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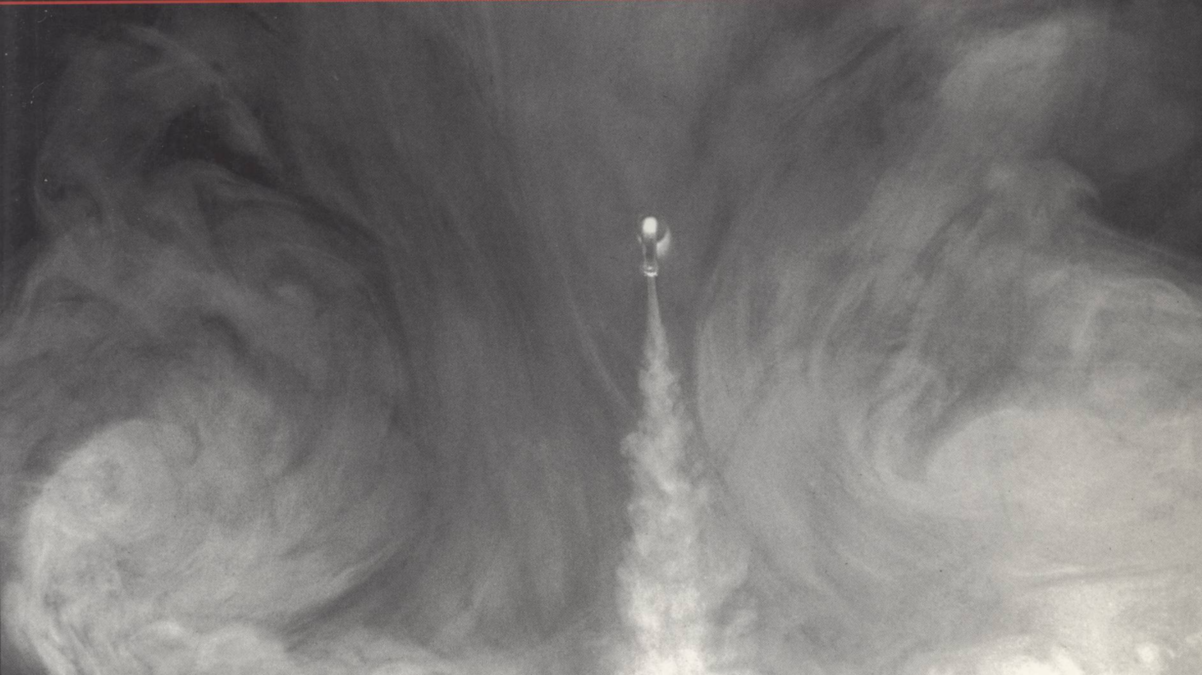
STATISTICAL THEORY AND MODELING FOR TURBULENT FLOWS



P. A. Durbin

B. A. Pettersson Reif

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P. A. Durbin
Stanford University, Stanford, USA

B. A. Petterson Reif
Norwegian Defence Research Establishment
Norway

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John Wiley & Sons (Canada) Ltd, 22 Worcester Road,
Rexdale, Ontario M9W 1L1, Canada

Library of Congress Cataloging-in-Publication Data

Durbin, P. A.
Theory and modeling of turbulent flows / P. A. Durbin, B. A. Petterson Reif
p.cm
Includes bibliographical references and index.
ISBN 0 471 49736 3 (ppc) ISBN 0 471 49744 4 (paper)
1.Turbulence — Mathematical models. I. Reif, B. A. Petterson. II. Title

QA913. D94 2000
532'. 0527—dc21 00-043687

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

ISBN 0 471 49736 3 (ppc) ISBN 0 471 49744 4 (paper)

Produced from LaTeX files supplied by the author
Printed and bound in Great Britain by Antony Rowe Ltd, Chippenham, Wilts
This book is printed on acid-free paper responsibly manufactured from sustainable forestry, in which at least two trees are planted for each one used for paper production.

