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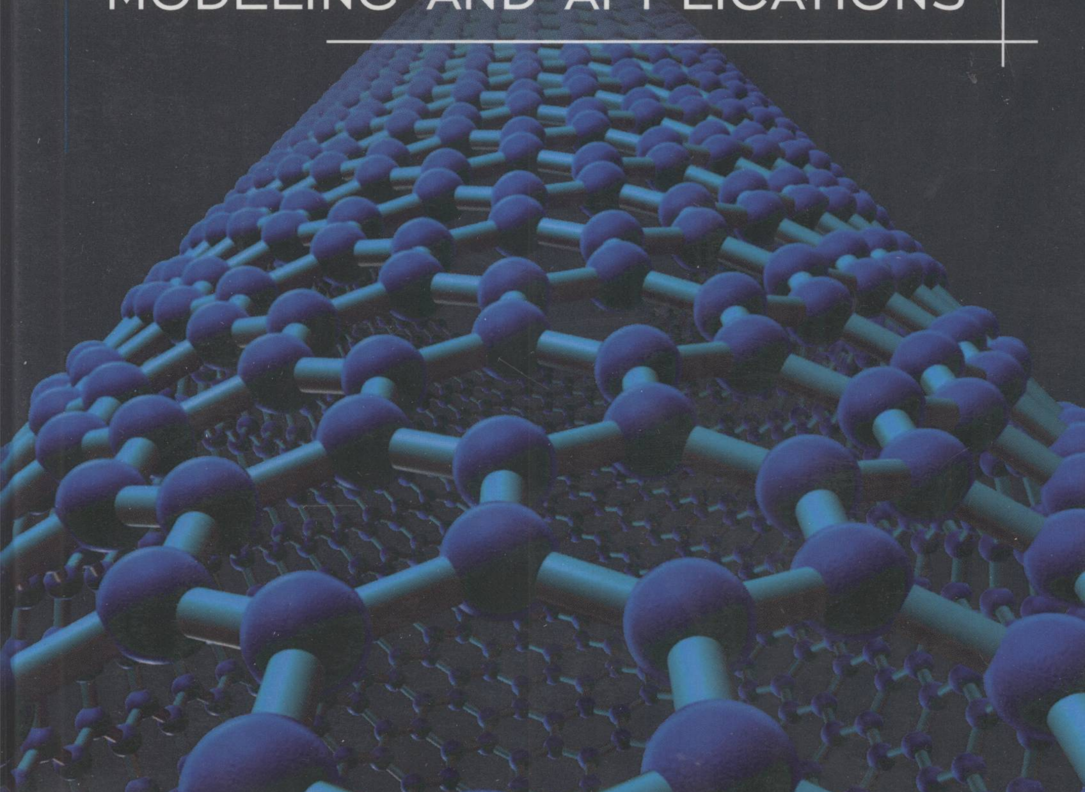
# MECHANICAL DESIGN OF MICRORESONATORS

## MODELING AND APPLICATIONS



NICOLAE O. LOBONTIU

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Nicolae Lobontiu



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# Mechanical Design of Microresonators

*With love to my wife Simona*
*and daughters Diana and Ioana*

# Preface

Mechanical microresonators are fundamental components in a host of MEMS applications covering the automotive sector (safety systems, stability and rollover, occupant detection, tire pressure monitoring, biometric sensors for comfort programs), the telecommunication industry [especially the radio-frequency (RF) domain with implementations such as switches, tunable capacitors and mechanical filters implemented in wavelength division multiplexing (WDM) and mobile communication, variable attenuators in cell phones, frequency reference, digital micromirror devices (DMD), laser tuning or radar systems], the bio/medicine domain [detection and tracking of various substances including hazardous and explosive ones at the femtogram level, magnetic resonance imaging (MRI), surgical instrumentation for corneal resurfacing or hair/tattoo removal], the material/surface characterization area [scanning probe microscopy (SPM) and atomic force microscopy (AFM), resonant strain gauges, residual stress measurements], and motion sensing (gyroscopes and other resonant accelerometers for navigational systems and platform stabilization). Applications of mechanical micro-resonators are also implemented in virtual reality, people-to-device communication (gloves, helmets and haptic systems for remote surgical intervention), optical beam scanners, laser printers, inertial mouse devices in computers, CD players, video cameras, fluid density and mass/pressure flow sensors, low acceleration (low-*g*) sensors, and light modulators.

