SIMULATION OF CONTROL SYSTEMS

Edited by

I. TROCH, P. KOPACEK

and

F. BREITENECKER

IFAC Proceedings Series, 1987. Number 13

SIMULATION OF CONTROL SYSTEMS

Selected Papers from the IFAC Symposium, Vienna, Austria, 22–26 September, 1986

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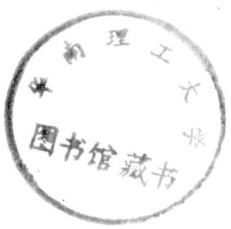

Published for the

INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL

by

PERGAMON PRESS

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U.K.	Pergamon Press, Headington Hill Hall, Oxford OX3 0BW, England
U.S.A.	Pergamon Press, Maxwell House, Fairview Park, Elmsford, New York 10523, U.S.A.
PEOPLE'S REPUBLIC OF CHINA	Pergamon Press, Room 4037, Qianmen Hotel, Beijing, People's Republic of China
FEDERAL REPUBLIC OF GERMANY	Pergamon Press, Hammerweg 6, D-6242 Kronberg, Federal Republic of Germany
BRAZIL	Pergamon Editora, Rua Eça de Queiros, 346, CEP 04011, Paraiso, São Paulo, Brazil
AUSTRALIA	Pergamon Press Australia, P.O. Box 544, Potts Point, N.S.W. 2011, Australia
JAPAN	Pergamon Press, 8th Floor, Matsuoka Central Building, 1-7-1 Nishishinjuku, Shinjuku-ku, Tokyo 160, Japan
CANADA	Pergamon Press Canada, Suite No. 271, 253 College Street, Toronto, Ontario, Canada M5T 1R5

First edition 1987

Library of Congress Cataloging-in-Publication Data

Simulation of control systems.

(IFAC proceedings series ; 1987, no. 13)
"IFAC Symposium on Simulation of Control Systems . . . sponsored by IFAC—Committee on Theory"—P.
Includes indexes.
1. Automatic control—Mathematical models—Congresses. 2. Automatic control—Data processing—Congresses. 3. Computer-aided design—Congresses. I. Troch, I. II. Kopacek, Peter. III. Breitenecker, F. (Felix). IV. IFAC Symposium on Simulation of Automatic Control (1986 : Vienna, Austria). V. International Federation of Automatic Control. Technical Committee on Theory. VI. Series.
TJ212.2.S555 1987 629.8'072'4 87–25718
ISBN 0–08–034349–X

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Printed in Great Britain by A. Wheaton & Co. Ltd., Exeter

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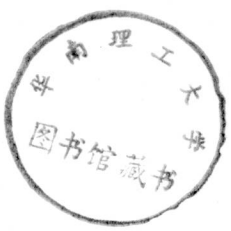

IFAC Proceedings Series, 1987. Number 13

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PREFACE

Simulation has been for a long time a valuable and widely used tool for the analysis of systems. Especially the control engineer has been familiar with this means for a long time. Simulation was used not only for the analysis of plants and controlled systems but also - and may be primarily - as a valuable remedy for controller design.

Further, simulation has turned out to be a valuable tool in the education of under-graduate and post-graduate students of control engineering. Simulation may equally well be used in continuing education programs. This is due to the fact that it allows a quick and instructive presentation and discussion of phenomena and methods. At the same time, simulation can be used quite easily in a laboratory allowing the student to test various types of controllers in connections with a 'real' plant. To some extent, simulation may even replace laboratory equipment allowing the analysis and design of systems at relatively low costs and without risks by a large number of students.

Whereas in old times of control engineering, analogue hardware was the main instrument for performing such simulation, nowadays mainly digital equipment is used because the latter has become more comfortable due to special simulation software and languages. Further, in most cases digital simulations can be performed now with sufficient speed.

Nevertheless, the development of new software as well as the improvement of existing software in order to meet in a better way the needs of control engineers is still going on and will continue for some years.

For these reasons it is not amazing that the IFAC committee on Theory decided to sponsor a Symposium on 'SIMULATION OF CONTROL SYSTEMS' and to ask the IFAC committees on Applications, on Computers and on Education as well as IMACS (the International Association for Mathematics and Computers in Simulation) to act as co-sponsors.