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Preface

This book evolved out of lecture notes for a course taught in the Mechanical Engineering department at Stanford University. The students were at M. S. and Ph. D. level. The course served as an introduction to turbulence and to turbulence modeling. Its scope was single point statistical theory, phenomenology, and Reynolds averaged closure. In preparing the present book the purview was extended to include two-point, homogeneous turbulence theory. This has been done to provide sufficient breadth for a complete introductory course on turbulence.

Further topics in modeling also have been added to the scope of the original notes; these include both practical aspects, and more advanced mathematical analyses of models. The advanced material was placed into a separate chapter so that it can be circumvented if desired. Similarly, two-point, homogeneous turbulence theory is contained in part III and could be avoided in an M. S. level engineering course, for instance.

No attempt has been made at an encyclopedic survey of turbulence closure models. The particular models discussed are those that today seem to have proved effective in computational fluid dynamics applications. Certainly, there are others that could be cited, and many more in the making. By reviewing the motives and methods of those selected, we hope to have laid a groundwork for the reader to understand these others. A number of examples of Reynolds averaged computation are included.

It is inevitable in a book of the present nature that authors will put their own slant on the contents. The large number of papers on closure schemes and their applications demands that we exercise judgement. To boil them down to a text requires that boundaries on the scope be set and adhered to. Our ambition has been to expound the subject, not to survey the literature. Many researchers will be disappointed that their work has not been included. We hope they will understand our desire to make the subject accessible to students, and to make it attractive to new researchers.

An attempt has been made to allow a lecturer to use this book as a guideline, while putting his or her personal slant on the material. While single point modeling is decidedly the main theme, it occupies less than half of the pages. Considerable scope exists to choose where emphasis is placed.

Motivation

It is unquestionably the case that closure models for turbulence transport are finding an increasing number of applications, in increasingly complex flows. Computerised fluid dynamical analysis is becoming an integral part of the design process in a growing number of industries: increasing computer speeds are fueling that growth. For instance, computer analysis has reduced the development costs in the aerospace industry by decreasing the number of wind tunnel tests needed in the conceptual and design phases.

As the utility of turbulence models for computational fluid dynamics (CFD) has increased, more sophisticated models have been needed to simulate the range of phenomena that arise. Increasingly complex closure schemes raise a need for computationalists to understand the origins of the models. Their mathematical properties and predictive accuracy must be assessed to determine whether a particular model is suited to computing given flow phenomena. Experimenters are being called on increasingly to provide data for testing turbulence models and CFD codes. A text that provides a solid background for those working in the field seems timely.

The problems that arise in turbulence closure modeling are as fundamental as those in any area of fluid dynamics. A grounding is needed in physical concepts and mathematical techniques. A student, first confronted with the literature on turbulence modeling, is bound to be baffled by equations seemingly pulled from thin air; to wonder whether constants are derived from principles, or obtained from data; to question what is fundamental and what is peculiar to a given model. We learned this subject by ferreting around the literature, pondering just such questions. Some of that experience motivated this book.

Epitome

The prerequisite for this text is a basic knowledge of fluid mechanics, including viscous flow. The book is divided into three major parts.

Part I provides background on turbulence phenomenology, Reynolds averaged equations and mathematical methods. The focus is on material pertinent to single point, statistical analysis, but a chapter on eddy structures is also included.

Part II is on turbulence modeling. It starts with the basics of engineering closure modeling, then proceeds to increasingly advanced topics. The scope ranges from integrated equations to second moment transport. The nature of this subject is such that even the most advanced topics are not rarefied; they should pique the interest of the applied mathematician, but should also make the R & D engineer ponder the potential impact of this material on her or his work.

Part III introduces Fourier spectral representations for homogeneous turbulence theory. It covers energy transfer in spectral space and the formalities of the energy cascade. Finally rapid distortion theory is described in the last section. Part III is intended to round out the scope of a basic turbulence course. It does not address the intricacies of two-point closure, or include advanced topics.

A first course on turbulence for engineering students might cover part I, excluding the section on tensor representations, most of part II, excluding chapter 8, and a brief mention of selected material from part III. A first course for more mathematical students might place greater emphasis on the latter part of chapter 2 in part I, cover a limited portion of part II — emphasizing chapter 7 and some of chapter 8 — and include most of part III. Advanced material is intended for prospective researchers.

