SECOND EDITION

Vibration Analysis

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Vibration Analysis

To my wife, Myrtle

Preface

The purpose of this revision is to bring the treatment of the subject of vibration analysis into line with present practice. At the same time, it is hoped that the clearness of presentation and understandability which was characteristic of the previous edition have been retained. Basic principles and thoroughness of explanation are again emphasized, keeping the student's viewpoint firmly in mind.

The changes occur mainly in the latter half of the book. This makes the text suitable for a second, more advanced, course and does not disturb its usefulness for a first, basic, course.

An explanation of SI units has been introduced in Chapter 1, and both engineering units and SI units are used throughout the text. This double system is followed because the United States will be in a period of transition to the metric system which will extend for several years. In this connection, it is recognized that the system of units followed, while not unimportant, is not a major concern in the subject of vibrations and, accordingly, should not be overemphasized.

For the most part, the changes in the first six chapters are not major in nature and are mainly for the purpose of clarity and understandability. In Chapter 5 the section on response to irregular forcing conditions has been replaced and represents a numerical procedure currently in use.

In Chapter 7, on two-degree-of-freedom systems, matrix notation has been introduced separately as an alternate representation of equation systems, and thus may be employed or not, depending on course arrangement and length.

The applicability of the first seven chapters to a beginning course in vibrations has been maintained. The adaptability to a first course of varying length, depending on the number of course credits available, has also been retained. Accordingly, a most abbreviated course might cover Chapters 1 through 4, treating single-degree-of-freedom systems. If more time is available, Chapter 5 on response to general forcing conditions and/or Chapter 6 on instrumentation may be included. Chapter 7 covering two-degrees-of-freedom systems, which is the simplest form of multidegree systems, can also be included to improve the adequacy of coverage of a beginning course. For a course of greater extent, Chapter 8 on multidegree-of-freedom systems would be included, in which case, Chapter 7 could be omitted if desired.

A second, more advanced, course would cover the latter portion of the text, starting with Chapter 8.

Revisions in Chapter 8 are extensive, involving the use of matrices and the resulting matrix concepts and procedures, and thus follows current treatment for such material. Numerical methods for multidegree systems have been separated from Chapter 8 and are placed in a new Chapter 9 on numerical analysis for lumped systems. The numerical analyses have also been revised and rearranged. Transfer matrices have been introduced and followed reasonably extensively, forming the basis for certain of the numerical procedures, including the Myklestad method.

The main change in Chapter 10 (formerly Chapter 9) is the simplification of the treatment of boundary conditions for continuous systems.

In Chapter 11 (formerly Chapter 10), on computer techniques, a section on electromechanical analogies has been included. The section on analog computers and computation has been rearranged and simplified. The numerical analyses related to digital computation and programs have been coordinated with corresponding developments in other parts of the text. A section on digital computation for the Continuous System Modeling Program has also been included.

An appendix has been added. This includes parts on matrices and matrix algebra, the development of Lagrange's equation, and units and dimensional values and conversions between systems of units.

Throughout the text, new illustrative examples have been introduced, problems have been revised, and many new problems have been added.

I am grateful to those who aided in the revision of this book, particularly to Dr. Vernon Neubert, The Pennsylvania State University, who is mainly responsible for the revision of Chapter 11 on computer methods; to Dr. John G. Bollinger, University of Wisconsin, and Dr. O. E. Adams, Ohio University, for their considerate reviews and suggestions, and to many students for their acceptance and response; to Mrs. Marilyn Day for the capable typing of manuscript; and especially to my wife, Myrtle, for her help and understanding.

ROBERT K. VIERCK

Preface to the First Edition

The aim of this book is to present the analysis of vibrations in a basic manner so that the reader may gain a fundamental understanding of this field and the methods which are peculiar to its investigation. The book is intended for a first course in vibrations for college seniors or beginning graduate students. The text was composed from class notes, written and revised over a period of several years and used in teaching courses in vibrations. It presumes no previous knowledge of the subject, but does expect the student to have a good foundation in calculus, differential equations, and dynamics. Although a prior course in strength of materials would be helpful, it is not necessary.

