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Qualitative Theory of Hybrid Dynamical Systems

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Preface

Hybrid dynamical systems have attracted considerable attention in recent years. In general, hybrid dynamical systems are those that combine continuous and discrete dynamics and involve both continuous and discrete state variables. From an engineering viewpoint, a hybrid system is a network of digital and analog devices or a digital device that interacts with a continuous environment. The emerging area of hybrid dynamical systems lies at the crossroads of control theory and computer science: control theory contributes to the analog aspects of hybrid systems, and computer science contributes to the digital aspects. Driven by rapid advances in digital controller modern technology, hybrid dynamical systems are objects of increasing relevance and importance. However, at the present there is no systematic qualitative theory of hybrid dynamical systems. This book is concerned with development of such a theory. Although numerous journal and conference papers have appeared on the topic of hybrid systems, this book is one of the first monographs on this field.

This book is primarily a research monograph that presents in a unified fashion, some recent research on hybrid dynamical systems. The book is intended for both researchers and advanced postgraduate students in control engineering, theoretical computer science, and applied mathematics with an interest in the field of hybrid dynamical systems. The book consists mainly of the authors' original results and is essentially self-contained. Many of these results have not been published previously. The material presented in the monograph derives from a period of research collaboration between the authors from 1997 to 1999.

The authors are very grateful to Rob Evans who attracted the second author's attention to the area of hybrid systems and always strongly encouraged the research presented in the monograph. His advice, knowledge, and deep insight have been invaluable. This book would not have been possible without Rob's support. Our special thanks also go to our colleagues Ian Petersen, Andrey Barabanov, and Matthew James who have provided useful comments and suggestions. The stimulating research environment at the Department of Electrical and Electronic Engineering, The University of Western Australia, with its amazing academic culture has been an ideal setting for the development of the book. Also, the authors wish to acknowledge the support they have received from the Australian Research Council. Finally, the first author is grateful for the enormous support he has received from his wife Elena and daughter Julia.

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1
Introduction

1.1 Hybrid Dynamical Systems

The hybrid dynamical systems (HDS) of interest in this book are those that involve the interaction of discrete and continuous dynamics. These systems typically contain variables that take values from a continuous set (usually, the set of real numbers) and also variables that take values from a discrete set (e.g., the set of symbols $\{q_1, q_2, \ldots, q_n\}$).

There are many examples of hybrid dynamical systems. One well-known instance of a hybrid system is a dynamical system described by a set of ordinary differential equations with discontinuous or multivalued right-hand sides. Such mathematical models can be used to describe various engineering systems with relays, switches, and hysteresis. Properties of these hybrid systems have been studied in great detail for the past fifty years, especially in the Soviet literature (see e.g. [4, 23, 24, 40, 80]). Another existing area that has recently been brought under the hybrid systems framework is the study of sliding mode control [83].

In the linear control area, a typical example of a hybrid system is that which is created when a continuous-time plant described by differential equations is controlled by a digital regulator described by difference equations. These types of systems are studied in modern control engineering courses under the name of computer-controlled systems or sampled-data systems [7, 21, 37]. This is an extremely important area, because a consequence of the revolutionary advances in microelectronics is that practically all control systems implemented today are based on microprocessors and sophisticated microcontrollers. If we consider quantization of the continuous-valued variables, then the hybrid systems contain not only continuous-valued signals, but the discrete-valued variables as well.

A typical hybrid system is a logical discrete-event decision-making controller interacting with a continuous-time process. This model can be used to accurately describe a wide range of real-time industrial processes and their associated supervisory control and monitoring systems. A simple example is a home climate-control system. Due to its on-off nature, the thermostat is modelled as a discrete-event system, whereas the furnace and air-conditioner are modelled as continuous-time systems. Some other instances of such systems include automotive power train systems, computer disk drives, robotic systems, automotive engine management,high-level flexible manufacturing systems, intelligent vehicle/highway systems, sea/air traffic management, modern spacecraft control systems, job scheduling, interconnected power systems, chemical processes (see e.g. [9, 12, 25, 32, 44, 48, 60, 79, 86]).

