Donald L. Stancl
Mildred L. Stancl

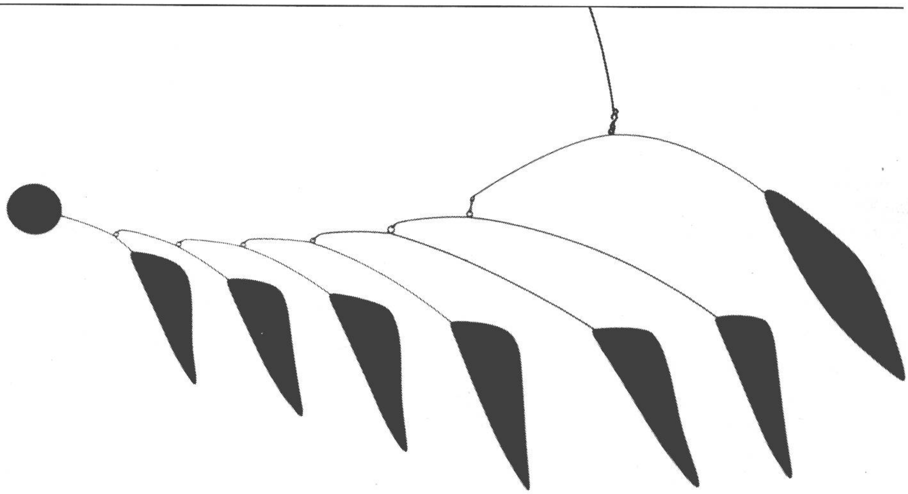
Applications of College Mathematics

MANAGEMENT, LIFE, AND SOCIAL SCIENCES

Donald L. Stancl
Mildred L. Stancl
Nichols College

Applications of College Mathematics

MANAGEMENT, LIFE, AND SOCIAL SCIENCES



D. C. Heath and Company Lexington, Massachusetts Toronto

To
George A. Stancl
with love

Photo Credits

Cover: Yellow Whale, 1958 by Alexander Calder. Painted sheet metal, wire, 45 inches wide. Collection of Howard and Jean Lipman, New York. Photograph by Arthur Vitols.

Title page, Chapter 7, and Chapter 11: Details of *Dots and Dashes,* 1959 by Alexander Calder. Painted sheet metal, wire, 60 inches wide. Collection of Howard and Jean Lipman, New York. Photograph by Jerry L. Thompson.

Chapters 1 and 2: Details of *Mobile,* 1958 by Alexander Calder. Hirshhorn Museum and Sculpture Garden, Smithsonian Institution.

**Chapter 3:* Detail of *The Pistil,* 1931 by Alexander Calder. Wire, brass, 38 inches high. Collection of Whitney Museum of American Art. Gift of the Howard and Jean Lipman Foundation.

Chapter 4: Detail of *Zarabanda (Un Disco Blanco),* 1967 by Alexander Calder. Hirshhorn Museum and Sculpture Garden, Smithsonian Institution.

Chapters 5 and 12: Details of *Big Red,* 1959 by Alexander Calder. Painted sheet metal and steel wire. 74 × 114 inches. Collection of Whitney Museum of American Art. Gift of the Friends of the Whitney Museum of American Art.

**Chapter 6:* Detail of *The Chariot,* 1961 by Alexander Calder. Painted sheet metal, wire, 63 inches wide. Collection of Howard and Jean Lipman, New York. Photograph by Geoffrey Clements.

**Chapter 8:* Detail of *Agnes's Ring (The Circle),* 1935 by Alexander Calder. Vassar College Art Gallery, Poughkeepsie, New York. Gift of Agnes Rindge Claflin.

**Chapter 9:* Detail of cover

Chapter 10: Detail of *Effects of Red,* 1967 by Alexander Calder. Hirshhorn Museum and Sculpture Garden, Smithsonian Institution.

**Chapter 13:* Detail of *Performing Seal,* c. 1950 by Alexander Calder. Collection of Mr. and Mrs. Leonard J. Horwich.

**Chapter 14:* Detail of *Little Spider,* c. 1940 by Alexander Calder. Collection of Mr. and Mrs. Klaus G. Perls, New York.

