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GEOMETRIC OPTICS:
An Introduction

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To PARK H. MILLER, Jr.,
Teacher and Friend

PREFACE

The usual way of teaching geometrical optics involves tracing light rays from one surface to another in the lens system. It is necessary to determine an image for each surface, and this image acts as the object for the succeeding one. Location of the final image may be a fairly complicated process for a system having several components.

The purpose of this small book is to show that the analysis of a lens system—no matter how complex—can be simplified, and made virtually automatic by combining elementary matrix algebra with some reasonable conventions. Matrix methods have the additional advantage of being easily adaptable to computers, and some typical programs are given as examples. It is hoped that this new approach to optics will not only make the basic principles easy to apply, but show the fascinating and exciting nature of this subject.

The arrangement of material that has been used here is adaptable to the reader's mathematical background. The first five chapters, which cover the elementary theory of lens systems, assume only a knowledge of algebra and trigonometry. Hence this portion of the book should be useful to students, such as those in technical institutes, who may not be familiar with calculus. Further, Chapters 1 through 4 cover topics which are usually included in the geometrical optics portion of college-level physics sequences. These four chapters would thus serve as a supplementary or alternative approach in such courses.

The last part of this book—Chapters 6, 7, and 8—require a knowledge of partial differentiation and multiple integration; topics considered in this part include light as a form of energy and the theory of aberrations; this material is appropriate to the course in geometrical optics customarily offered to physics majors.

As mentioned above, computer methods are used for examples which are tedious when worked by hand. For those readers who are unfamiliar with programming, a brief and self-contained introduction to FORTRAN will be found in Appendix 1. We have chosen this programming language, since it is the one in common use. Because FORTRAN, as normally taught, gives the impression of consisting of a large number of confusing rules, it is presented in this appendix in an unusual way. A specific problem—the multiplication of several 2×2 matrices—is posed, and then the step-by-step solution is given, permitting the reader to teach himself the rules as he needs them.

The matrix approach to optics used in this book is based primarily on the treatments given in the following books:

Brouwer, W., *Matrix Methods in Optical Instrument Design.* New York: W. A. Benjamin, Inc., 1964.

O'Neill, E. L., *Introduction to Statistical Optics.* Reading, Mass.: Addison-Wesley Publishing Co., 1963.

Leatham, G. G., *The Elementary Theory of the Symmetrical Optical Instrument.* New York: Hafner Publishing Co., 1960.

This material was originally developed in connection with a course offered at the IBM Company in Rochester, Minnesota. I am grateful for the suggestions received from this group of students, and especially the help of Dr. Robert Kulterman and Dr. Milton Chace.

The discussion of zoom lenses has benefited greatly from my correspondence with Dr. Klaus Halbach and from discussions with Professor Clayton Giese.

The manuscript for this book was reviewed by Dr. Adrian Walther. It is my great fortune to benefit from his experience as a teacher of optics and his practical knowledge.

Finally, I would like to acknowledge my debt to Professor James E. Holte, Director of Continuing Education in Science at the University of Minnesota, who encouraged me to become involved in this project, and to Mrs. Sharon Johnson, who did the typing.

