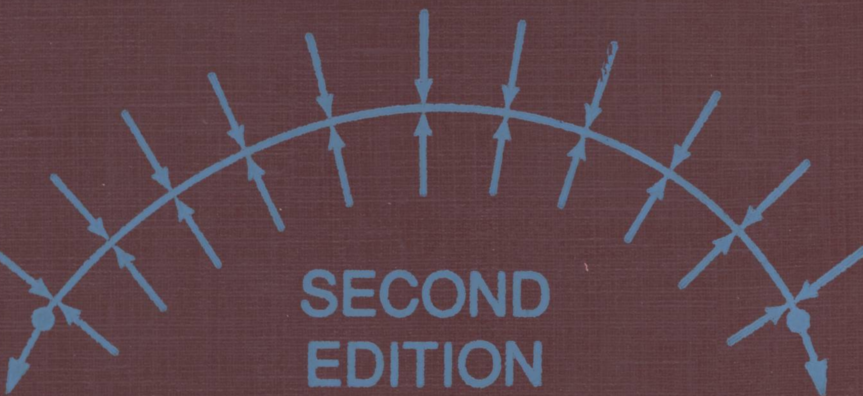
Porous Media

Fluid Transport and Pore Structure



SECOND EDITION

F.A.L. DULLIEN

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Porous Media
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To my wife Ann
for her moral and professional support

Preface

Since the publication of the first edition of this book a great deal of research has been done in the author's field of interest. As a result, major parts of the book have been completely revised and many additions have been made. As this text is very strongly research oriented, it is fair to say that the second edition is a new progress report on this field. Owing to limitation of space, time, and the author's own interests, this review of the field of transport in porous media is not exhaustive. The author apologizes to those colleagues whose work, regrettably, receives less attention than their importance warrants.

The unique property of a porous medium, the one that distinguishes it from other solid bodies on the one hand and from simple conduits on the other, is its complicated pore structure. The vast majority of porous media contain an interconnected three-dimensional network of capillary channels of nonuniform sizes and shapes, commonly referred to as *pores*.

Fluid flow, diffusion, and electrical conduction in porous media take place within extremely complicated microscopic boundaries that in the past made a rigorous solution of the equations of change in the capillary network practically impossible. However, this situation has changed recently because high-powered digital computers now permit the solution of these equations in small samples of the pore network. The past state of affairs is one of the reasons why some of the brilliant and successful practitioners in the field of "flow through porous media" have tried, as much as possible, to stick with the continuum approach in which no attention is paid to pores or pore structure. Another reason is that the continuum approach is often adequate for the phenomenological description of macroscopic transport processes in porous media. The continuum approach, however, fails to provide a clue to help explain any of a multitude of observations that depend on the properties of the microscopic channels and the behavior of the fluids on the microscopic scale. The desire of scientists and engineers to be able to understand and then explain their observations has always been a powerful driving force for progress. Therefore there have been numerous attempts over the past sixty years or so to explain the flow phenomena in terms of the microscopic structure as accurately as possible. The results have seldom been entirely satisfactory, but with every step a further penetration into an immensely complicated territory has been achieved. There is a great deal of information available in the technical literature on the role played by pore structure in determining transport phenomena in pore spaces.

This book has been written with the primary purpose of presenting in an organized manner the most pertinent information available on the role of pore structure and then putting it to use in the interpretation of experimental data and the results of model calculations.

Pore structure is inseparable from the convective, diffusive, and interfacial effects that take place in the pores; these effects are all interrelated so that there is little point in trying to evaluate their relative merits with the aim of deciding which of them is most important.

Existing books on "flow through porous media" have been written with an emphasis on the fluid mechanical aspects. Interfacial effects, such as interfacial tension and wettability, have been under intensive and productive investigation for quite some time. In this book the author has made an attempt to show that there are benefits to be gained by trying to think about the phenomena in porous media in terms of interactions among the three main factors, i.e., transport phenomena, interfacial effects, and pore structure. The book contains many examples of applications of this concept, and it is the hope of the author that many readers will find this approach useful as well as an inspiration and motivation to do more fundamental research on the role played by pore structure.