* Complete reproductions of these mobiles appear in *Calder's Universe* by Jean Lipman, 1976. Whitney Museum of American Art, New York and Viking Press.

Published simultaneously in Canada.

Printed in the United States of America.

International Standard Book Number: 0-669-03860-1

Library of Congress Catalog Card Number: 82:82277

Applications of College Mathematics

Preface

Applications of College Mathematics is intended for a first course in college-level mathematics for students of management and the life and social sciences. The only prerequisite is intermediate algebra. Chapter 1 of the text provides a brief review of algebra for those students who need it.

An important feature of this book is the organization of the material into sections and exercise sets. Most numbered sections are divided into subsections, and the exercises at the end of each section are grouped according to the subsections. We do this for two reasons. First, by reading a subsection and working the exercises assigned to it before proceeding to the next subsection, the student can master the concepts presented one at a time in the order in which they are needed. We think this will contribute to a more positive and less frustrating learning experience than is often the case. Second, the arrangement offers the instructor a great deal of flexibility in the choice of topics to be covered and exercises to be assigned.

Other features of this text include:

Emphasis on Applications. Many topics are introduced via their applications, and applications are given whenever possible. An index of applications is given on the inside covers of the text.

Emphasis on Graphing. We believe that students at this level must be encouraged to draw and interpret graphs. Toward this end we have included more figures than are usually encountered in a text of this type and we have emphasized graphing in the exercises.

Many Examples. Every mathematical concept and technique used is illustrated with at least one example.

Many Exercises. We believe that students at this level need many drill and applied exercises. Thus, there are over 3000 exercises in this text. Many of the exercises, particularly those of an applied nature, have several parts. Answers to all odd-numbered exercises can be found at the back of the book.

Review Exercises. Each chapter concludes with a set of review exercises designed to test the student's knowledge of the topics covered in the chapter. Answers to all review exercises can be found at the back of the book.

Class-testing. This text has been class-tested for three years at Nichols College.

There are two supplements to this book: an *Instructor's Guide* and a *Computer Problem-Solving Guide*. The *Instructor's Guide* contains answers to all the even-numbered exercises in the text, as well as two sample examinations for each chapter. Answers are provided for the sample examinations. The *Computer Problem-Solving Guide* contains over 50 computer programs designed to aid in the solutions of certain types of problems in the text. The programs are written in minimal

BASIC and are exceptionally user-friendly. Each program is described, illustrated with a sample run, and listed in full. The Guide also contains exercises and suggestions for additional programs.

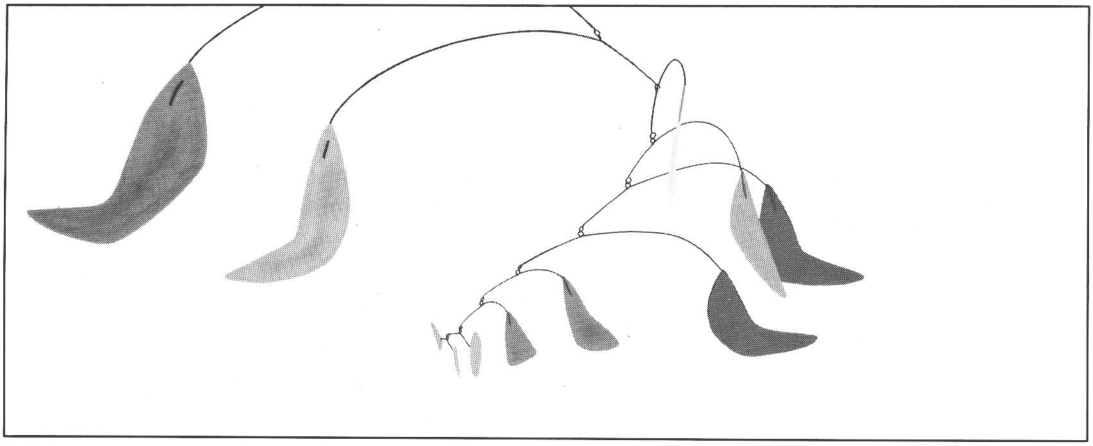
We would like to thank our colleagues at Nichols College, Professors Paul Creegan and William Steglitz, for helping us class-test this book. Our thanks also to Professor James Conrad and Mr. Gordon Benson of Nichols for their assistance. We appreciate the comments and criticisms of the following reviewers: Leonard D. Fountain, San Diego State University; Emma Garnett, Ball State University; Eugene Jacobs, Illinois State University; Terry L. Jenkins, University of Wyoming; Leonard J. Lipkin, University of North Florida; Robert A. Moreland, Texas Tech University; R. H. Rodine, Northern Illinois University; Erik A. Schreiner, Western Michigan University; Martha L. Stewart, University of North Carolina; and Robert L. Taylor, University of South Carolina.