Another example of a hybrid control system is a switched controller dynamical system. There are several theoretically interesting and practically significant problems concerning the use of switched controllers. In some situations it is possible to design several controllers and then switch between them to provide a performance improvement over a fixed controller, as well as new functionality [22, 53]. In other situations the choice of linear or nonlinear controllers available to the designer is limited and the design task is to use the available set of controllers in an optimal fashion [68–71, 75–77]. The latter problem includes, for example, the optimal switching between gears in a gear-box and the optimal switching between heating and cooling modes of operation in an air-conditioning plant.

Recently there has been a great deal of research activity in the area of hybrid control systems (see e.g. [5,6,8,10,13–15,17,26,27,30,33,34,39,42,46, 47,50,56,64,66,67,70,72,74,84,85,89]). This activity has been motivated in part by the development of the theory of discrete-event dynamical systems in the 1980s and 1990s [16, 31, 52, 57, 63]. At the same time there has been growing interest in hybrid dynamical systems among theoretical computer scientists and mathematical logicians [1, 2, 6]. In this literature, the most common example is a timed automaton. This is a hybrid system consisting of a set of simple integrators (clocks) coupled with a finite state automaton. Such systems can be used, for example, to model protocols with timing requirements and constraints. The main issue there is the verification that a hybrid system exhibits a desired behaviour. The verification problem is nontrivial and in many cases may be undecidable.

This book consists of original authors' results and is essentially self-contained. We apologize in advance to the many authors whose contributions have not been mentioned. The coverage in this brief overview is by no means complete. The literature in the field of hybrid systems is vast, and we limited ourselves to references that we found most useful, or that contain material supplementing the text.

In conclusion, the area of hybrid systems is a new, fascinating discipline bridging control engineering, theoretical computer science, and applied mathematics. In fact, many problems facing engineers and scientists, as they seek to use computers to control complex physical systems, naturally fit into the HDS framework. The study of hybrid dynamical systems represents a difficult and exciting challenge in control engineering. This field is referred to as "The Control Theory of Tomorrow" by SIAM News [28]. There is now an emerging literature on this topic describing a number of mathematical models, heuristic algorithms and stability criteria. However, at present there is no systematic qualitative theory of hybrid systems. This book is concerned with development of such a theory.

1.2 Two Contrasting Examples of Discretely Controlled Continuous Variable Systems

The research presented in this book has been motivated in part by two very interesting examples of the discrete control of a continuous variable system introduced in the paper [17] by Chase, Serrano, and Ramadge. These examples exhibit what may be regarded as two extremes of complexity of the behaviour of hybrid dynamical systems: one is eventually periodic, and the other is chaotic. They are of interest in their own right but have also been used to model certain aspects of flexible manufacturing systems [51, 60]. In this section, we describe these two examples following [17].

Example 1.2.1: A switched server system

Consider a system consisting of three buffers and one server. We refer to the contents of buffers as "work"; it will be convenient to think of work as a fluid, and a buffer as a tank. However, in manufacturing applications, work can represent a continuous approximation to the discrete flow of parts in a flexible manufacturing system [60]. Work arrives to the buffer j at a constant rate $p_j > 0$. The server removes work from a selected buffer at the unit rate. We assume that the system is closed, so that

$$p_1 + p_2 + p_3 = 1. \qquad (1.2.1)$$

The location of the server is a discrete control variable, and may be selected using a feedback policy.

The switched server system with three buffers and the server in the location 2 are shown in Fig.1.2.1.

This example can also be thought of as a simple instance of the switched controller problem (see e.g. [70, 75]).

The location of the server is selected based on quantized observation of the state, and the movement of the server is triggered by a "discrete event."

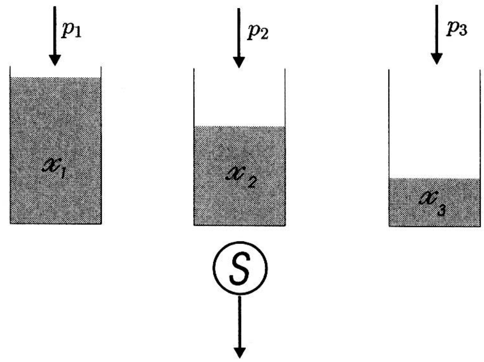

FIGURE 1.2.1. The switched server system with three buffers.