Speaking about SIMULATION means a good deal more than solving some more or less complicated equations or computing more or less useful and meaningful numbers. Simulation consists of performing experiments with a system - in most cases on a computer - of different physical nature but with analogous behaviour with respect to the problem to be investigated and solved.

One of the main goals of a simulation study is to get INSIGHT IN A SYSTEMS BEHAVIOUR, no matter whether a controlled or an uncontrolled system is investigated. Consequently, the task of MODELLING a given system in a satisfactory way is of primary concern in all simulations. Such a model has to be established in an adequate way. This means that the model must account for all properties of the real system which are of importance for the problem to be solved. But at the same time the model should not be to complicated in order not only to facilitate the necessary computations but also to allow a sufficiently accurate estimation of all required parameters and exogeneous functions of time.

Vienna, Autumn 1986

Consequently, questions of modelling, of model simplification but also theoretic aspects of modelling were discussed during the Symposium to some extent. Papers dealing primarily with this subject are collected in Session 1 of these Proceedings but the interested reader should consult also Session 2 where he may find papers dealing basically with systems analysis and design, but some of them deal in a short way also with interesting modelling aspects. Last but not least modelling aspects are addressed too in some of the invited papers, especially in those prepared by Lennart Ljung and Spyros Tzafestas.

Session 2 is devoted to the analysis and design of systems. In control engineering the ANALYSIS of SYSTEMS is of importance in two respects. Firstly, the analysis of the plant has to be performed, a topic which is covered by many papers on modelling, especially by papers presenting a case study on this subject. Secondly, the analysis of the controlled system is of intrinsic importance. This is due to the fact that it allows to judge whether or not the proposed controller will be adequate for the given control task.

The problem of CONTROLLER DESIGN can be termed as the central question within the papers collected in Session 2. Especially the use of so-called advanced control concepts as e.g. ADAPTIVITY and OPTIMALITY often require detailed simulation studies. This is not only due to the high theoretic requirements of such concepts but also to the complexity of the plants for which such controllers have to be designed.

Session 2 deals with theoretic aspects as well as with practical applications - as it was the case in Chapter 1. Yet, the reader interested in systems analysis and design should not forget to consult also the invited papers especially those by Katsuhira Furuta et al., P.M. Bruijn et al., D.P. Atherton and Gislain Vansteenkiste.

As already mentioned the improvement of existing and the development of new simulation tools is of actual and growing interest. Such tools exist mainly in the form of software packages and more or less specialised simulation languages. Nevertheless, also the development of new hardware or the combination of existing hardware by new interfaces are of interest.

Session 3 collects papers devoted to the question of adequate and improved SIMULATION TOOLS. It is completed by the invited paper prepared by Walter Ameling. The invited paper by A. Fischlin et al. should also be mentioned in this connection which emphasizes also on questions of engineering education.

The editors like to thank all who assisted in the organization of the Symposium and in the preparation of these Proceedings. We hope that these Proceedings will provide a unique source of information on both the state-of-the-art and new developments in the field of SIMULATION OF CONTROL SYSTEMS.

Inge Troch
Peter Kopacek
Felix Breitenecker

CONTENTS

BUILDING MODELS FOR A SPECIFIED PURPOSE USING SYSTEM IDENTIFICATION

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Abstract. Models and model quality are prime concerns for most design issues in control and systems analysis. The success of a simulation study hinges upon the reliability of the model used. In this contribution we discuss how to build mathematical models that given certain constraints, are of optimum quality for a prespecified (simulation) application. We then take into account the influence of both bias errors and random errors on the model. It turns out that for a fairly broad class of identification methods in the prediction error family, the optimal choices of design variables can be given in an explicit form.

1 INTRODUCTION

Building mathematical models of dynamical system involves many possibilities and choices of design variables. The particular route taken may have a substantial influence on the quality of the resulting model, and it is of course desirable to make the choices so that a model of "optimal quality" is achieved, given the constraints. A complication here is that there will typically be no "uniformly good" designs, so the model quality concept must be tied to the intended application.

A completely general treatment of this problem is no doubt difficult. In this contribution we shall formulate and solve a subproblem, where the first of possibilities have been constrained as follows:

- Only linear models will be considered.
- The true system will be assumed to be linear (but may be much more complex than the models considered)
- The model construction will be by system identification in a class of prediction error methods (to be precisely defined in Section 4)
- The intended model application will be simulation with input signals of given frequency characteristics (spectra).