The book, in essentially its present context, was classroom tested by a rather large number of students, with good response and feedback later from graduates in industry. Subsequently, some revisions and additions were made and incorporated into the manuscript.

In writing this book, I have tried to retain the student's viewpoint. For the student's sake, each topic is presented in greater detail and length than is customary. In derivations and developments, steps which appear to be essential for continuity of understanding have been included. Reasons and meaning which seem necessary in developing an idea have been readily stated. The temptation to leave out intermediate steps and omit meaningful statements has been resisted. Naturally, these features increase the length of treatment for each topic. However, brevity of presentation is a questionable virtue in an introductory text. It is hoped that the student will find the text to be useful in his or her study, and that, indirectly, this will aid the instructor.

Some subject matter is included which is not found in the usual text on vibrations. Also, certain material is given greater coverage than is common. For example, the basic theory of instrumentation is presented in detail and explained more fully than is customary. The criteria for dynamic stability are developed for relatively simple conditions and are then extended to the general case for systems of several degrees of freedom. The principle of orthogonality is derived for the general case involving both dynamic and static coupling, and then is modified to apply to the case of static coupling only.

In the arrangement of exercises for assignment, in general, several problems in which similar situations occur are grouped together. This enables problems to be assigned in succeeding semesters, or terms, without duplication.

No claim is made of having adequately covered the entire field of vibrations. Indeed, this is impossible in a single book of reasonable length. The difficult choice faced in writing a general text on vibrations is deciding what important topics one can afford to omit. A major hope is that the text will enable readers to gain a basic understanding of the subject so that they can progress to more advanced material, and that their interest will have been stimulated to do so.

I am grateful to those who contributed toward the preparation of this book, especially to the late Dr. Joseph Marin, who encouraged the undertaking; to Dr. John G. Bollinger, University of Wisconsin, for the preparation of the material on computer methods; to a host of former students for their response and stimulating interest—particularly to Mr. M. C. Patel and Mr. J. S. Patel for their willing assistance in checking manuscript and proof; to Mrs. R. F. Trufant for the competent typing of the manuscript; and to my wife, Myrtle, for her encouragement and assistance.

ROBERT K. VIERCK

List of Symbols

Symbol	Meaning	U.S.-British Engineering Units	SI Units
a_{ij}	flexibility coefficient	in./lb	m/n
a, b, c	coefficients, constants, lengths, dimensions		
b	hysteresis damping constant		
b	natural frequency ratio		
A	area	in.^{2}	m^2
A	amplifier gain		
$B = EI$	section stiffness in bending	lb in.^{2}	N · m^2
A, B, C, D	constants, amplitudes		
c	wave velocity	in./sec	N/s
c	viscous damping constant	lb sec/in.	N · s/m
c_c	critical viscous damping constant	lb sec/in.	N · s/m
c_e	equivalent viscous damping constant	lb sec/in.	N · s/m
C	electric capacitance	farad	F
$\bar{C}$	complex amplitude		
d	dimension, diameter	in.	m
d_{ij}	tangential deviation	in.	m
D	determinant		
e	eccentricity	in.	m
e, E	electric voltage (electromotive force)	volt	V
E	tension-compression modulus of elasticity	lb/in.^{2}	Pa = N/m^2
E	error or residue of Holzer calculations		
f	frequency	Hz	Hz
F	force, transmitted force	lb	N
F_d	damping force	lb	N