Now we show that this system can be described by a set logic-differential equations. Indeed, let $Q := \{q_1, q_2, q_3\}$ where $q_1, q_2, , q_3$ are symbols. The discrete state q_j where $j = 1, 2, 3$ corresponds to the case when the server is removing work from the buffer j, and the discrete state variable $q(t) \in Q$ describes the state of the server at time t. Let $x_j(t)$ be the amount of work in the buffer j at time t, and let

$$x(t) := \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}.$$

The state of the system at time t can be described by the pair $[x(t), q(t)]$. Furthermore, introduce the following vectors:

$$a(q_1) := \begin{pmatrix} p_1 - 1 \\ p_2 \\ p_3 \end{pmatrix}, \quad a(q_2) := \begin{pmatrix} p_1 \\ p_2 - 1 \\ p_3 \end{pmatrix}, \quad a(q_3) := \begin{pmatrix} p_1 \\ p_2 \\ p_3 - 1 \end{pmatrix}.$$

Then the above switched server system can be described by the following logic-differential equation:

$$\textbf{if} \quad q(t) = q_j \quad \textbf{then} \quad \dot{x}(t) = a(q_j). \tag{1.2.2}$$

In [17] a certain parametric class of server switching policies was considered. This class includes, in particular, the following quite natural policy.

SP1.2.1 The server switches as soon as the current buffer is emptied at time t and to the buffer j with the largest scaled content $\zeta_j(t) := c_j^{-1} x_j(t)$. (The coefficients $c_1 > 0, c_2 > 0$ and $c_3 > 0$ are given.)

SP1.2.2 Likewise, the server starts with the buffer that has the largest scaled content at $t = 0$.

This control policy does not specify what to do if the largest content is attained at two buffers. In this event, the server can be switched to the buffer with the least index j.

Note that the control policy **SP1.2.1, SP1.2.2** is a generalization of the *Clear-the-Largest-Buffer-Level Policy* studied in [60].

Let $\gamma > 0$ be a given constant. Introduce the set

$$K_\gamma := \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbf{R}^3 : \begin{array}{l} x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \\ x_1 + x_2 + x_3 = \gamma \end{array} \right\}.$$

Then it immediately follows from (1.2.1) that the set K_γ is invariant: if $x(0) \in K_\gamma$ for a solution $[x(t), q(t)]$, then $x(t) \in K_\gamma$ for all $t \geq 0$.

It was shown in [17], that for almost all values of the parameters c_1, c_2, c_3, the closed-loop system (1.2.2) with the switching policy **SP1.2.1, SP1.2.2** is eventually periodic in the following sense: For any $\gamma > 0$, there exists a finite number (no more than six) limit cycles lying in K_γ, and any trajectory from K_γ converges to one of them. As usual, "almost all" means "all but a set with zero Lebesgue measure."

Example 1.2.2: A switched arrival system

Like the switched server system, the second system to be considered consists of three buffers and one server (see Fig. 1.2.2). However, work is removed from the buffer j at a given constant rate $\rho_j > 0$. To compensate, the server delivers material to any selected buffer at the unit rate. As in the previous example, the location of the server is a control variable that can be chosen using a feedback policy. Again, we assume that the system is closed, i.e.,

$$\rho_1 + \rho_2 + \rho_3 = 1.$$

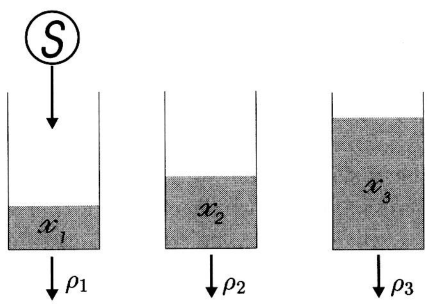

FIGURE 1.2.2. The switched arrival system with three buffers.

Introduce the following vectors:

$$a(q_1) := \begin{pmatrix} 1-\rho_1 \\ -\rho_2 \\ -\rho_3 \end{pmatrix}, \quad a(q_2) := \begin{pmatrix} -\rho_1 \\ 1-\rho_2 \\ -\rho_3 \end{pmatrix}, \quad a(q_3) := \begin{pmatrix} -\rho_1 \\ -\rho_2 \\ 1-\rho_3 \end{pmatrix}.$$

Then this system can be described by the equation (1.2.2). The control policy introduced in [17] consists in switching the server to an empty buffer when some buffer becomes empty. The singular case when more than one buffer is empty was ignored. It can be easily seen that the set of initial conditions that give rise to such singular trajectories is of zero Lebesgue measure. It was shown in [17] that the switched arrival system with this switching policy exhibits a chaotic behaviour.