The author wishes to acknowledge an operating grant from the National Research Council of Canada. The perfect typing of the manuscript of the second edition and a great deal of dedicated secretarial help by Susie Bell and the outstanding professional artwork of Rinze Koopmans are gratefully acknowledged. Many discussions with the author's colleagues, especially Professor I. Chatzis and L. Catalan, have aided the process of formulating the author's ideas. The moral as well as professional support throughout all the author's work and life by his wife Ann, to whom this book is dedicated, has been invaluable.

List of Symbols

Latin Letters

a	cross-sectional area; speed of sound; edge length of cube
a	$= (1/8)R^2$ [Eq. (6.3.59)]
a_{ijmn}	medium's dispersivity tensor [Eq. (6.3.101)]
a_{L}	$= D_{\mathrm{L}}/v_{\mathrm{DF}} = \sigma_x^2/2x =$ geometrical dispersivity
$a_{\mathrm{L,eff}}$	effective dispersivity
a_{T}	average transverse dispersivity
a_{w}	activity of water
$2a$	wall-to-wall distance
A	cross-sectional area, constant in Eq. (3.2.18)
$(A_{\mathrm{v}})_i$	unfilled cross-sectional area in pore corner
A_A	area fraction
A_∞	longitudinal macrodispersivity [Eq. (6.3.5)]
δA	incremental surface area
b	constant; constant in Eq. (1.1.7); number of branches
$2b$	wall-to-wall distance
B	constant in Eq. (3.2.19); hydraulic conductivity; channel conductance
Bd	bulk density (Table 1.5)
c	compressibility; constant parameter; molar concentration; concentration
$\bar{c}$	average tracer concentration
c_{m}^*	molar tracer concentration
c_0	initial tracer concentration
C	connectivity; dimensionless coefficient in Eqs. (6.3.15) and (6.3.16)
C	$= (R_{\mathrm{p}} - R)R_{\mathrm{p}}$ [Eq. (2.5.17)]
C'	$\equiv T =$ shift factor

Ca	capillary number
CA_{imb}	capillary number defined by Eq. (5.3.83)
CA_{mob}	capillary number defined by Eq. (5.3.88)
C_D	drag coefficient
$C(L)$	covariance
d	distance
D	pore or capillary diameter; dimension; darcy
Da	Darcy number
DI	difficulty index [Eq. (5.3.71)]
D_b	breakthrough diameter
D_{ct}	dispersion coefficient in a capillary tube
D_e	pore entry diameter
D_f	fiber diameter
D_F	fractal dimension
D_H	hydraulic diameter
D_ℓ	diameter of large capillary
D_L	longitudinal dispersion coefficient
D_p	sphere or particle diameter
rD_p	interparticle distance
$\overline{D}_p$	effective average particle or fiber diameter
$\overline{D}_{p2}$	surface average sphere diameter
$\overline{D}_{pr}$	average sphere diameter defined by Eq. (3.7.10)
D_s	diameter of small capillary
D_T	container diameter; transverse dispersion coefficient
D_v	volume average diameter
D_v^+	dimensionless volume average diameter
$\underline{D}_2$	diameter of 3-D object
$\overline{\overline{D}}$	hydraulic dispersion tensor
$\mathscr{D}$	tracer or mutual diffusion coefficient
$\mathscr{D}_{AB}$	mutual diffusion coefficient
$\mathscr{D}_{eff}$	$= \mathscr{D}/X =$ effective diffusion coefficient
$\mathscr{D}_{KA}$	Knudsen diffusion coefficient of A
$\mathscr{D}_m$	moisture or hydraulic diffusivity
$(\mathscr{D}_{eff})_{ss}$	$= \mathscr{D}\phi/X =$ effective diffusion coefficient measured in steady-state experiment
E	modulus of elasticity
E_D	inherent efficiency of conversion of work to the creation of surface
$E(-x)$	$= -\int_x^\infty \frac{e^{-u}\,du}{u}$ Eq. (1.4.1)
E[...]	expected value
f	separation factor (Eq. 4.2.10)
f_i	frequency of nodes of type i
f_p	friction factor
f_r	relative frequency of pores of type r in the network
$f_T(P'')$	adsorption isotherm
f_ℓ	friction factor defined by Eq. (3.3.10)
f_w	fractional flow function of water
f_ϕ	non-Newtonian friction factor
$f(a, b, c)$	joint density function of distribution of lengths a, b, and c
$f(A, \ell, \alpha)$	number of tubes per unit volume in the intervals $A \to A + dA$, $\ell \to \ell + d\ell$ and $\alpha \to \alpha + d\alpha$
$f(D)$	density function of distribution of pore diameters
$f(R)$	pore size distribution defined by Eq. (3.4.10)