Symbol	Meaning	U.S.-British Engineering Units	SI Units
F_T	maximum transmitted force	lb	N
$[F]$	field transfer matrix		
g	acceleration of gravity	in./sec^2	m/s^2
G	modulus of rigidity (shearing modulus of elasticity)	lb/in.^{2}	Pa = N/m^2
h	dimension, height	in.	m
$h(t)$	response to unit impulse		
H	determinant		
i	$\sqrt{-1}$		
i, I	electric current	amp	A
i, j, k	indices		
I	moment of inertia of area	in.^{4}	m^4
I, J	mass moment of inertia	lb in./sec^2	kg · m^2
j	interval number		
J	polar moment of inertia	in.^{4}	m^4
k	spring constant or modulus, elastic constant	lb/in.	N/m
k_{ij}	stiffness coefficient	lb/in.	N/m
k_T, K_T	torsional spring constant	lb in./radian	N · m/rad
l	length	in.	m
L	electric inductance	henry	H
m	mass	lb sec^2/in.	kg
m_o	eccentric mass	lb sec^2/in.	kg
M	mass	lb sec^2/in.	kg
M	bending moment	lb in.	N · m
M_t	twisting moment	lb in.	N · m
MF	magnification factor		
M, N	coefficients, constants		
n	general number, modal number		
n	gear ratio		
N	normal force	lb	N
p, q	real numbers		
$P, P(t)$	force	lb	N
P_o	constant force, force amplitude	lb	N
$[P]$	point transfer matrix		
q	time-interval width or length	sec	s
q	normal coordinate		
q	electric charge	coulomb	C
q_k	generalized coordinate		
Q_k	generalized force		
$[Q]$	transfer matrix		
$r = \omega_f/\omega$	frequency ratio		
r, R	radius	in.	m
R	electric resistance	ohm	Ω
$\mathscr{R}$	real part of		
s	exponential coefficient, root of equation		
S	scale factor		
t	time	sec	s
t', t''	time offset	sec	s
T	kinetic energy, kinetic energy function	in. lb	J = N · m

Symbol	Meaning	U.S.-British Engineering Units	SI Units
T	torque	lb in.	$N \cdot m$
$T = T(t)$	function of t only		
$\bar{T}$	function of amplitudes		
TR	transmissibility		
u	displacement	in.	m
U	work or energy	in. lb	$J = N \cdot m$
$U = U(x)$	displacement function		
ΔU	energy change	in. lb	$J = N \cdot m$
V	potential energy, potential-energy function	in. lb	$J = N \cdot m$
V	shear force	lb	N
$\bar{V}$	function of amplitudes		
w	intensity of loading	lb/in.	N/m
w, W	weight	lb	N
x	displacement	in.	m
x_o	initial displacement	in.	m
x_a	complementary function		
x_b	particular solution		
x_λ	wavelength	in.	m
$\dot{x}$	velocity	in./sec	m/s
$\dot{x}_o$	initial velocity	in./sec	m/s
$\ddot{x}$	acceleration	in./sec^2	m/s^2
$\ddot{x}_o$	initial acceleration	in./sec^2	m/s^2
$(\text{sgn } \dot{x})$	sign of $\dot{x}$		
X	displacement amplitude	in.	m
$X_o = P_o/k$	displacement reference	in.	m
$X = X(x)$	function of x only		
$\bar{X}$	complex amplitude		
$\dot{X}$	velocity amplitude	in./sec	m/s
$\ddot{X}$	acceleration amplitude	in./sec^2	m/s^2
y	displacement	in.	m
Y	displacement amplitude	in.	m
z	relative displacement	in.	m
$z = \omega_f t$	time related variable	radians	rad
Z	relative displacement amplitude	in.	m
Z	electric impedance	volt/amp	V/A
α_{ij}	slope influence coefficient in bending	radian	rad
α, β, γ	coefficients, constants, angles, exponents		
β	hysteresis damping coefficient		
β, γ	phase angle	radians	rad
γ	specific mass		
γ	hysteresis exponent		
δ	deflection, displacement, deformation	in.	m
δ	logarithmic decrement		
Δ	static displacement	in.	m
ε	constant		
ε	unit strain	in./in.	m/m
$\zeta = \dfrac{c}{c_c}$	damping factor		