Based on the resonant beam technology, mechanical microresonators are capable of high accuracy and sensitivity (order of magnitudes over conventional-technology counterparts), very good signal-to-noise (S/N) ratio, relatively large bandwidth, compatibility with the integrated-circuit (IC) technology, simplified digital interface, and miniaturization. The last feature in this enumeration is crucial, particularly in detecting minute amounts of substances (at the cell level in biodetection, for instance) where very small quantities of extraneous matter can be detected only by very small mechanical resonators which operate at frequencies in the gigahertz domain.

The large spectrum of current and foreseeable micromechanical resonator applications has sparked a wide interest in advancing the practical and theoretical knowledge in this area. Inroads have been made at all component levels that are involved in developing high-performance mechanical resonator systems, including the fabrication, electronic, mechanical, and control subdomains. This book is dedicated to the mechanical modeling and design of microresonators. The book addresses the main methods and procedures which can be utilized in evaluating the behavior of mechanical microresonators by means of lumped- and distributed-parameter modeling. It also contains a database offering comprehensive characterization of mechanical resonator components and systems (including hinges, cantilevers and bridges)—many of them novel—in the frequency domain. It is hoped that professionals with various expertise levels and backgrounds, who are involved with the study, research, and development of mechanical microresonators will find this book useful. Many fully solved, real-life resonator examples accompany and complement the basic material. Although many of today's mechanical resonators are fabricated in the nanometer range, the prefix *micro* has been used in this book to keep the nomenclature unitary and short-form.

Chapter 1 introduces the main traits of modeling and designing mechanical microsystems which operate at resonance. Single- and multiple-degree-of-freedom systems are characterized in terms of their free and forced response. The damping in mechanical microsystems is discussed including loss mechanisms such as those produced by fluid-structure interaction and internal dissipation. Methods enabling us to formulate the dynamic equations of motion and to determine the resonant response, both exact and approximate, are also presented in Chapter 1, which concludes with notions of mechanical-electrical analogies, transfer functions, complex impedances, and micromechanical resonator filters.

Chapter 2 focuses on basic components that are the backbone of mechanical microresonators, such as line members, circular rings, thin plates, and membranes. Lumped-parameter modeling is presented together with the methods enabling derivation of stiffness properties (Castigilano's displacement theorem) and inertia fractions (Rayleigh's principle), which are usually combined to yield the relevant resonant frequencies. Basic microcantilever shapes such as constant cross-section, trapezoid and corner-filleted are fully defined in terms of their axial, torsional, and bending resonant frequencies. The distributed-parameter modeling approach targets the resonant characterization of line members under the action of axial loads, circular rings, thin plates and membranes.

Chapters 3 and 4 are dedicated to microhinges, microcantilevers, and microbridges. These compliant members can be utilized as either stand-alone resonator systems (such as in mass detection or switches) or components of more complex resonators (such as elastic suspensions). The lumped-parameter stiffness, inertia and corresponding resonant frequencies are derived for various configurations including paddle, filleted (circular and elliptic), notched, hollow, and multimorph (sandwiched). Generic formulations are also provided which facilitate modeling and designing of components with geometric profiles other than the ones presented in these chapters.

Chapter 5 studies resonant mechanical microsystems such as beam type, spring type, microgyroscopes, tuning forks, and microaccelerometers. Various models are proposed and compared, which characterize the dynamic response and performance of mechanical microresonators at different levels of accuracy. The main methods of transduction (actuation and/or sensing) which are implemented in microresonator applications such as electrostatic, electromagnetic, piezoelectric and piezomagnetic are also discussed in Chapter 5.