$f(2a), f(2b), f(2c)$	cumulative size distributions in the x-, y-, and z-directions, respectively
$f(\phi)$	porosity function [Eq. (3.2.6)]
F	flow contribution to molar flux
F	$= R_o/R_w$ = formation factor
F_c	capillary force
F_e	effective formation factor at partial saturation
F_s	fractional flow of solvent
F_v	viscous force
F_w	fractional flow of water
$F(a, b, c)$	cumulative joint distribution of lengths a, b, and c
$F(D_2)$	density function of distribution of diameters of 3-D objects
$F(D_e)$	density function in model of formation factor
$F(t)$	surface force potential function [Eq. (2.5.26)]
$F(\alpha^2R^2, \alpha R)$	function in Eq. (3.3.62)
$F(\tau_\phi)$	function defined by Eq. (3.5.2)
$F(\tau_w)$	function defined by Eq. (3.5.1)
g	gravitational acceleration constant
g_1	conductance of capillary segment of length ℓ_1
g_3	$= g_1/\sqrt{3}$ = conductance of capillary segment of length $\ell_3 = \ell_1\sqrt{3}$
$g(r^*)$	density function of distribution of r^*
$g(\delta; \theta; \phi; \Omega, D_2)$	density function of distribution of section lengths δ of an object of diameter D_2 in an orientation (θ, ϕ, Ω)
G	Gibbs energy; molar flow rate; genus
G_m	mass flow rate per unit area
$G(p_b)/G^o$	relative conductivity of network
h	variable height [Eq. (1.8)]; net thickness [Eqs. (1.4.1), (1.4.2)]; elevation; half width of channel
h_c	$\equiv \psi$ = capillary pressure head
H	intermittency (Hurst) exponent
H	$= R_{max}/R_{min}$ = heterogeneity factor
I	$= F_e/F$ = resistivity index
$\mathbf{j}_i^v$	mass flux of i with respect to $\mathbf{v}^v$
J_m^*	molar flux of tracer
$J(S_w)$	Leverett J-function [Eq. (2.3.6)]
k	Darcy permeability coefficient [Eq. (3.1.1)]; mass transfer coefficient
k'	Kozeny constant [Eq. (3.3.7)]
$\bar{\bar{k}}$	permeability tensor
$\tilde{k}$	"pure water permeability constant" [Eq. (4.2.14)]
$\underline{k}$	mass transfer coefficient (Eq. (4.2.23))
k_{CK}	permeability predicted by Carman–Kozeny model
k_H	hydraulic conductivity [Eq. (3.1.4)]
k_i	effective or phase permeability
k_{ij}	component of permeability tensor
k_M	$= \mathscr{D}K_H$ = permeability coefficient [Eq. (4.2.8)]
k_n', k_n''	directional permeabilities
k_{ri}	relative permeability
k_0	shape factor [Eq. (3.3.7)]
k_1	permeability predicted by 1-D capillaric model
$k_{1,2,3}$	constants in Eqs. (3.2.38), (3.2.39), and (3.2.40)
k_2/μ	2-D network conductivity [Eq. (3.3.46)]
k_3	permeability predicted by 3-D pseudo capillaric network model
$k_{11}, k_{22}; k_{11}', k_{22}'$	principal phase permeability coefficients
$k_{12} = k_{21}$; $k_{12}' = k_{21}'$	interaction phase permeability coefficients