Acknowledgments

Finally, we would like to thank those who have provided encouragement for us to write this book. Doubts over whether to write it at all were dispelled by Cinian Zheng-Durbin; she was a source of support throughout the endeavor.

We gratefully acknowledge the conducive environment created by the Stanford/NASA

Center for Turbulence Research, and its director Prof. P. Moin. This book has benefited from our interactions with visitors to the CTR and with its post-doctoral fellows. Our thanks to Dr. L. P. Purtell of the Office of Naval Research for his support of turbulence modeling research over the years. We have benefited immeasurably from many discussions and from collaboration with the late Prof. C. G. Speziale. Interactions with Prof. D. Laurence and his students have been a continual stimulus. Prof. J. C. R. Hunt's unique insights into turbulence have greatly influenced portions of the book.

Stanford, California 2000

Contents

Where under this beautiful chaos can there lie a simple numerical structure?
— Jacob Bronowski

1

Introduction

Turbulence is an ubiquitous phenomenon in the dynamics of fluid flow. For decades, comprehending and modeling turbulent fluid motion has stimulated the creativity of scientists, engineers and applied mathematicians. Often the aim is to develop methods to predict the flow fields of practical devices. To that end, analytical models are devised that can be solved in computational fluid dynamics codes. At the heart of this endeavor is a broad body of research, spanning a range from experimental measurement to mathematical analysis. The intent of this text is to introduce some of the basic concepts and theories that have proved productive in research on turbulent flow.

Advances in computer speed are leading to an increase in the number of applications of turbulent flow prediction. Computerised fluid flow analysis is becoming an integral part of the design process in many industries. As the use of turbulence models in computational fluid dynamics increases, more sophisticated models will be needed to simulate the range of phenomena that arise. The increasing complexity of the applications will require creative research in engineering turbulence modeling. We have endeavored in writing this book both to provide an introduction to the subject of turbulence closure modeling, and to bring the reader up to the state of the art in this field. The scope of this book is certainly not restricted to closure modeling, but the bias is decidedly in that direction. To flesh out the subject a broader presentation of statistical turbulence theory is provided in the chapters that are not explicitly on modeling. In this way an endeavor has been made to provide a complete course on turbulent flow. We start with a perspective on the problem of turbulence that is pertinent to this text. Readers not very familiar with the subject might find some of the terminology unfamiliar; it will be explicated in due course.

1.1 The Turbulence Problem

The turbulence problem is an age-old topic of discussion among fluid dynamicists. It is not a problem of physical law; it is a problem of description. Turbulence is a state of fluid motion, governed by known dynamical laws — the Navier-Stokes equations in cases of interest here. In principle turbulence is simply a solution to those equations. The turbulent state of motion is defined by the complexity of such hypothetical solutions. The challenge of description lies in the complexity: how can this intriguing behavior of fluid motion be represented in a manner suited to the needs of science and engineering?

Turbulent motion is fascinating to watch: it is made visible by smoke billows in the atmosphere, by surface deformations in the wakes of boats, and by many laboratory techniques involving smoke, bubbles, dyes, etc. Computer simulation and digital image processing show intricate details of the flow. But engineers need numbers as well as pictures, and scientists need equations as well as impressions. How can the complexity be fathomed? That is the turbulence problem.

Two characteristic features of turbulent motion are its ability to stir a fluid and its ability to dissipate kinetic energy. The former mixes heat or material introduced into the flow. Without turbulence these substances would be carried along streamlines of the flow and slowly diffuse by molecular transport; with turbulence they rapidly disperse across the flow. Energy dissipation by turbulent eddies increases resistance to flow through pipes and it increases the drag on objects in the flow. Turbulent motion is highly dissipative because it contains small eddies that have large velocity gradients, upon which viscosity acts. In fact, another characteristic of turbulence is its continuous range of scales. The largest size eddies carry the greatest kinetic energy. They spawn smaller eddies via non-linear processes. The smaller eddies spawn smaller eddies, and so on in a cascade of energy to smaller and smaller scales. The smallest eddies are dissipated by viscosity. The grinding down to smaller and smaller scales is referred to as the *energy cascade*. It is a central concept in our understanding of stirring and dissipation in turbulent flow.