The final chapter, Chapter 6, focuses entirely on microcantilever and microbridge systems which are designed for mass detection. Static detection of extraneous substance attachment is treated here but the emphasis falls on resonant methods and devices enabling mass detection by means of resonant frequency shift monitoring.

The book contains quite a few novel designs and associated models, and although a lot of effort and time has been spent at making sure that the mathematical apparatus is correct, errors might have slipped in—I would appreciate signaling of such occurrences.

My thanks go to Dr. Rob Ilic of Cornell NanoScale Facility for allowing me to present pictures of his work on microresonators, for his enthusiastic and thorough review of the chapter on mass detection, and for the precious suggestions on the introduction to this chapter, which have been included almost *ad literam*.

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# Contents

Chapter

# 1

# Design at Resonance of Mechanical Microsystems

## 1.1 Introduction

This chapter is an introduction to the main aspects encountered in modeling and designing mechanical microresonators.

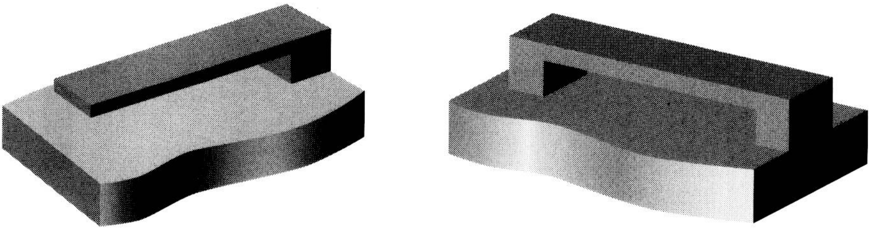
Aside from the technological reasons for realizing systems that integrate the mechanical structure and the associated silicon/semiconductor electronic circuitry, the drive toward smaller-scale, nano-domain mechanical resonators is motivated by the need for pushing the limits to the resonant frequencies in the gigahertz domain. It is known that the stiffness of a mechanical resonator varies with the inverse of the length (because the basic definition of stiffness is force divided by length):

$$k \sim \frac{1}{l} \tag{1.1}$$

and that the resonant frequency is proportional to the square root of the stiffness:

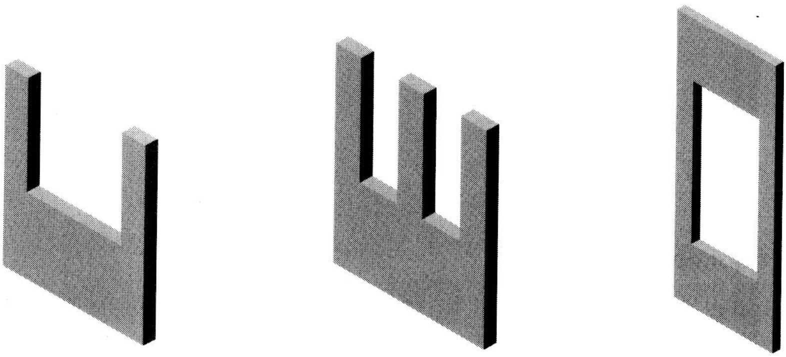
$$\omega_r \sim \sqrt{k} \tag{1.2}$$

As a consequence, increasing the resonant frequency of a mechanical device implies miniaturization, and therefore very high frequencies are achieved by very small resonator dimensions. In addition, as this chapter discusses, higher resonant frequencies (which are achieved



(a) (b)

**Figure 1.1** Single-component mechanical microresonators: (*a*) cantilever; (*b*) bridge.



(a) (b) (c)

**Figure 1.2** Tuning-fork microresonators: (*a*) classical; (*b*) trident; (*c*) double-ended.

with small-dimension resonators) also contribute to increasing the quality factor of a system, which is a measure of its resonant performance. Smaller is also better, as Chap. 6 will demonstrate, in detecting minute amounts of deposited substances as the capacity of capturing the effects of mass at the cell level is inversely proportional to the geometric dimensions of a mechanical resonator.