K	quantity defined by Eq. (4.2.19); K-factor [Eq. (6.3.11)]
K_H	Henry's law constant
ℓ	chord length; length of a step [Eq. (6.3.57)]; length of capillary; characteristic pore scale; oil blob length
ℓ	$= v_p/n$ [Eq. (6.3.48)]
ℓ_ℓ	aggregate length of large capillaries in the model
ℓ_s	aggregate length of fine capillaries in the model
ℓ_1	lattice constant of 1-D capillaric model
ℓ_3	lattice constant of 3-D capillaric model
$\delta\ell$	elemental length
L	length of sample; intercept length, length, length scale of permeability correlation
L_A	length of lines in a plane per unit test area
L_e	average effective path length of flow
L_L	length fraction
L_m	mixing length, i.e., the distance in the macroscopic flow direction over which $\bar{c}/c_0$ changes from 0.9 to 0.1 or from 0.8 to 0.2
L'_m	dimensionless mixing length defined by Eq. (6.2.14a)
L_v	length of lines per unit test volume
$L(R)$	total length of pores with radii between R and $R + dR$ [Eq. (2.5.11)]
m	mass; cementation factor; molality
m	$= k_{2r}\mu_1/k_{12}\mu_2$ = mobility ratio
mD	millidarcy
$(\Sigma m)_r$	number of pores connected to a pore of type r at both ends
M	Mach number; molecular weight
$M(\phi)$	effective viscosity
M	$= (3/2)(v_p \ell \mu / D_{ct})$ [Eq. (6.3.83)]
n	number of capillaries in parallel; mole number; number of nodes in a network; number of lines per unit area; number of points; number of dust particles; exponent in Archie's law; number of steps taken in unit time [Eq. (6.3.48)]
$\mathbf{n}$	unit normal vector of surface
$\mathbf{n}_i$	mass flux of i with respect to solid matrix
n_x	flux of tracer
$n(L)$	density function of distribution of intercept length
$n(\delta)$	density function of distribution of circle diameters
N	number of steps; number of separate networks; total number of bonds; number of occurrences of an event; Avogadro number
N_A	number of features per unit test area
N_A, N_B	molar flux of A and B, respectively
N_{As}	molar flux of A per unit cross section of sample
N_d	deflection number [Eq. (3.2.44)]
N_e	effective pore number [(Eq. (3.2.37)]
N_p	number of pores per unit section
N'_{Re}	non-Newtonian Reynolds number
N_s	number of spheres in unit volume of bed
N_T	total molar flux
N_w	molar flux of water
$N(D)$	density function of distribution of sphere diameters or pore diameters
$N(D_p)$	density function of distribution of particle diameters
p	probability of a particle traveling a distance x_p in the characteristic time τ [Eq. (6.3.52)]; fraction of void space that is accessible through pores of diameters less than a given value (p. 50)
$\tilde{p}$	random quantity in Eq. (6.3.59)

p_b	fraction of open bonds
p_{cr}	critical percolation probability
p_s	fraction of open sites
$p(u)\,du$	probability that a molecule has velocity between u and $u + du$
$p(u, t/u_0/t_0)\,du$	probability that a molecule has velocity u at time t if it had velocity u_0 at time t_0
$p(x, t)\,dx$	probability of tracer to be between x and $x + dx$ at time t
P	hydrostatic pressure; number of points; denoting a point on a surface
P'	hydrostatic stress intensity; absolute value of macroscopic pressure gradient; pressure on the convex side of the interface
P''	pressure on the concave side of the interface
P_A	number of points per unit test area
P_c	capillary pressure
P_c^*	$= P_c/P_{cb}$
P'_{cb}	$= P_{cb}/4\sigma \cos\theta$ reduced breakthrough or bubbling capillary pressure
Pd	particle density (Table 1.5)
P_{I1}	capillary pressure defined by Eq. (5.3.69)
P_{I2}	capillary pressure defined by Eq. (5.3.70)
P_L	number of intersections per unit length of test lines with features in the plane of polish
P_m	arithmetic mean pressure
P_0	vapor pressure of bulk liquid
P_p	capillary pressure in piston type displacement
P_s	snap-off capillary pressure
P_v	number of points of intersection between surfaces and lines per unit test volume
Pe	$= v_{DF}\bar{D}_p/\mathscr{D}$ = Peclet number in porous media
Pe′	$= v_{DF}\bar{D}_p/D_L$ = dynamic Peclet number in porous media
Pe_{ct}	$= D\bar{u}/\mathscr{D}$ = Peclet number in capillary tube
Pe_R	$= \bar{R}v_{DF}/\mathscr{D}$ (p. 516)
Pe'_{ct}	$= \bar{u}\bar{D}_p/D$ = dynamic Peclet number in capillary tube
Pe'_{ft}	Peclet number in Eq. (6.3.56)
Pe'_L	$= v_p L/D_L$ = a dynamic Peclet number
Pe'_t	Peclet number for transverse dispersion in Eq. (6.3.56)
Pe'_T	$= v_p \ell/D_T$ = dynamic Peclet number for transverse dispersion
$\mathcal{P}$	$= P + \rho g z$
$\mathcal{P}^*$	$= \mathcal{P}/\rho v^2$ = dimensionless pressure
q	$= \ell/t$ = velocity of tracer particle in a step; probability that a randomly chosen pore will not exceed a given size (p. 50); production rate
q_{av}	$= dQ/dN$ [Eq. (5.2.10)]
q_i	particle size distribution parameter [Eq. (3.2.1)]
q_k	number of fraction of penetrated pores with no exit (dendritic pores)
Q	volumetric flow rate
δQ	incremental volume flow
r	radial coordinate; radius; principal radius of curvature; ratio of true-to-apparent area of solid surface
$\mathbf{r}$	position vector
r_1, r_2	radii of rotation
$r(c, t)$	rate of production of solute per unit volume of solution [Eq. (6.3.31)]
R	radius of curvature
R	$= D/2$
$\mathbf{R}$	force
R_{eq}	equivalent pore radius defined by Eq. (5.3.86)
R_o	resistance of saturated sample