Symbol	Meaning	U.S.-British Engineering Units	SI Units
η	constant, coefficient		
θ	slope	radians	rad
θ	angular displacement	radians	rad
$\dot{\theta}$	angular velocity	radians/sec	rad/s
$\ddot{\theta}$	angular acceleration	radians/sec^2	rad/s^2
θ_o	angular-displacement amplitude	radians	rad
λ	inverse frequency factor		
μ	coefficient of friction		
μ	mass ratio, constant		
ν	number of intervals		
$[\nu]$	state vector (matrix)		
ξ	direct frequency ratio		
ρ	weight or mass density		
σ	exponential coefficient		
σ	unit stress	lb/in.^{2}	Pa = N/m^2
τ	period	sec	s
τ	time at which impulse segment is imposed	sec	s
ϕ	phase, phase angle	radians	rad
ϕ	scale factor		
ϕ	magnetic flux	weber	Wb
χ	frequency factor		
χ	phase angle	radians	rad
ψ	phase angle	radians	rad
ω	natural circular frequency	radians/sec	rad/s
ω_d	damped natural circular frequency	radians/sec	rad/s
ω_f	forced circular frequency	radians/sec	rad/s
ω_R	Rayleigh circular frequency	radians/sec	rad/s

Contents

chapter 1

Introduction

1-1. PRELIMINARY REMARKS

The primary aim of this chapter is to introduce, in a relatively simple manner, the subject of vibrations. Some essential definitions are set down and general concepts are established. No attempt is made to be complete, as many ideas and the related terms and definitions become meaningful only when they are developed as significant features of specific topics and conditions, and consequently are left until later.

The present chapter also contains a partial review of kinetics. Again, no effort is made to be complete, but some of the important principles and relations of motion are recorded. A discussion of units and dimensional values is also included.

1-2. THE NATURE OF VIBRATIONS

The study of vibrations treats the oscillatory motion of mechanical systems and the dynamic conditions related thereto. This motion may be of regular form and repeated continuously, or it may be irregular or of a random nature. Vibrations are accompanied by, or are produced by, forces that vary in an oscillatory manner.

Although the term "vibration" usually implies a mechanical oscillation, similar conditions prevail in other areas, such as for alternating electric circuits, electromagnetic waves, and acoustics. This condition may be related, in some manner, in different fields; for example, a mechanical vibration may cause an acoustical vibration or sound. A mechanical vibration may produce an electric oscillation, or vice versa. The basic principles, analyses, mathematical formulations, and terminology for oscillatory phenomena are similar in the various fields. The vibration of mechanical systems *only* will be considered in this text.

1-3. VIBRATORY MOTION AND SYSTEMS

In order for a mechanical vibration to occur, a minimum of two energy-storage elements is required—a mass which stores kinetic energy, and an elastic member which stores potential energy. These can be represented as in Fig. 1-1, where m and k are the mass and the elastic elements, respectively. Assuming that horizontal movement is prevented, if m is displaced vertically from its equilibrium position and released, it will exhibit an oscillatory vertical motion. Such motion is repeated in equal time intervals and hence is said to be cyclic or periodic. If the elastic element is linear (that is, if the spring force is proportional to its deformation), then the motion curve of the mass displacement against time will be sinusoidal in form. This is called *harmonic motion* and is shown in Fig. 1-2, where x is displacement and t is time. The difference between the motions of parts (a), (b), and (c) is entirely due to the initial conditions of displacement and velocity. The maximum displacement X is generally referred to as the displacement *amplitude*; the term ϕ represents the *phase* or *phase angle*, and ω is a constant called the *circular frequency*.

Certain conditions produce cyclic or periodic motion that is not harmonic. A motion of this type is represented in Fig. 1-3. One complete movement of any repeated motion is called a *cycle*. The time for one cycle is termed the *period*. It is designated by τ and is generally measured in seconds. The *frequency* is the number of cycles of motion occurring in unit time. The symbol for frequency is f, and the most common unit is cycles per second, which are called hertz (Hz). Note that τ is the reciprocal of f. Thus

$$f = \frac{1}{\tau} \quad \text{and} \quad \tau = \frac{1}{f} \tag{1-1}$$

A vibration can also be of an irregular nature, such as that shown in Fig. 1-4. Here there is no repeated part to the movement, although many of the peak displacement values may occur again and again. This type of motion, for which there is no apparent pattern in the vibration record, is called a *random vibration*. A random vibration is produced by input forces of an irregular nature acting on the vibratory system. Such random forces occur in missiles and space vehicles, due to aerodynamic buffeting during launching. Packaged assemblies of structural and mechanical equipment are subjected to random forces while they are being shipped or transported.

A type of motion related to vibrations is the short-time response of a