Finally, we would like to express to Mary Lu Walsh, Cathy Cantin, Elizabeth Van de Kerkhove, and the rest of the staff at D. C. Heath and Company our sincere thanks for their suggestions and support.

Donald L. Stancl
Mildred L. Stancl

Contents

1 Review



This chapter consists of a brief review of number systems and algebra. A knowledge of these topics is basic to an understanding of the remainder of this text, and we urge you to read this chapter carefully in preparation for the material to come.

1.1 The Real Numbers

In this section we review the structure and properties of the real number system.

Number Systems

The **natural numbers** are the numbers

$$1, 2, 3, 4, 5, \ldots$$

(The three dots . . . mean "and so on.") The collection of all natural numbers is called the **natural number system** and is symbolized by the letter N. It is the simplest of the number systems we will use.

Another number system we will use is the **integers.** The integers are the numbers

$$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$$

The collection of all integers is the **system of integers,** symbolized by the letter Z. Note that every natural number is also an integer, and hence the system N of natural numbers is contained within the system Z of integers. For this reason the system N is sometimes called the system of **positive integers.**

Another useful number system is the **rational numbers.** A rational number is a number that can be written in the form m/n, where m and n are integers and $n \neq 0$. Some examples of rational numbers are the following:

$$\frac{1}{2}, \quad \frac{3}{17}, \quad -\frac{21}{5}, \quad -\frac{7}{3}, \quad \frac{824}{137}, \quad -\frac{741}{1000}$$

The collection of all rational numbers forms the **system of rational numbers,** symbolized by the letter Q. Since every integer m may be written as $m/1$ and hence is a rational number, it follows that the system Z of integers is contained in the system Q of rational numbers.

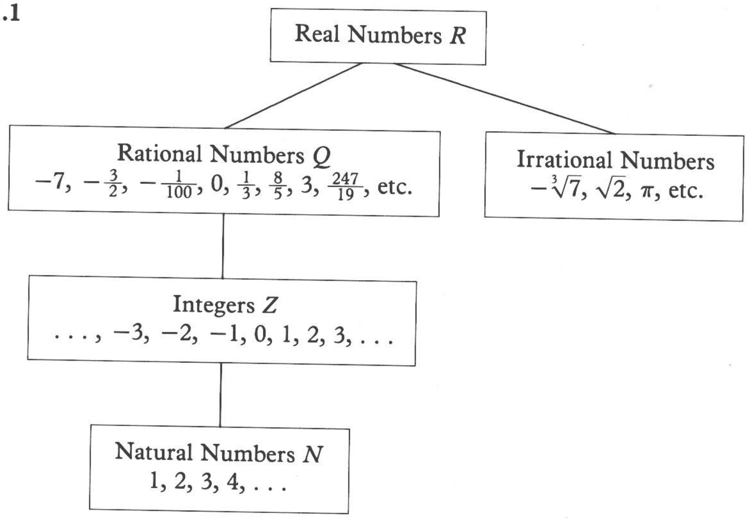
Many numbers cannot be written in the form m/n, where m and n are integers. Such numbers are not rational and hence are referred to as **irrational numbers.** Some examples of irrational numbers are the following:

$$\sqrt{2}, \quad \sqrt{5}, \quad -\sqrt[3]{7}, \quad \sqrt[4]{3}, \quad \pi$$

The **system of real numbers** consists of all rational numbers and all irrational numbers. Thus, every real number is either a rational number or an irrational number. We symbolize the real number system by the letter R. The relationships among the system R and its various subsystems are depicted in Figure 1.1.