The energy that cascades is first produced from orderly, mean motion. Small perturbations extract energy from the mean flow and produce irregular, turbulent fluctuations. These are able to maintain themselves, and to propagate by further extraction of energy. This is referred to as production, and transport of turbulence. A detailed understanding of such phenomena does not exist. Certainly these phenomena are highly complex and serve to emphasize that the true problem of turbulence is one of analyzing an intricate phenomenon.

The term ‘eddy’, used above, may have invoked an image of swirling motion round a vortex. In some cases that may be a suitable mental picture. However, the term is usually meant to be more ambiguous. Velocity contours in a plane mixing layer display both large and small scale irregularities. Figure 1.1 illustrates an organization into large scale features with smaller scale random motion superimposed. The picture consists of contours of a passive scalar introduced into a mixing layer. Very often the image behind the term ‘eddy’ is this sort of perspective on scales of motion. Instead of vortical whorls, eddies are an impression of features seen in a contour plot. Large eddies are the large lumps seen in the figure, small eddies are the grainy background. Further examples of large eddies are discussed in the chapter of this book on coherent and vortical structures.

A simple method to produce turbulence is by placing a grid normal to the flow in a wind tunnel. Figure 1.2 contains a smoke visualization of the turbulence downstream of the bars of a grid. The upper portion of the figure contains velocity contours from a numerical simulation of grid turbulence. In both cases the impression is made that, on average, the scale of the irregular velocity fluctuations increases with distance downstream. In this sense the average size of eddies grows larger with distance from the grid.

Analyses of turbulent flow inevitably invoke a statistical description. Individual eddies occur randomly in space and time and consist of irregular regions of velocity or vorticity. At the statistical level, turbulent phenomena become reproducible and subject to systematic study. Statistics, like the averaged velocity, or its variance, are orderly and develop regularly in space and time. They provide a basis for theoretical descriptions and for a diversity of prediction methods. However, exact equations for the statistics do not exist. The objective of research in this field has been to develop mathematical models and physical concepts to

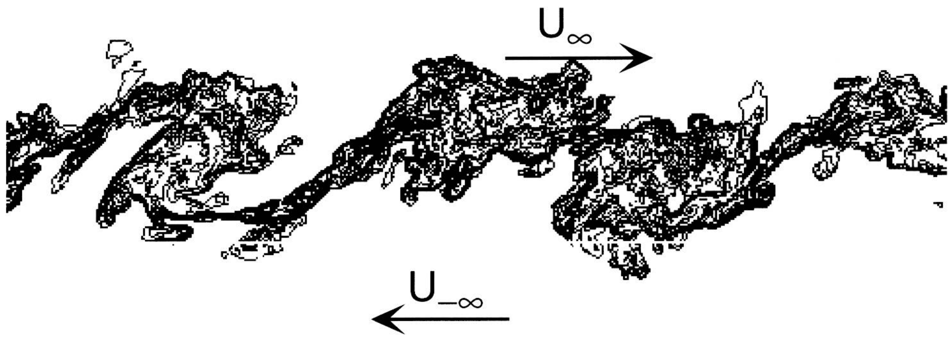


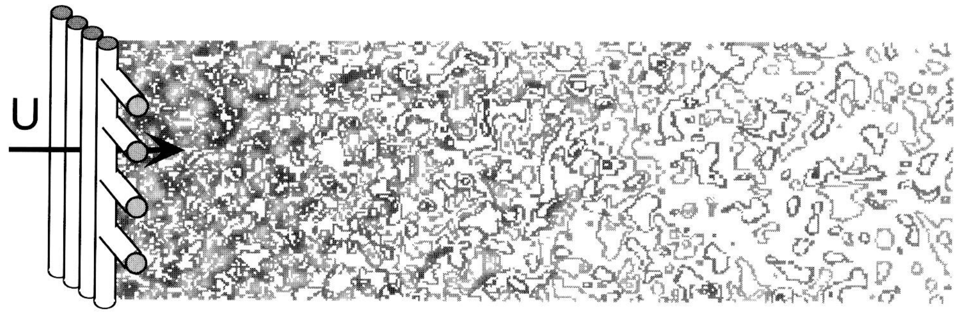
Figure 1.1 *Turbulent eddies in a plane mixing layer subjected to periodic forcing. From Rogers & Moser (1994), reproduced with permission.*