The analysis is based on general asymptotic results given in Ljung (1985ab) and Wahlberg and Ljung (1986). Related discussions are given in Yuan and Ljung (1985), Gevers and Ljung (1986) and Ljung (1986). For comprehensive treatment, see Ljung (1987). For general discussions on Systems Identification, see also Goodwin and Payne (1977), Eykhoff (1974, 1981) and Åström and Eykhoff (1971).

2 PROBLEM SETUP

In this contribution we shall assume that there is true linear systems S, that generates the observed data. If y(t) and u(t) denote the output and the input, respectively at time instant t we thus assume that

$$y(t)=G_0(q)u(t)+v_0(t) \qquad (1)$$

Here $G_0(q)$ is the transfer operator

$$G_0(q)u(t)=\Big[\sum_{k=1}^{\infty} g_0(k)q^{-k}\Big]u(t)=\sum_{k=1}^{\infty} g_0(k)u(t-k) \qquad (2)$$

in the shift operator $q[qu(t)=u(t+1);\ q^{-1}u(t)=u(t-1)]$.

We thus describe the system in discrete time and, for simplicity, the sampling interval is taken to be the time unit. In (1), $v_0(t)$ is an additive disturbance, which is supposed to be a stationary stochastic process with spectrum

$$\Phi_v(\omega)=\lambda_0\left|H_0(e^{i\omega})\right|^2 \qquad (3)$$

This means that $\{v_0(t)\}$ can be regarded as generated by

$$v_0(t)=H_0(q)e_0(t) \qquad (4)$$

where $\{e_0(t)\}$ is white noise with variance λ_0.

For the system (1) we may generate an input $\{u(t)\}$, such that

$$\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N} u(t)u(t-\tau)=R_u(\tau)$$

exist for all τ, and the spectrum is

$$\Phi_u(\omega)=\sum_{\tau=-\infty}^{\infty} R_u(\tau)r^{-i\tau\omega}. \qquad (5)$$

We allow the possibility of output feedback, in which case the cross-spectrum $\Phi_{ue}(\omega)$ is non-zero. Thus collecting the data set

$$Z^N=\{u(1),y(1),\ldots,u(N),y(N)\} \qquad (6)$$

we may proceed to estimate the transfer functions G_0 and H_0 in (1), (4). Let the result be denoted by

$$\begin{matrix}\hat{G}_N(q)\ \ (=\hat{G}q,Z^N)\\ \hat{H}_N(Q)\ \ (=\hat{H}(q,Z^N))\end{matrix} \qquad (7)$$

We shall discuss procedures for this in Section 4.

3 MEASURES OF MODEL QUALITY

The true system and the model.

Suppose that the true system is subject to (1)-(4), i.e. that

$$y(t)=G_0(q)u(t)+H_0(q)e_0(t) \tag{8}$$

where $\{e_0(t)\}$ is white noise with variance λ_0.

For simpler notation, we shall also use

$$T_0(q)=[G_0(q)\ H_0(q)] \tag{9}$$

Suppose that we have decided upon all the design variables $\mathcal{D}$, associate with the model construction and as a result obtained the model

$$\hat{T}(q,\mathcal{D})=[\hat{G}(q,\mathcal{D})\hat{H}(q,\mathcal{D})] \tag{10}$$

$\mathcal{D}$ will contain, among other things N, the number of collected data.

A scalar design criterion

It is of course desirable that the model $\hat{T}(q,\mathcal{D})$ is close to $T_0(q)$. The difference

$$\tilde{T}(e^{i\omega},\mathcal{D}) \triangleq \hat{T}(e^{i\omega},\mathcal{D})-T_0(e^{i\omega}) \tag{11}$$

should, in other words, be small. Let us develope a formal measure of the size of $\tilde{T}$. Depending on the intended use of the model a good fit in some frequency ranges may be more important than in others. To capture this fact, we introduce a frequency weighted scalar criterion

$$J_1(\tilde{T}(\cdot,\mathcal{D}))=\int_{-\pi}^{\pi}\tilde{T}(e^{i\omega},\mathcal{D})C(\omega)\tilde{T}^T(e^{-i\omega},\mathcal{D})d\omega \tag{12}$$