Minneapolis, Minnesota A. N.
July 1968

TABLE OF CONTENTS

Chapter 1

ELEMENTARY PROPERTIES OF LENSES

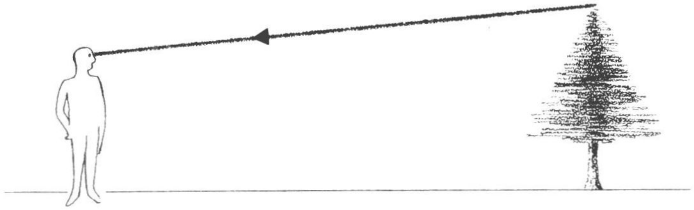

Fig. 1.1

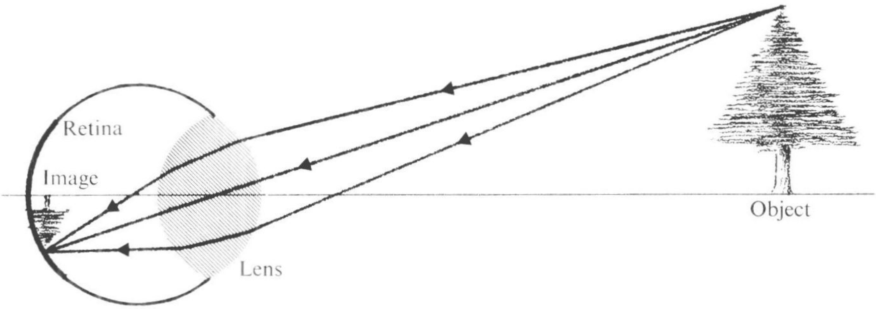

Fig. 1.2

1.1 THE CONCEPT OF AN IMAGE

Let us assume that light travels in straight lines or *rays*. Suppose that a person is looking at a tree (Fig. 1.1), and we want to know why he is able to see the tree. As the figure shows, at least one light ray from each portion of the tree strikes his eye. However, if his eye received only a single ray from each place, the tree would appear very dim; for him to see the tree clearly, a great many rays must leave each point of the tree (which we call an *object*) and come together on the retina of the observer's eye, as shown in Fig. 1.2. The intersection point of the rays from the top of the tree is called the *image* of the top. The lens of the eye bends or *refracts* almost all the rays leaving the object point so that they meet approximately at a single point to form the image; the eye's lens is said to *focus* the incoming rays. Figure 1.2 shows only three of a large number of the rays that are focused to produce a bright sharp image.*

Thus one function of any lens or optical system is to take all the rays leaving each point of the object and focus them to a point in the image. We shall see later what we mean by a "point" in the image, and we shall also see that an optical system must meet other requirements as well.

* Although we have been regarding the human eye as a simple camera, it is actually a very complicated organ; even the lens has a rather involved structure.

1.2 THE LENS EQUATIONS

It is possible to predict the size and position of the image formed by a thin lens if we make two reasonable assumptions about the properties of this lens:

1. The lens takes all rays of light parallel to its axis, such as the ray PQ at a distance o above the axis AA' of Fig. 1.3, and causes them to pass through a single point F', the *focal point* or *focus* of the lens.

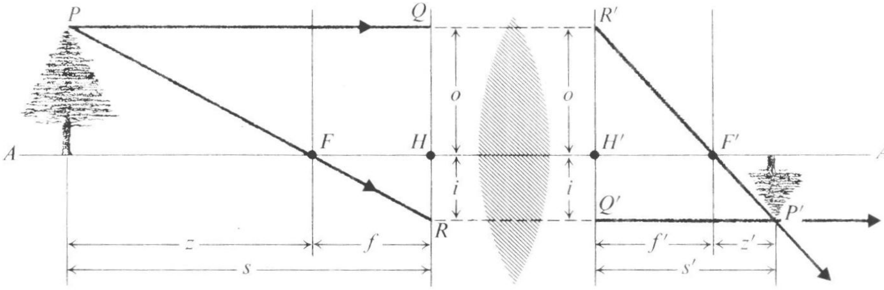

Fig. 1.3

2. An object located at point H at a distance f, the *focal length*, to the right of F will have an image of exactly the same size; for this condition, we say that the *magnification* is unity, and point H is called the *unit point*.

These two assumptions make it possible to trace the path of the light ray PQ from point R' on. In the first place, the ray emerges at R' on the plane $R'Q'$ at a distance o from the axis. This plane is the *unit plane*, corresponding to the position of an image for which the magnification is unity. Although we do not know the behavior of the light ray as it goes from Q through the lens to R', we do know that any ray leaving Q must cross the unit plane specified by H' at a distance $H'R' = o$ from the axis.

The other fact that we know about this ray is that it must pass through the focal point F' after it leaves R', since the original section PQ is parallel to the axis. Hence the path of this ray is $PQR'F'P'$ as shown.

Since a ray should trace the same path going in either direction, we may follow a second ray from P' back to P via Q', R, and F in the same fashion and then reverse its direction to obtain the path shown. This procedure may be applied to every point on the tree to obtain the image, which we note is inverted and smaller than the object.