stand in place of exact laws of motion. Statistical theory is a way to fathom the complexity. Mathematical modeling is a way to predict flows. Hence the title of this book: statistical theory and modeling for turbulent flows.

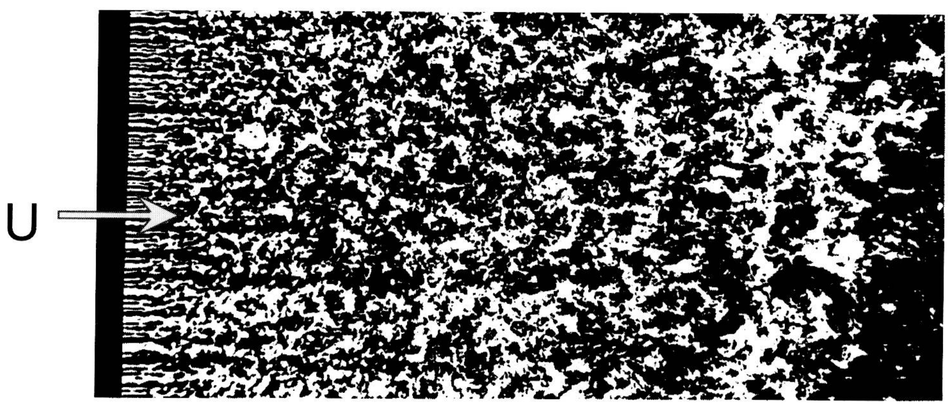
The alternative to modeling would be to solve the three-dimensional, time-dependent Navier-Stokes equations to obtain the chaotic flow field, and then to average the solutions in order to obtain statistics Such an approach is referred to as direct numerical simulation (DNS). Direct numerical simulation is not practical in most flows of engineering interest. Engineering models are meant to bypass the chaotic details and to predict statistics of turbulent flows directly. A great demand is placed on these engineering closure models: they must predict the averaged properties of the flow without requiring access to the random field; they must do so in complex geometries for which detailed experimental data do not exist; they must be tractable numerically and not require excessive computing time. These challenges make statistical turbulence modeling an exciting field.

The goal of turbulence theories and models is to describe turbulent motion by analytical methods. The particular methods that have been adopted depend on the objectives: whether it is to understand how chaotic motion follows from the governing equations, to construct phenomenological analogues of turbulent motion, to deduce statistical properties of the random motion, or to develop semi-empirical calculational tools. The latter two are the subject of this book.

The first step in statistical theory is to greatly compress the information content from that of a random field of eddies to that of a field of statistics. In particular, the turbulent velocity consists of a three component field (u_1, u_2, u_3) as a function of four independent variables (x_1, x_2, x_3, t). This is a rapidly varying, irregular flow field, such as might be seen embedded in the billows of a smoke stack, the eddying motion of the jet in figure 1.3, or the more explosive example of figure 1.4. In virtually all cases of engineering interest, this is more information than could be used, even if complete data were available. It must be reduced to a few useful numbers, or functions, by averaging. The picture to the right of figure 1.4 has been blurred to suggest the reduced information in an averaged representation. The small-scale structure in smoothed by averaging. A true average in this case would require repeating the explosion many times and summing the images; even the largest eddies would be lost to smoothing. A stationary flow can be averaged in time, as illustrated by the time-lapse



(a)



(b)

Figure 1.2 *a) Grid turbulence schematic, showing contours of streamwise velocity from à numerical simulation. b) Turbulence produced by flow through a grid. The bars of the grid would be to the left of the picture, and flow is from left to right. Visualization by smoke wire of laboratory flow, courtesy of T. Corke & H. Nagib.*

photograph at the right of figure 1.3. Again, all semblance of eddying motion is lost in the averaged view.