where the 2×2 matrix function

$$C(\omega)=\begin{bmatrix}C_{11}(\omega) & C_{12}(\omega)\\ C_{21}(\omega) & C_{22}(\omega)\end{bmatrix} \tag{13}$$

describes the relative importance of a good fit at different frequencies as well as the relative importance of the fit in G and H, respectively. We shall generally assume that $C(\omega)$ is Hermitian, i.e. that

$$C_{21}(\omega)=\overline{C_{12}(\omega)}\ [=C_{12}(-\omega)]$$

(the last equality follows when the dependence on ω is via $e^{i\omega}$.) We shall shortly give an example of how such weighting functions can be determined.

Now, the scalar $J_1(\tilde{T}(\cdot,\mathcal{D}))$ is a random variable due to the randomness in $\tilde{T}$. To obtain a realization independent quality measure, it is natural to take the expectation of J_1 and form the criterion

$$\boxed{\begin{aligned}J(\mathcal{D}) &= \int_{-\pi}^{\pi} E\,\tilde{T}(e^{i\omega},\mathcal{D})C(\omega)\tilde{T}^T(e^{-i\omega},\mathcal{D})d\omega = \\ &= \int_{-\pi}^{\pi}\mathrm{tr}[\Pi(\omega,\mathcal{D})C(\omega)]d\omega\end{aligned}} \tag{14}$$

where the 2×2 matrix Π is given by

$$\Pi(\omega,\mathcal{D}) = E\,\tilde{T}^T(e^{-i\omega},\mathcal{D})\tilde{T}(e^{i\omega},\mathcal{D}) \tag{15}$$

The problem of choosing design variables can now be stated as

$$\boxed{\min_{\mathcal{D}\in\Delta}\ \bar{J}(\mathcal{D})} \tag{16}$$

where Δ denotes the constraints associated with our desire to do at most "a reasonable amount of work". These will typically include a maximum number of samples, signal power constraints, not too complex numerical procedures etc. The constraints Δ could also include that certain design variables simply are not available to the user in the particular application in question.

Model application: Simulation

Suppose that the transfer function G is used to simulate the input-output part of the system with input $u^*(t)$. The model $G(q,\mathcal{D})$ then produces the output

$$y_{\mathcal{D}}(t) = \hat{G}(q,\mathcal{D})u^*(t)$$

while the true system would give the correct output

$$y_0(t) = G_0(q)u^*(t).$$

The error signal

$$\tilde{y}_{\mathcal{D}}(t) = y_{\mathcal{D}}(t) - y_0(t) = [\hat{G}(q,\mathcal{D})-G_0(q)]u^*(t)$$

has the spectrum

$$\Phi_{\tilde{y}}(\omega,\mathcal{D})=\left|\hat{G}(e^{i\omega},\mathcal{D})-G_0(e^{i\omega})\right|^2\Phi_u^*(\omega) \tag{17}$$

where $\Phi_u^*(\omega)$ is the spectrum of $\{u^*(t)\}$. This, again, is a random function, and its expectation w.r.t G

$$\Psi_{\tilde{y}}(\omega,\mathcal{D})=E\left|\hat{G}(e^{i\omega},\mathcal{D})-G_0(e^{i\omega})\right|^2\Phi_u^*(\omega) \tag{18}$$

is a measure of the average performance degradation due to errors in the model G. Note that, with (15) and

$$C(\omega) = \begin{bmatrix}\Phi_u^*(\omega) & 0\\ 0 & 0\end{bmatrix} \tag{19}$$

we can rewrite (18) as

$$\Psi_{\tilde{y}}(\omega,\mathcal{D}) = \mathrm{tr}\,\Pi(\omega,\mathcal{D})C(\omega) \tag{20}$$

Finally, the average variance $E\tilde{y}^2(t)$ (averaged over $u^*(t)$ as well as over $\hat{G}$) will be

$$E\tilde{y}^2(t)=\bar{J}(\mathcal{D})=\frac{1}{2\pi}\int_{-\pi}^{\pi}\Psi_{\tilde{y}}(\omega,\mathcal{D})d\omega \tag{21}$$

which is a special case of (14).