Since every rational number has a decimal representation which either terminates, such as $\frac{5}{8} = 0.625$, or repeats, such as $\frac{1}{3} = 0.3333\ldots$, and since every irrational number has a decimal representation which neither terminates nor repeats, such as $\pi = 3.14159\ldots$, the system R includes all decimal numbers. On

Figure 1.1

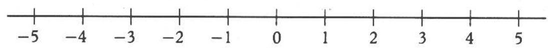


the other hand, it can be shown that every decimal number is the representation of either a rational or an irrational number. Hence we may think of the real number system as consisting of all possible decimal numbers.

The Real Line

The system of real numbers may be represented geometrically by a horizontal line called the **real line.** To construct the real line we choose a convenient point on a horizontal line to represent the number 0. This point is called the **origin.** We then choose a convenient **unit length** and, starting at the origin, mark off this length on the line over and over in both directions. The resulting points on the line represent the integers. See Figure 1.2.

Figure 1.2



The Real Line

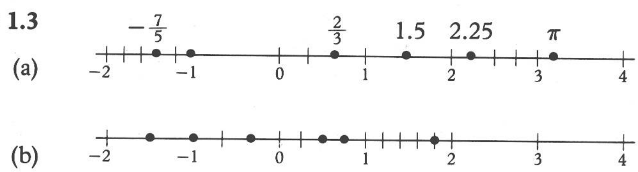
Every real number corresponds to a point on the real line, and, conversely, every point on the line corresponds to a real number.

EXAMPLE 1

(a) Plot the real numbers $-\frac{7}{5}$, -1, $\frac{2}{3}$, 1.5, 2.25, and π on the real line. These numbers are plotted in Figure 1.3(a).

(b) What real numbers are represented by the points plotted in Figure 1.3(b)? From left to right, the points plotted in the figure represent the numbers $-\frac{3}{2}$, -1, $-\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{9}{5}$.

Figure 1.3



Exercises: 1 and 2.

Inequalities

The real line makes it possible to compare real numbers using inequalities. If r and s are real numbers and if r lies to the left of s on the real line, we say that **r is less than**

s, written

$$r < s,$$

or that **s is greater than r,** written

$$s > r.$$

If $r < 0$, then r is a **negative** real number; if $s > 0$, then s is a **positive** real number. If **r is less than or equal to s,** we write

$$r \leq s;$$

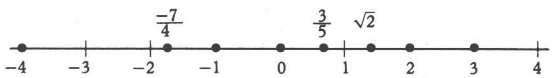
and if **s is greater than or equal to r,** we write

$$s \geq r.$$

EXAMPLE 2

Consider the real numbers, -4, $-\frac{7}{4}$, 2, 0, $\frac{3}{5}$, -1, $\sqrt{2}$, and 3. We plot these numbers on the real line. See Figure 1.4.

Figure 1.4



The following inequalities are *true*.

$$-4 < -\tfrac{7}{4} \qquad 0 \leq 0 \qquad 3 > 1 \qquad -1 < 0 \qquad -1 \leq 3$$
$$-1 > -4 \qquad 0 \geq 0 \qquad -1 \leq -1 \qquad \tfrac{3}{5} \geq 0 \qquad \tfrac{3}{5} < \sqrt{2}$$

The following inequalities are *false*.

$$-1 < -4 \qquad 3 \leq 2 \qquad 3 < 3 \qquad 0 \geq 2 \qquad -4 \geq -1$$
$$-1 > 0 \qquad -\tfrac{7}{4} > -\tfrac{7}{4} \qquad \tfrac{3}{5} > \sqrt{2} \qquad 3 \leq -1 \qquad -1 \geq \tfrac{3}{5}$$

Also, -4, $-\frac{7}{4}$, and -1 are negative real numbers, and $\frac{3}{5}$, $\sqrt{2}$, 2, and 3 are positive real numbers. The real number 0 is neither positive nor negative.