There is one other feature in Fig. 1.3 that needs clarification: the size of the lens, which is shown as being twice as large as the tree. This makes it possible for the ray PQ to pass through the lens and reach R'. Actually, we need not worry about what happens between Q and R'; therefore, our choice of lens diameter can be arbitrary. We have chosen this unrealistically large lens to make the behavior of the light rays easier to visualize. Also, as we shall see in a later chapter, the unit planes for a moderately thin lens are closer together than Fig. 1.3 indicates; the points H and H' lie where the glass surfaces cross the axis AA'. For a very thin lens, H and H' both lie at the center of the lens.

We should note that Fig. 1.3 shows how we determine P' by finding the intersection of two specific rays leaving P and passing through the lens. We shall see that *all* the rays leaving the object point P and passing through the lens will meet again at P'. This is, of course, what we usually want a lens or optical instrument to accomplish.

Using the distances indicated on the diagram and the relations between corresponding sides of similar triangles, we can show that

$$\frac{i+o}{s} = \frac{o}{z} = \frac{i}{f}, \qquad \frac{o+i}{s'} = \frac{i}{z'} = \frac{o}{f'}, \tag{1.1}$$

from which it follows that

$$o = \frac{zi}{f} = \frac{if'}{z'}$$

or

$$zz' = ff'. \tag{1.2}$$

This is *Newton's form of the lens equation.*

Let us define the *magnification* β as the ratio of the image size to the object size, or

$$\beta = i/o. \tag{1.3}$$

Then, by using (1.1) and (1.2), we find that

$$\frac{f}{z} = \frac{i}{o} = \beta = \frac{z'}{f'} \tag{1.4}$$

and

$$i + o = \frac{si}{f} = \frac{s'o}{f'}.$$

Hence

$$\beta = i/o = s'f/sf'. \tag{1.5}$$

Further,

$$\frac{i+o}{i} = \frac{s}{f} = 1 + \frac{1}{\beta} = 1 + \frac{sf'}{s'f}$$

or

$$\frac{f'}{s'} + \frac{f}{s} = 1. \tag{1.6}$$

This is *Gauss' form of the lens equation*, and it is completely equivalent to (1.2).

Problem 1.1 Obtain Newton's equation from Gauss' equation.

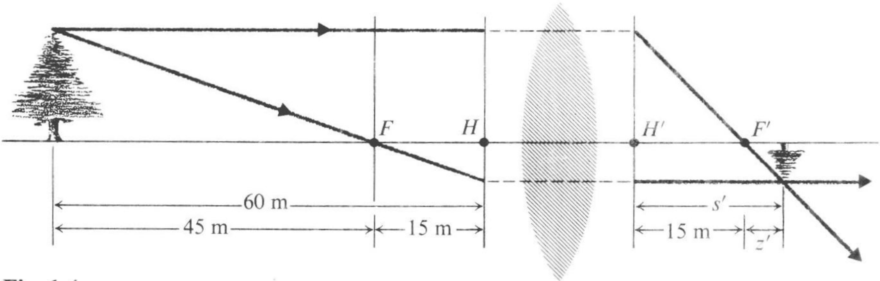

Fig. 1.4

Example 1.1 Symmetrical Convex Lens

A symmetrical lens of focal length equal to 15 m has its unit plane 60 m from a tree 10 m high. To find the position of the image and the magnification (Fig. 1.4), we use (1.6), where $f = f'$ for a symmetrical lens, since parallel light coming from either side of the lens will be focused in the same way. Hence

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f'}. \tag{1.7}$$

Then

$$\frac{1}{s'} = \frac{1}{f'} - \frac{1}{s} = \frac{1}{15} - \frac{1}{60} = \frac{3}{60}$$

or

$$s' = 20 \text{ m}.$$

Using (1.2), we have

$$z' = f'^2/z = \tfrac{225}{45} = 5 \text{ m},$$

which checks, since $s' = 15 + z'$. By using (1.5), we can calculate the magnification:

$$\beta = s'/s = \tfrac{20}{60} = \tfrac{1}{3}.$$

We therefore know that the image is reduced (and inverted).

Example 1.2 Symmetrical Concave Lens

To deal with concave lenses or lens systems, we must introduce *sign conventions.* Those which we shall use at this point are temporary; a permanent set of rules will be established in Chapter 4. The purpose of the present discussion is to give the reader a feeling for the behavior of systems which are more complicated than a simple lens, but less involved than those we shall eventually learn to treat.