Since our focus in this contribution is on the simulation application (19) we shall confine our interest to the following special case of (13):

$$C(\omega) = \begin{bmatrix}C_{11}(\omega) & 0\\ 0 & 0\end{bmatrix} \tag{22}$$

See Ljung (1986) for more general treatment.

4 PREDICTION ERROR IDENTIFICATION METHODS

The model set

The perhaps most common approach in modern identification is to postulate that the transfer function is to be sought within a certain set:

$$\mathcal{G} = \{G(e^{i\omega},\theta)\,|\,\theta\in D_M\} \tag{23}$$

Here D_M typically is a subset of R^d. In order to improve the result, it is customary to also include assumptions about the disturbance spectrum

$\Phi_v(\omega)$ (see (1)-(4)). It is assumed to belong to a set

$$\Phi_v(\omega) = \lambda\left|H(e^{i\omega},\theta)\right|^2;\ H(e^{i\omega},\theta) \in \mathcal{H} \qquad (24)$$

$$\mathcal{H} = \{H(e^{i\omega},\theta)\,|\,\theta\in D_M\}.$$

This means that the system is assumed to be described as

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t) \qquad (25)$$

for some $\theta\in D_M$. Here $\{e(t)\}$ is a sequence of independent random variables with zero mean values and variances λ, and G and H are functions of the shift operator q;

$$G(q,\theta) = \sum_{k=1}^{\infty} g_k(\theta)q^{-k} \qquad (26a)$$

$$H(q,\theta) = 1 + \sum_{k=1}^{\infty} h_k(\theta)q^{-k}. \qquad (26b)$$

There are several ways by which the transfer functions in (15) can be parametrized. Common ones include state-space models, ARMAX models, output-error models, etc.

The estimation method

Given the model (25) and input-output data up to time t-1, we can determine the prediction output at time t as

$$\hat{y}(t|\theta)=\left(1-H^{-1}(q,\theta)\right)y(t)+H^{-1}(q,\theta)G(q,\theta)u(t) \qquad (27)$$

At time t, when y(t) has been recorded we can compute the prediction error that the model (25) led to:

$$\varepsilon(t,\theta)=y(t)-\hat{y}(t|\theta)=H^{-1}(q,\theta)\left(y(t)-G(q,\theta)u(t)\right) \qquad (28)$$

We may say that the model (25) is "good" if the sequence $\varepsilon(t,\theta)$, t=1,2,...,N is "small". In a very common class of identification methods, the squared sum of prediction errors is minimized to find the "best" model:

$$\hat{\theta}_N = \arg\min_{\theta\in D_M} \frac{1}{N}\sum_{t=1}^{N} \varepsilon^2(t,\theta) \qquad (29)$$

With $\hat{\theta}_N$ determined in this way, the transfer function estimate becomes

$$\hat{G}_N(e^{i\omega}) = G(e^{i\omega},\hat{\theta}_N) \qquad (30)$$

Among methods that can be expressed as (29) we find the "maximum likelihood method" (assuming Gaussian disturbances), the "least squares method" and others. See Ljung (1987) and Åström (1980) for further discussions.

Some extensions

It may often be worthwhile to consider a modified criterion (29) where the prediction errors $\varepsilon(t,\theta)$ (or, equivalently the input-output sequences) first are filtered through a filter L(q):

$$\varepsilon_F(t,\theta) = L(q)\varepsilon(t,\theta) \qquad (31)$$

This is, however, equivalent to replacing the noise model $H(q,\theta)$ by $H(q,\theta)/L(q)$. See (28) and, for a further discussion, Wahlberg and Ljung (1986). Prefiltering the data thus corresponds to selecting another noise model set.

Also the use of k-step ahead predictors in (27) might be useful. As elaborated on in Wahlberg and Ljung (1986), k-step ahead prediction methods are equivalent to replacing $H(q,\theta)$ by

$$H(q,\theta)M_k^{-1}(q,\theta) \qquad (32)$$

where $M_k(q,\theta)$ are the first k terms in the Laurent expansion of $H(q,\theta)$. The use of k-step ahead predictors is thus equivalent to prefiltering ($L(q) = M_k(q,\theta)$) or to selecting another noise model set.