1.3 The Main Goal of This Book

The examples in Section 1.2 explain what types of hybrid systems are studied in this book. It should be pointed out that in [17] only the case of systems with three buffers was considered. Because the set K_γ is invariant and planar, the systems with three buffers can be reduced to two-dimensional systems, which makes their analysis a much easier task. To extend the results of [17] to the case of systems with an arbitrary number of buffers is a quite nontrivial problem. Another interesting problem is to study various server switching strategies. Furthermore, a natural generalization of a switched server system is a switched flow network consisting of a number of interconnected buffers. Such networks can be used to model flexible manufacturing assembly/disassembly systems [60]. They can also be interpreted as models for various computer and communication systems, especially those with time-sharing schemes. The main goal of this research monograph can be stated as follows: *To develop a general qualitative theory of hybrid dynamical systems that will provide effective tools to analyze and describe the dynamics of various complex multidimensional generalizations of Examples 1.2.1 and 1.2.2.*

As a general mathematical model for flow networks, we employ the concept of a *differential automaton* introduced by Tavernini [78]. We should point out that a very similar mathematical model was considered by Witsenhausen in 1966 [87]. Roughly speaking, a differential automaton operates as follows. While the discrete state remains constant, the continuous one obeys a definite dynamical law. Transition to another discrete state implies a change of this law. In its turn, the discrete state evolves as soon as a certain event occurs, with both the evolution and the event depending on the continuous state.

Examples 1.2.1 and 1.2.2 show that some of differential automata exhibit chaotic behaviour whereas, under certain assumptions, the dynamics of other automata is eventually periodic. It is quite typical for differential automata to have no equilibrium points. Therefore, the simplest attractor in such systems is a limit cycle. The main results of this book describe some broad and important classes of hybrid dynamical systems such that any system from these classes satisfies the following properties:

(i) There exist a finite number of limit cycles.

(ii) Any trajectory of the system converges to one of these limit cycles.

Hence any trajectory of the system is asymptotically periodic and the system always exhibits a regular stable predictable behavior. This conclusion is very important for applications. We believe that the systems satisfying the properties (i) and (ii) play the same role in the field of hybrid systems as the globally stable systems do in the conventional continuous-time control theory.

Obtaining criteria for existence of self-excited oscillations or limit cycles is a very old and challenging problem of the classic qualitative theory of differential equations originated in the work of Poincaré and Lyapunov. Few constructive results are known for nonlinear systems of order higher than two, and it is even harder to study stability of limit cycles (see e.g. [3,45]). Our results show that constructive criteria for existence and global stability of limit cycles can be proved for quite general classes of hybrid dynamical systems. This appears to be surprising and gives us a hope that it is possible to develop a qualitative theory of some classes of hybrid dynamical systems that will be even more constructive than the classic qualitative theory of differential equations. We view this book as the first major step towards the development of such a theory.

Furthermore, we study switched flow networks with time-varying arrival rates and transportation delays. Such models are much more realistic, especially in the case of computer or communication networks. For these networks, we propose a decentralized control policy implementable in real time that guarantees a regular behavior of the closed-loop system.

1.4 Organization of the Book

The body of the book is organized as follows.

Chapter 2

In this chapter, we present a number of relatively simple examples to explain the intuitive ideas underlying the topics of this book. For this purpose, we introduce a special class of hybrid dynamical systems. We call these hybrid systems *cyclic linear differential automata (CLDA)*. We show that any CLDA can be reduced to a linear discrete-time system with periodic coefficients. Hence, qualitative analysis of such a hybrid system is a relatively easy task. We call a CLDA *globally periodic* if it has a limit cycle that attracts all other trajectories. A necessary and sufficient condition for global periodicity of a CLDA is given. Furthermore, we consider several switched server systems and prove existence and stability of limit cycles. Finally, we prove that the switched arrival system with an arbitrary number of buffers