An example of the greatly simplified representation invoked by statistical theory is provided by grid turbulence. When air flows through a grid of bars the fluid velocity produced is a complex, essentially random, three-component, three-dimensional, time-dependent field, that defies analytical description (figure 1.2). This velocity field might be described statistically by its variance, q^2 as a function of distance downwind of the grid. q^2 is the average value of $u_1^2 + u_2^2 + u_3^2$ over planes perpendicular to the flow. This statistic provides a smooth function that characterizes the complex field. In fact, the dependence of q^2 on distance downstream of the grid is usually represented to good approximation by a power-law: $q^2 \propto x^{-n}$ where n is about 1. The average length scale of the eddies grows like $L \propto x^{1-n/2}$. This provides a simple formula that agrees with the impression created by figure 1.2 of eddy size increasing with x.

The catch to the simplification which a statistical description seems to offer is that it is only

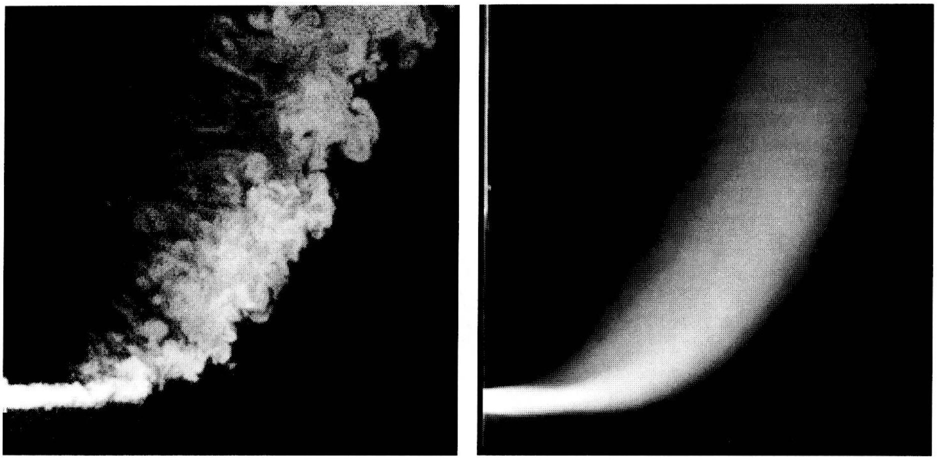


Figure 1.3 *Instantaneous and time averaged views of a jet in cross flow. The jet exits from the wall at left into a stream flowing from bottom to top (Su & Mungal, 1999).*

a simplification if the statistics somehow can be obtained without having first to solve for the whole, complex velocity field and then compute averages. The task is to predict the smooth jet at the right of figure 1.3 without access to the eddying motion at the left. Unfortunately there are no exact governing equations for the averaged flow, and empirical modeling becomes necessary. One might imagine that an equation for the average velocity could be obtained by averaging the equation for the instantaneous velocity. That would only be the case if the equations were linear, which the Navier-Stokes equations are not.

The role of non-linearity can be explained quite simply. Consider a random process generated by flipping a coin, assigning the value 1 to heads and 0 to tails. Denote this value by ξ. The average value of ξ is 1/2. Let a velocity, u, be related to ξ by the linear equation

$$u = \xi - 1. \tag{1.1.1}$$

The average of u is the average of $\xi - 1$. Since $\xi - 1$ has probability $1/2$ of being 0 and probability $1/2$ of being -1, the average of u is $-1/2$. Denote this average by $\overline{u}$. The equation for $\overline{u}$ *can* be obtained by averaging the exact equation: $\overline{u} = \overline{\xi} - 1 = 1/2 - 1 = -1/2$. But if u satisfies a non-linear equation

$$u^2 + 2u = \xi - 1 \tag{1.1.2}$$

then the averaged equation is

$$\overline{u^2} + 2\overline{u} = \overline{\xi} - 1 = -1/2. \tag{1.1.3}$$

This is not a *closed** equation for $\overline{u}$ because it contains $\overline{u^2}$: squaring, then averaging, is

* The terms 'closure problem' and 'closure model' are ubiquitous in the literature. Mathematically this means that there are more unknowns than equations. A closure model simply provides extra equations to complete the unclosed set.