Design variables

Let us list the available design variables:

- $\Phi_u(\omega)$: spectrum of the extra input u in (5) (33a)
- $\Phi_{ue}(\omega)$ cross spectrum between u and e (resulting from output fedback (33b)
- $\mathcal{H}=\{H(q,\theta)\,|\,\theta\in D_M\}$: set of noise models. This includes, as we noted, the possibility of prefiltering with L in (31) and the use of k-step ahead predictors (see (32)).
- In this study we shall confine ourselves to fixed noise models, i.e. the set $\mathcal{H}$ is a singelton:

  $$\mathcal{H} = \{M_*(q)\} \qquad (33c)$$

  The coice of M_* is however included among the design variables.

These three items will henceforth be denoted collectively by the symbol $\mathcal{D}$.

Other design variables, such as N, the number of collected data, and $\mathcal{G}$, the set of transfer function models (including the model order n) will be regarded as fixed in this study.

5 ASYMPTOTIC PROPERTIES OF THE ESTIMATED TRANSFER FUNCTIONS

Convergence

Under weak conditions it can be shown that

$$\hat{\theta}_N \to \theta^* = \arg\min_{\theta\in D_M} \bar{V}(\theta) \text{ w.p.1 as } N\to\infty \qquad (34)$$

where

$$\bar{V}(\theta) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N} E\varepsilon^2(t,\theta) \qquad (35)$$

See, e.g. Ljung (1978).

Applying Parseval's relationship to (35) gives, after some calculations (see Ljung (1987)), using also (33c).

$$\theta^*=\arg\min_{\theta}\int_{-\pi}^{\pi} \mathrm{tr}\left[R(\omega,\theta)\cdot Q(\omega)\right]d\omega \qquad (36)$$

with

$$R(\omega,\theta)=\tilde{T}^T(e^{-i\omega},\theta)\tilde{T}(e^{i\omega},\theta) \qquad (37)$$

$$Q(\omega,\theta)=\Phi_\chi(\omega)/\left|H_*(e^{i\omega})\right|^2 \qquad (38)$$

Note that for open loop operation ($\Phi_{ue}(\omega)\equiv 0$),

$$\tilde{T}(q,\theta)=\left[G(q,\theta)H_*(q)\right]-T_0(q) \qquad (39)$$

and

$$\Phi(\omega) = \begin{bmatrix}\Phi_u(\omega) & \Phi_{ue}(\omega)\\ \Phi_{ue}(\omega) & \lambda_0\end{bmatrix} \qquad (40)$$

this expression specializes to

$$\theta^*= \arg\min_{\theta}\int_{-\pi}^{\pi} \left|\tilde{G}(e^{i\omega},\theta)\right|^2\cdot\frac{\Phi_u(\omega)}{\left|H_*(e^{i\omega})\right|^2}d\omega \qquad (41)$$

Variance

Let

$$T^*(q) = T(q,\theta^*) \tag{42}$$

with θ^* defined as above.

Under fairly general conditions it can then be shown that

$$\sqrt{N}\,[\hat{T}_N(e^{i\omega})-T^*(e^{i\omega})] \in AsN\,(0,P_n(\omega)) \tag{43}$$

Here (43) means that the random variable on the left converges in distribution to the normal distribution with zero mean and covariance matrix $P_n(\omega)$. Here the index n denotes the order of the model used in $T(q,\theta)$.

Results, such as (43) go back to the asymptotic normalities of the parameter estimate θ_N, established, e.g. in Ljung and Caines (1979). The expression for $P_n(\omega)$ is in general complicated. For models that are parametrized as "black boxes" we have, however the following general result, Ljung (1985b).

$$\lim_{n\to\infty} \frac{1}{n} P_n(\omega)=\Phi_v(\omega)[\Phi_\chi(\omega)]^{-1} \tag{44}$$

with Φ_v and Φ_χ defined by (3) and (40), respectively.

A pragmatic interpretation

Even though the covariance of $\hat{T}_N$ need not converge (convergence in distribution does not imply convergence in L_2), we shall allow ourselves to use the result (43) - (44) in the following more suggestive version:

$$\mathrm{Cov}\ \hat{T}_N(e^{i\omega}) \sim \frac{n}{N} \Phi_v(\omega)[\Phi_\chi(\omega)]^{-1} \tag{45}$$

We shall also allow the approximation

$$E\ \hat{T}_N(e^{i\omega}) \approx T^*(e^{i\omega}) \tag{46}$$

(See Ljung (1987) for justifications.)

