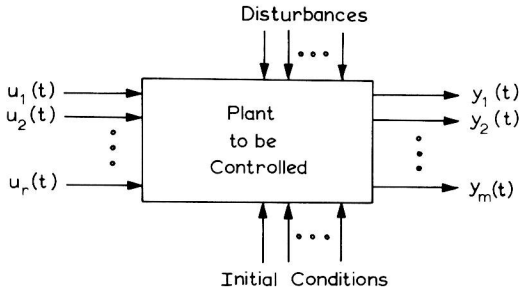


Fig. 1.1 Open-loop-controlled plant.

output variables (those we want to control) $y_1(t) \dots y_k(t)$ ($k \leq m$) and, after measurement, comparing them with what we would like each of them to be at time t and calling the difference between the desired value $r_i(t)$ and the actual value $y_i(t)$ the "error." Using the errors in each of the variables we can generate the control variables or efforts so as to drive the errors to zero. This situation is depicted in Fig. 1.2 and referred to as "feedback control." Due to friction, inertia, and other dynamic properties, it is impossible to drive a system instantaneously to zero error. Thus the system outputs $y_i(t)$ will "lag" the desired inputs $r_i(t)$ and sometimes they will overshoot or oscillate about a zero error condition.

Often we are not able to have the physical output variables available nor may it be convenient to compare physical variables. For example, if one of these were the temperature in the core of a nuclear reactor, it would not be physically accessible nor would it be convenient to actually compare temperatures. It would be much more convenient to measure the output with a transducer or sensor which generates a signal which is easily compared with a reference signal to generate the control effort. This is often, but not always, done electronically. Again, due to the dynamic character of sensors, their outputs are not always exactly proportional to the variables which they are designed to measure, and hence the signal fed back is not

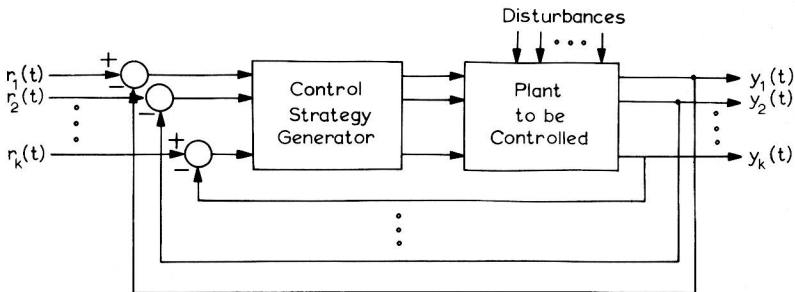


Fig. 1.2 Feedback-controlled multivariable system.

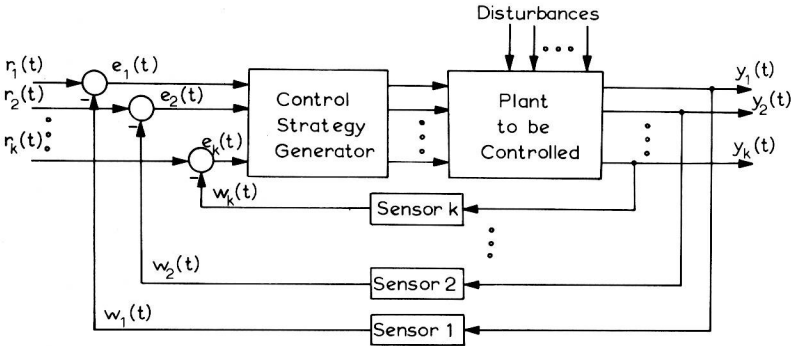


Fig. 1.3 Feedback-controlled multivariable system with sensors.

always a true replica of the variable which is being measured. We hope at the outset that the desired information to accomplish control is present in the sensor output or the situation becomes one of "open-loop" control. The feedback-controlled system with sensors is shown in Fig. 1.3 with the sensor output signals denoted by $w_1(t)$, $w_2(t)$, \dots , $w_k(t)$.

For many years the sensors and the synthesis of the control signal generator have been the center of considerable engineering activity, and in general these devices are electrical and electronic in nature—more specifically, filters, amplifiers, power amplifiers, motors, and actuators. In all these cases the signals have been continuous functions of the temporal variable t . We commonly refer to such systems as continuous-time control systems. As systems have become more and more complex with a multitude of control variables and output variables, the hardware synthesis job becomes a difficult task. Typical systems are airplane autopilot controls, chemical-processing controls, nuclear power plant controls, and a host of others generally involving large-scale systems.

With the advent of the digital computer, engineers began to explore the possibilities of having the computer keep track of the various signals and

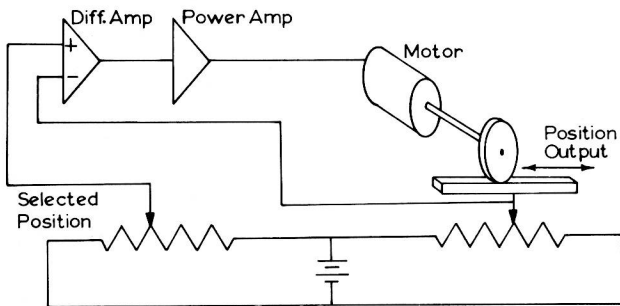


Fig. 1.4 Position servomechanism with continuous signals.

4 Introduction to Digital Control

make logical decisions about control signals based on the measured signals and the desired values of the outputs. We know, however, that a digital computer is capable only of dealing with numbers and not signals, so if a digital computer is to accomplish the task, the sensor signals must be converted to numbers while the output control decisions must be output in the form of continuous-time control efforts.

As an example let us consider a single-loop position servomechanism in continuous-time form as shown in Fig. 1.4. The reference signal is in the form of a voltage, as is the feedback signal, both generated by mechanically driven potentiometers.

1.2 The Computer as a Control Element

Let us now investigate how this relatively simple task, outlined in the previous section, might be accomplished by employing a digital computer to generate the signal to the power amplifier. We must first postulate the existence of two devices. The first of these devices is the analog-to-digital (A/D) converter which will sample the output signal periodically and convert this sample to a digital word to be processed by the digital computer and thus generate a control strategy in the form of a number. The second device is a digital-to-analog (D/A) converter which converts the numerical control strategy generated by the digital computer from a digital word to an analog signal. The position servomechanism is shown in Fig. 1.5 controlled by a digital computer.

This system was not chosen because it was realistic but rather to show the principle of digital control of a single-loop system. Generally the A/D and D/A converters operate periodically and hence the closer together in time the samples are taken and the more often the output of the D/A converter is updated, the closer the digital control system will approach the continuous-

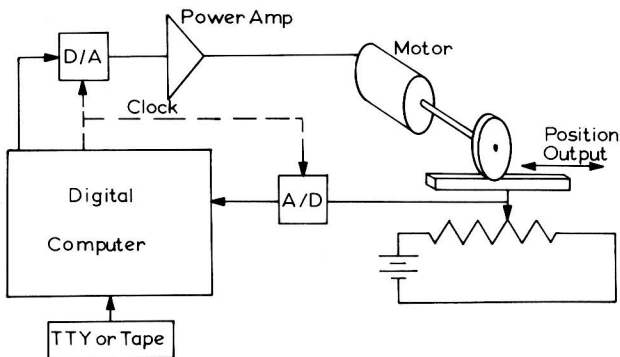


Fig. 1.5 Digitally controlled positioning system.