Replacing the lens of Example 1.1 with the similar double concave lens of Fig. 1.5, we use the rules that were implicit in the first example. That is, s and s' are positive as measured from the center of the lens in the direction away from the center C; however, f' is taken as negative for a concave lens. Also, we shall label the unit planes from now on by using the single letters H and H'. Hence, by (1.7), we have

$$\frac{1}{s'} = \frac{1}{-15} - \frac{1}{60} = \frac{-5}{60}$$

or

$$s' = -12\text{ m}.$$

This means that the image must be to the *left* of the unit plane H'. To see how this comes about, we shall introduce the simplification that the lens is thin enough so that—as mentioned just before Eq. (1.1)—the planes H and H' fall at the center of the lens.

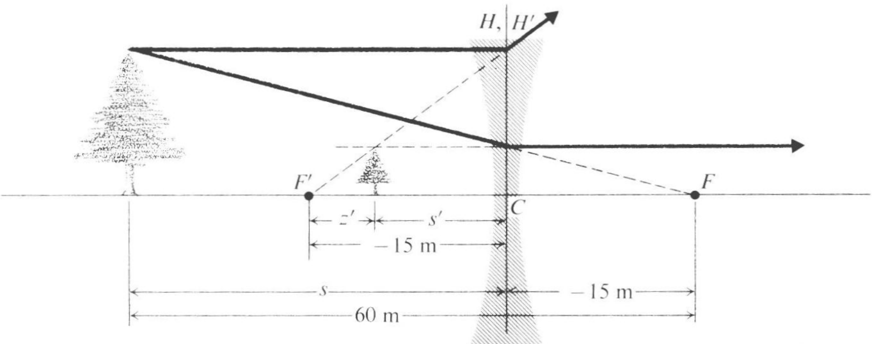

Fig. 1.5

We regard a concave lens as one for which the focal points F and F' are reversed from Fig. 1.4. Hence the light ray from the object which is parallel to the axis has its projection pass through F', and the other ray has its projection pass through F. Thus the image is on the same side of the lens as the object. The magnification is

$$\beta = s'/s = -12/60 = -1/5.$$

The negative value corresponds to an upright image. Figure 1.4 shows that

$$z = 75 \text{ m}, \qquad z' = 3 \text{ m}$$

and

$$(75)(3) = (-15)^2,$$

which again verifies Newton's formula.

Problem 1.2 An object with a height of 10 cm is 6 cm to the left of a symmetrical convex thin lens with a diameter of 5 cm and a focal length of 12 cm. Find the position of the image and the magnification by (a) making a sketch to scale and (b) using an equation.

Example 1.3 A Compound Lens

The combination of a convex and a concave lens, with focal lengths as indicated in Fig. 1.6, will form an image of the object shown. To find the position and size of this image, we shall consider the effect of each lens separately. The image in the convex lens has a position s', which Eq. (1.7) gives as

$$\frac{1}{s'} = \frac{1}{15} - \frac{1}{60} = \frac{1}{20}.$$

This image is therefore 20 cm to the right of the first lens and 9 cm to the right of the second one. Using Eq. (1.7) again, this time we have $s = -9$ cm and

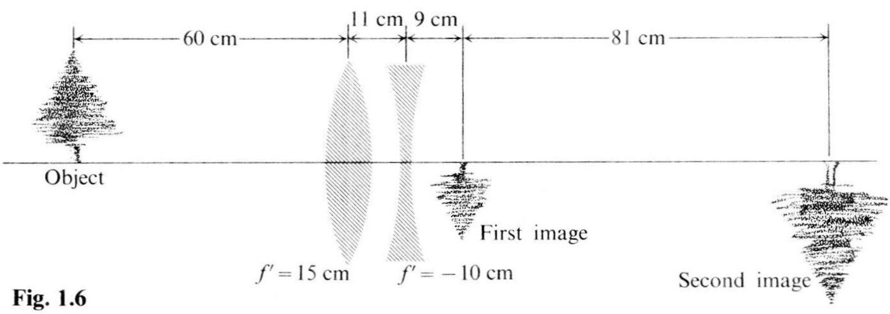

Fig. 1.6