With (45) and (46) the expression (15) can be rewritten

$$\Pi(\omega,\mathcal{D})=E\ \tilde{T}^T(e^{-i\omega},\mathcal{D})\tilde{T}(e^{i\omega},\mathcal{D})= R(\omega,\theta^*(\mathcal{D}))+$$

$$+\mathrm{Cov}\ \hat{T}_N(e^{i\omega},\mathcal{D}) \tag{47}$$

where the bias contribution R was defined in (37). We have here appended the argument $\mathcal{D}$ ($\theta^*=\theta^*(\mathcal{D})$, $\theta_N=\theta_N(\mathcal{D})$) to stress the dependence on the design variables.

The criterion (14) can thus be split into a bias and a variance contribution:

$$J(\mathcal{D})\approx J_B(\mathcal{D})+J_P(\mathcal{D}) \tag{48}$$

where

$$J_B(\mathcal{D})= \int_{-\pi}^{\pi} \mathrm{tr}\, R(\omega,\theta^*(\mathcal{D}))\,C(\omega)\,d\omega \tag{49}$$

$$J_P(\mathcal{D})= \int_{-\pi}^{\pi} \mathrm{tr}[\,\mathrm{Cov}\ \hat{T}_N(e^{i\omega},\mathcal{D})\cdot C(\omega)\,d\omega]\approx$$

$$\approx \frac{n}{N} \int_{-\pi}^{\pi} \Phi_v(\omega)\cdot \mathrm{tr}[\Phi_\chi^{-1}(\omega,\mathcal{D})C(\omega)]\,d\omega \tag{50}$$

In the following two sections we shall discuss the minimization of these two contributions to the design criterion.

6 MINIMIZING THE BIAS CONTRIBUTION

Consider now the problem of minimizing the bias distribution, i.e.

$$\min_{\mathcal{D}\in\Delta} J_B(\mathcal{D}) \tag{51}$$

where $J_B(\mathcal{D})$ is defined by (37), (49). The function $J_B(\mathcal{D})$ depends on $\mathcal{D}$ via $\theta^*(\mathcal{D})$. The dependence on $\mathcal{D}$ of the latter function, in turn, is defined by (36), which we write as

$$\theta^*(\mathcal{D})=\arg\min_{\theta} \int_{-\pi}^{\pi} \mathrm{tr}[\,R(\omega,\theta)\cdot Q(\omega,\mathcal{D})]\,d\omega \tag{52}$$

$R(\omega,\theta)$ is defined by (37) and Q by (38). We have appended the argument $\mathcal{D}$ to Q, to stress hat it is made up from the design variables (33). See also Ljung (1986) and Gevers and Ljung (1986).

Comparing (52) with the minimization problem (51), (49) it is intuitively clear that the best choice of $\mathcal{D}$ should be one that makes $C(\omega)$ and $Q(\omega,\mathcal{D})$ proportional. That this is indeed the case is proven in Yuan and Ljung (1985) and Ljung (1986b). We thus have the following result.

Theorem 1. Consider the problem to minimize (51) with respect to $\mathcal{D}=\{\Phi_u(\omega), \Phi_{ue}(\omega), L(e^{i\omega}), H_*(e^{i\omega})\}$ (see (33)) under the assumptions (22). Then $\mathcal{D}_{opt}$ is such that

$$\Phi_{ue}^{opt}(\omega)\equiv 0 \tag{53a}$$

$$\frac{|L(e^{i\omega})|^2\Phi_u^{opt}(\omega)}{|H_*^{opt}(e^{i\omega})|^2} = \alpha\cdot C_{11}(\omega) \tag{53b}$$

where α is any constant that makes $\mathcal{D}_{opt}$ belong to the admissible set.

□

Here are included the prefilter L in (31) as an explicit option. Notice that there are several ways of obtaining the optimal design. Any combinations of input spectrum and noise model that obey (53b) will give the optimal bias distribution. Also recall that the choice of noise model $H_*(q)$ contains the option of prediction horizon k (see (32)).