R_p	cumulative oil recovery
R_w	resistance of electrolytic solution of the same geometry as the sample
R_1	constriction radius
R_1^*	dimensionless constriction radius
$\mathscr{R}$	universal gas constant
Re	Reynolds number
Re_c	Reynolds number defined by Eq. (4.1.7)
Re_f	fiber Reynolds number
Re_k	Reynolds number defined by Eq. (6.3.20)
Re_p	particle or superficial Reynolds number
Re_v	Reynolds number based on D_v
Re_{50}	Reynolds number defined by Eq. (6.3.19)
s	distance along axis of pore
S	specific surface area per gram adsorbent; saturation; surface area of solid spreading coefficient
S'	constriction factor
S_{eff}	saturation based on drainable porosity
S_k	saturation predicted by model in step k
S_k^*	saturation predicted by model in step k if all pores were independent domains
S_0	specific surface area per unit solids volume
S_p	aggregate surface area of particles
S_t	surface area defined by Eq. (2.5.25)
S_v	specific surface area per unit bulk volume
$S_v(\theta)$	unfilled fraction of pore cross section
S_w	water saturation
$S(D_k)$	cumulative volume fraction of pores of size $D \geq D_k$
S^2	function expressed by Eq. (6.3.68)
Sc	$= \nu/\mathscr{D}$, Schmidt number
t	time; multilayer thickness; geometrical factor in Eqs. (3.3.46) and (3.3.47); duration of a step [Eq. (6.3.58)]
t_1, t_0	the time in which the mean square displacement of molecules by molecular diffusion is equal to the radius R and the length ℓ of a tube, respectively [Eq. (6.3.62)]
t_D	$= \eta t/r^2$ (Fig. 1.61)
T	absolute temperature
T	$= L/\bar{u}$ [Eq. (6.2.10)]
T	$= t/N =$ time interval [Eq. (6.3.40)]
T	$\equiv (L_e/L)^2 =$ hydrodynamic tortuosity factor
T_{ij}	component of tortuosity tensor
T_{ij}^*	component of tensor related to the medium's tortuosity
T_n	time of displacement of a tracer particle [Eq. (6.3.58)]
u^*	ratio of average velocity of ganglion to the interstitial velocity of water
$\mathbf{u}^*$	$= \mathbf{u}/\mathbf{v} =$ dimensionless velocity in a capillary
$\bar{u}$	average velocity in individual pore
u_i	$= u(d\xi_i/d\sigma)$ [Eq. (6.3.91)]
$\bar{u}_m$	mean molecular speed
u_r^*	dimensionless r velocity component in a periodically constricted tube
u_z^*	dimensionless z velocity component in a periodically constricted tube
$\langle \bar{u}_z \rangle$	$\equiv v_p =$ average pore velocity in porous medium
$U(t)$	longitudinal component of the velocity of a marked particle relative to the mean velocity [Eq. (6.3.76)]