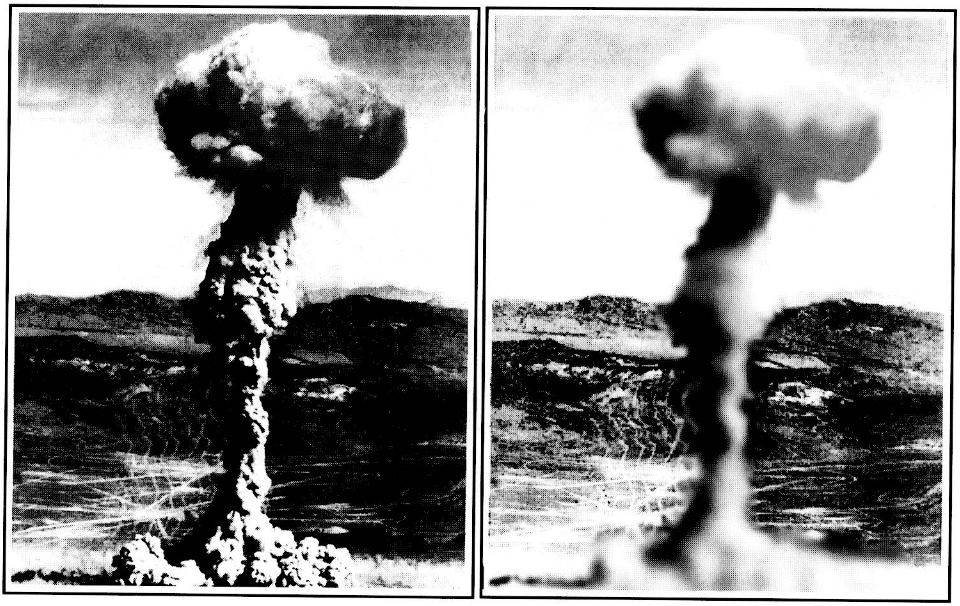


Figure 1.4 *Large and small scale structure in a plume. The picture at the right is blurred to suggest the effect of ensemble averaging.*

not equal to averaging, then squaring, $\overline{u^2} \neq \overline{u}^2$. In this example averaging produces a single equation with two dependent variables, $\overline{u}$ and $\overline{u^2}$. The example is contrived so that it first can be solved, then averaged: its solution is $u = \sqrt{\xi} - 1$; the average is then $\overline{u} = (1/2)(\sqrt{1} - 1) + (1/2)(\sqrt{0} - 1) = -1/2$. Similarly $\overline{u^2} = 1/2$, but this could not be known without first solving the random equation, then computing the average. In the case of the Navier-Stokes equations, one cannot resort to solving, then averaging. As in this simple illustration, the average of the Navier-Stokes equations are equations for $\overline{u}$ that contain $\overline{u^2}$. Unclosed equations are inescapable.

1.2 Closure Modeling

Statistical theories of turbulence attempt to obtain statistical information either by systematic approximations to the averaged, unclosed governing equations, or by intuition and analogy. Usually, the latter has been the more successful: the Kolmogorov theory of the inertial subrange and the log-law for boundary layers are famous examples of intuition.

Engineering closure models are in this same vein of invoking systematic analysis in combination with intuition and analogy to close the equations. For example, Prandtl drew an analogy between turbulent transport of averaged momentum by turbulent eddies and the kinetic theory of gasses when he proposed his 'mixing length' model. Thereby he obtained a useful model for predicting turbulent boundary layers.

The allusion to 'engineering flows' implies that the flow arises in a configuration that has technological application. Interest might be in the pressure drop in flow through a bundle of heat-exchanger tubes or across a channel lined with ribs. The turbulence dissipates energy and increases the pressure drop. Or the concern might be with heat transfer to a cooling