7 MINIMIZING THE VARIANCE CONTRIBUTION

The problem

Let us now turn to the problem

$$\min_{\mathcal{D}\in\Delta} J_P(\mathcal{D}) \tag{54}$$

where $J_P(\mathcal{D})$ is given by (50). We shall generally assume that the input power is constrained:

$$\Delta: \int_{-\pi}^{\pi} \Phi_u(\omega)d\omega \le \beta \tag{55}$$

Spelling out (50) gives

$$J_P(\mathcal{D}) = \int_{-\pi}^{\pi} \Psi(\omega,\mathcal{D})\,d\omega$$

where

$$\Psi(\omega,\mathcal{D})=\frac{\lambda C_{11}(\omega)-2\mathrm{Re}[C_{12}(\omega)\Phi_{eu}(\omega)]+C_{22}(\omega)\Phi_u(\omega)}{\lambda\Phi_u(\omega)-|\Phi_{ue}(\omega)|^2}\cdot$$

$$\cdot\Phi_v(\omega) \tag{56}$$

$$\mathcal{D}=\{\Phi_u,\Phi_{ue}\} \tag{57}$$

Here we dispensed with the scaling n/N, which is immaterial.

For the case (22) we obtain the problem

$$\min_{\Phi_u,\Phi_{ue}} \int_{-\pi}^{\pi} \frac{\lambda_0 C_{11}(\omega)}{\lambda_0\Phi_u(\omega)-|\Phi_{ue}(\omega)|^2}\Phi_v(\omega)d\omega \qquad (58)$$

subject to the constraint that

$$\int_{-\pi}^{\pi} \Phi_u(\omega)d\omega \le C \qquad (59)$$

From (58) and the fact that $\Phi_{ue}(\omega)$ does not enter the constraint, it follows that

$$\Phi_{ue}^{opt}(\omega) \equiv 0. \qquad (60)$$

It is thus optimal to use open loop experiments, and the optimal input is easy to compute using Schwarz's inequality:

Lemma 1 The solution to (58)-(59)

$$\Phi_u^{opt}(\omega) = \mu \cdot \sqrt{C_{11}(\omega) \cdot \Phi_v(\omega)} \qquad (61)$$

where μ is a constant, adjusted so that

$$\int_{-\pi}^{\pi} \Phi_u^{opt}(\omega)d\omega = C \qquad (62)$$

8 MINIMIZING THE DESIGN CRITERIA

Let us now turn to the full design criterion (13)-(16) in its pragmatic form (48)-(50). Our partial results on bias- and variance-minimization then show that it in certain cases is possible to minimize the two contributions simultaneously. Then of course the full criterion is also minimized. For the case of Theorem 1 we thus have the following result.

Theorem 2: Consider the problem to minimize (48) - (50) with respect to

$$= \{\Phi_u(\omega),\ \Phi_{ue}(\omega),\ L(e^{i\omega}),\ H_*(e^{i\omega})\}$$

under the assumptions (33)-(22), and subject to the constraint (59). Then $_{opt}$ is given by

$$\Phi_{ue}(\omega) \equiv 0$$

$$\Phi_u(\omega) = \mu_2\sqrt{C_{11}(\omega) \cdot \Phi_v(\omega)} \qquad (63)$$

$$\left| \frac{L(e^{i\omega})}{H_*(e^{i\omega})} \right|^2 = \mu_1 \sqrt{\frac{C_{11}(\omega)}{\Phi_v(\omega)}}$$

Here μ_1 is a constant, adjusted so that the left hand side has a Laurent expression that starts with a "1", and μ_2 is a constant adjusted so that the input power constraint is met. □

Note that the freedom in the choice of noise model and prefilter is imaginary, since they always appear in the combination L(q)/H*(q) in the criterion.

The case where our prime interest is in the transfer function G is probably the most common one, and therefore the optimal design variables offered by Theorem 2 should be of interest. The only drawback with this solution may be that the choice of constant noise model may lead to more calculations in the numerical minimization of the prediction error criterion.

9 CONCLUSIONS

In this contribution we have focused our interest on the design variables that are available for the estimation of transfer functions.

We have studied the family of prediction error identification methods for the parameter estimation, and made use of some recently derived asymptotic expressions for bias and variance of the transfer function estimate. Under certain assumptions some fairly explicit advice for the choice of input spectra, feedback mechanisms, prefilters and noise models have been derived.

Acknowledgement. The writing of this article has been supported in part by the Army Research Office under contract DAAG-29-84-K-005, and in part by the Swedish National Board for Technical Development (STUF).

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