$v, \mathbf{v}$	Darcy or superficial velocity
v^*	ganglion volume expressed in units of the number of pores occupied by the ganglion
$\mathbf{v}^*$	mass average particle velocity
v_{DF}	$\equiv v/\phi$ = average pore velocity defined by the Dupuit–Forchheimer assumption
$\mathbf{v}_i$	Darcy or superficial velocity of phase i
v_n	velocity component taken in the direction of ∂P
v_p	average pore velocity in porous media
$\mathbf{v}^V$	volume average velocity of mixture with respect to the solid matrix
$v(D)$	random part of velocity of a marked particle due to dispersion
V	volume; volume of pores with entry diameters $< D_e$
V'	molar volume of liquid
V_a	adsorbed volume of adsorbate in mL gas STP per grain of adsorbent
V_B	bulk volume of porous medium
$V^*_{\ell k}$	filled volume of a pore in class ℓ in penetration step k ($\ell = 1, 2, 3, \ldots, k$)
V_m	volume of gas in mL STP that should be able to cover the whole surface with a monolayer
$\bar{V}_m$	partial molar volume
V_o	STP number
V_p	aggregate volume of particles; pore volume
V_T	total pore volume of porous medium
V_v	volume fraction
$V(D)$	volume of pore of diameter D
$V'(D)$	volume of cylindrical capillary
$V(R)$	cumulative distribution of pore volume with R
$V(t)$	transverse component of velocity of a marked particle [Eq. (6.3.83)]
x	position coordinate
x	$\equiv D_\ell/D_s$ [Eq. (3.3.9)]
x	$\equiv P''/P_0$ [Eqs. (2.5.6) to (2.5.8)]
x'	$= x - ut$ or $x - v_{DF,x}t$
x_i	position coordinate
X	distance of plane from center of sphere [Eq. (1.2.12)]; adsorbed liquid volume [Eq. (2.5.23)]; "electrical" tortuosity factor
X_b	volume fraction of network consisting of bonds
X_n	coordinate of displacement of marked particle
y	position coordinate
y	$\equiv \nu_\ell/\nu_s$ [Eq. (3.3.9)]
y	$\equiv \ell_\ell/\ell_s$ [Eq. (3.3.10)]
y'	$= y - v_{DF,y}t$
y_A	mole fraction of A
y_k	fractional number of open bonds $j \leq k$ that are penetrated
Y_k	fractional number of bonds that are penetrated if bonds $j \leq k$ are open
Y_n	coordinate of displacement of marked particle
z	coordination number; average number of branches (bonds) meeting at a node [Eq. (2.5.30)]; position coordinate; distance measured vertically upward
z'	$= z - v_{DF,z}t$
z_i	number of bonds meeting at a node of type i [Eq. (2.5.30)]
Z	coordination number defined by Eq. (2.5.34a)
Z_r	quantity defined by Eq. (2.5.34b)
$Z(x_i)$	value of property at x_i