An inequality of the form $r < s < t$ is a **double inequality.** It is true provided that both $r < s$ and $s < t$ are true. In other words, the double inequality $r < s < t$ is true if and only if r is to the left of t and s is between r and t on the real line.

EXAMPLE 3

(a) The following double inequalities are *true*.

$$-2 < -1 < 1 \qquad -3 \leq 0 < 4 \qquad 1 < \tfrac{7}{4} \leq 2$$
$$-4 < 5 \leq 5 \qquad 1 < \sqrt{2} < 2 \qquad 3.12 < \pi < 3.20$$

(b) The following double inequalities are *false*.

$$-2 < -1 < -\tfrac{3}{2} \qquad 2 < 1 < 3$$
$$3 < 5 \leq 4 \qquad -1 < 4 < 4$$

(c) All inequality symbols in a double inequality should point in the same direction. Therefore, it is not permissible to write statements such as $2 < 3 > 1$. To express the inequality relationships among 2, 3, and 1 correctly, one should write $1 < 2 < 3$ or $3 > 2 > 1$.

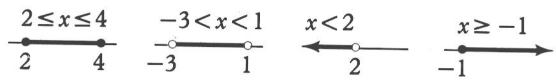
Exercises: 3 through 22.

Intervals

It is often convenient to consider all real numbers that lie between two given numbers or all those greater than or less than a given number. Collections of real numbers such as these are called **intervals.** For example, the collection of all real numbers x such that $0 \le x < 1$ is an interval because it consists of all real numbers between 0 and 1, including 0 but not including 1. Table 1.1 lists the various types of intervals. The line diagram corresponding to each interval shows how the interval is realized geometrically as a portion of the real line. Note that in the line diagrams an open circle at the endpoint of an interval indicates that the endpoint is not part of the interval, while a closed circle means that the endpoint is part of the interval.

EXAMPLE 4

The intervals correspond to the line diagrams below them.

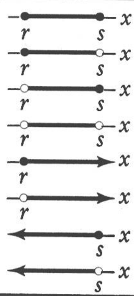


Exercises: 23 through 30.

Table 1.1 Intervals

Interval
$r \le x \le s$
$r \le x < s$
$r < x \le s$
$r < x < s$
$x \ge r$
$x > r$
$x \le s$
$x < s$

Line Diagram



Absolute Value

The absolute value of a real number is its distance from zero on the real line without regard to whether the number lies to the right or to the left of zero. For example, the real number 3 is at a distance of three from zero, as is the real number -3. Hence the absolute value of 3 and the absolute value of -3 are both equal to 3. More formally, we define the absolute value of a real number as follows:

> **Absolute Value**
>
> Let r be a real number. The **absolute value** of r, denoted $|r|$, is defined by
>
> $$|r| = \begin{cases} r, & \text{if } r \geq 0 \\ -r, & \text{if } r < 0. \end{cases}$$

Note that if r is nonnegative the absolute value is r, which is 0 if $r = 0$ and positive if $r \neq 0$. Furthermore, if r is negative, the absolute value is $-r$, which is positive. It follows that, for any real number r,

$$|r| = 0 \text{ if } r = 0$$

and

$$|r| \text{ is positive if } r \neq 0.$$

EXAMPLE 5

$$|2| = 2 \qquad |\sqrt{2}| = \sqrt{2}$$

$$|-2| = -(-2) = 2 \qquad |-1.729| = -(-1.729) = 1.729$$

$$|0| = 0 \qquad |6 - 4| = |2| = 2$$

$$\left|\tfrac{3}{5}\right| = \tfrac{3}{5} \qquad |4 - 6| = |-2| = 2$$

$$\left|-\tfrac{3}{5}\right| = -\left(-\tfrac{3}{5}\right) = \tfrac{3}{5} \qquad |(2)(-3)| = |(-2)(3)| = |-6| = 6$$

Exercises: 31 through 44.

Exercises 1.1

THE REAL LINE

1. Plot the following real numbers on a real line.