Greek Letters

α	bivariate density function [Eq. (1.2.16)]
α	$= 1/k$ [Eq. (3.2.17)]
$\alpha_p(D)$	volume density function of size distribution of capillaries in parallel [Eq. (3.3.20)]
$\alpha_s(D)$	volume density function of size distribution of capillaries in series [Eq. (3.3.21)]
$\alpha(D_e)\,dD_e$	fraction of pore volume characterized by entry diameters between D_e and $D_e + dD_e$
αR	porosity function [Eq. (3.3.62)]
β	bivariate density function Eq. (1.2.12), Eq. (1.2.23); dimensionless larger spacing Eq. (1.2.33); inertia parameter Eq. (3.2.17); exponent in Eq. (2.5.41); angle of directional cosine; constant in Eq. (7.3.50); second-order memory fluid parameter Eq. (3.5.4)
β	$= \gamma^2 - 1$ in Eq. (6.2.22)
$\beta(D, D_e)\,dD\,dD_e$	fraction of pore volume characterized by diameters between D and $D + dD$ and pore entry diameters between D_e and $D_e + dD_e$
$\Gamma(\tau)$	shear rate; function defined by Eq. (6.3.53)
γ	dimensionless coefficient in Eqs. (6.3.15) and (6.3.16); angle of directional cosine
γ	$= b/a$ in Fig. 6.9c
γ	$= c_p/c_v$
$\gamma(D_e)$	density function of pore entry diameter
$\gamma(L)$	variogram
Δ	difference
δ	diameter; diameter of 2-D features; length of "yardstick" in fractals
$\tilde{\delta}$	$= \overline{R}/\bar{\ell}$ [Eq. (6.3.101)]
η	hydraulic diffusivity [Eq. (1.4.2)]; individual fiber collection efficiency
η_ϕ	Darcy viscosity [Eq. (3.5.7)]
θ	contact angle; polar angle
θ	$= \phi S_w$ = volumetric moisture content
κ	$= R_i/R_o$ of an annulus
κ	$= \mu_o/\mu_w$ = viscosity of original fluid/viscosity of injection fluid = viscosity ratio
κ	$= \mu_2/\mu_1$ (2 = displacing fluid) (Fig. 5.82)
Λ	characteristic length
λ	ratio of interface velocities in the two branches of a pore doublet; mean free path; pore size distribution index [Eq. (5.2.21)]; dimensionless coefficient in Eqs. (6.3.15) and (6.3.16)
λ_i	kriging factor
μ	dynamic viscosity; chemical potential
μ	$\equiv \cos\theta$ in Eq. (6.3.80)
μ_{ik}	number of penetrated tubes of size i in Eq. (2.5.31); mean value
$\overline{\mu}_o$	"oil phase" viscosity in Eq. (6.3.114)
ν	kinematic viscosity
ν_ℓ, ν_s	number of large and fine capillaries, respectively
ν_p	$= t/T$ = number of pore volumes of effluent collected at time t
ξ	constant in Eq. (6.3.49)
ξ	$\equiv \ell \Big/ \left(\dfrac{R_e^2}{r_{dr}} - \dfrac{R^2}{r_{imb}} \right)$ Eq. (5.3.87)
ξ_i	local coordinate in the fixed coordinate system x_i [(Eq. (6.3.91)]
π	osmotic pressure
π	$= \sigma/\mu v$ (p. 424)

ρ	mass density; radial distance
ρ_w	resistivity of water (electrolyte)
Σ_i	total nimber of ions given by one mole of electrolyte (Eq. (4.2.13b))
σ	surface tension or interfacial tension; retention; distance measured along streamline
σ_k^2	variance of permeability
σ_x	standard deviation of tracer concentration in x-direction
σ^0	area of an adsorption site
σ^2	variance
τ	$= t\mathscr{D}/R^2$ [Eq. (6.2.14)]
τ	$= V/Q$ (mean) residence time
τ_w	wall shear stress in capillary tube
τ_ϕ	wall shear stress in bead pack
ϕ	porosity; azimuthal angle; piezometric head; angle; volume fraction; osmotic coefficient; half cone angle in Eq. (2.3.1)
ϕ^*	volume fraction
ϕ_c	capillary head
ϕ_{eff}	drainable porosity [Eq. (5.2.12)]
ϕ_i	angle defined in Fig. 2.60; volume fraction concentration of i
ψ	any point function [Eq. (3.3.57)]; viscosity level parameter [Eq. (3.5.11)]
Ω	angle of rotation
ω	angular velocity

Subscripts

a	denotes advancing; air; adsorbed layer; adsorption
b	denotes breakthrough or bubbling pressure; denotes bond; bulb
BL	denotes Buckley–Leverett
c	denotes capillary quantity; critical value; calculated quantity
ct	denotes capillary tube
d	denotes desorption
dr	denotes drainage
e	denotes entry
eff	denotes effective value
exp	denotes measured quantity
f	denotes pore fluid; front
g	denotes gas
H	denotes hydraulic diameter or conductivity
HP	denotes Hagen–Poiseuille
i	denotes "irreducible" saturation
i, j, k, ℓ	dummy index denoting pore size category
imb	denotes imbibition
ℓ	denotes liquid; large quantity
liq	denotes liquid
m	denotes mean value; monolayer
ma	denotes rock matrix
mt	denotes multilayer
n	denotes narrow
nw	denotes nonwetting phase
o	denotes entrance or inlet to tube
p	denotes pore

r	denotes receding; reference phase
s	denotes solid; site; small quantity
v	denotes vapor
w	denotes water or wetting phase; well; wide
1	denotes initial state
1, 2, 3	denotes dimension in space
2	denotes final state
λ	denotes wavelength

Special Symbols

∇	"del" or "nabla" operator
∇^*	$D\nabla$
∇^{*2}	$D^2\nabla^2$
δ	denotes a small quantity