Constructively, the mechanical microresonators can be cantilevers, as sketched in Fig. 1.1*a*; bridges, as in Fig. 1.1*b*; tuning forks, as shown in Fig. 1.2*a*, *b*, and *c*. Or they can be of a more complex geometry, such as the lateral resonator design with folded-beam suspensions illustrated in Fig. 1.3. More details regarding these mechanical resonators, as well as more resonator structures, are presented in subsequent chapters of this book.

This chapter analyzes the main aspects of single- and multiple-degree-of-freedom mechanical microresonators by discussing the models that are utilized to characterize and design these devices.

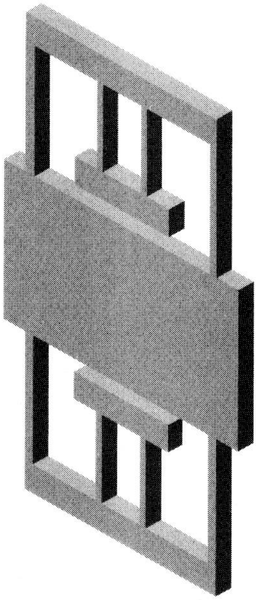


**Figure 1.3** Lateral mechanical microresonator with folded-beam suspensions.

## 1.2 Single-Degree-of-Freedom Systems

Many mechanical microresonators can be modeled as single-degree-of-freedom systems. A microcantilever, for instance, such as the one illustrated in Fig. 1.4, may only vibrate in bending and therefore can be modeled as a single-degree-of-freedom member by means of lumped-parameter properties (as shown in subsequent chapters in this book), namely, by allocating mass and stiffness fractions at the free end about the single motion direction. The free response of a mechanical system determines the resonant frequency in either the presence or the absence of damping. The forced response reveals the behavior of an undamped or damped mechanical system under the action of a sinusoidal (most often) excitation. In mechanical resonators, the phenomenon of resonance is important, and in such situations the excitation frequency matches the natural (resonant) frequency of the system.

### 1.2.1 Free response

For a single-degree-of-freedom (single-DOF) system formed of a body of mass $m$ and a spring of stiffness $k$, such as the one in Fig. 1.5, the dynamic equation of motion is

$$m\ddot{x} + kx = 0 \tag{1.3}$$

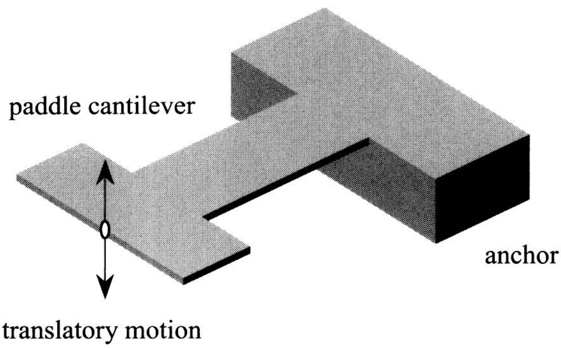



**Figure 1.4** Microcantilever as a single-degree-of-freedom system.

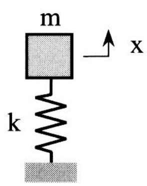



**Figure 1.5** Single-degree-of-freedom mass-spring system.

The solution to Eq. (1.3) is

$$x(t) = \frac{\dot{x}_0}{\omega_r}\sin(\omega_r t) + x_0\cos(\omega_r t) \tag{1.4}$$

where the natural (or resonant) frequency is

$$\omega_r = \sqrt{\frac{k}{m}} \tag{1.5}$$

and the initial displacement and velocity conditions are

$$x(0) = x_0 \quad \left.\frac{dx}{dt}\right|_{t=0} = \dot{x}_0 \tag{1.6}$$

Similarly, the equation of motion of a single-DOF system formed of a mass and a dashpot (mass-damper combination with viscous damping), such as the one in Fig. 1.6, is

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{1.7}$$

and the solution to this homogeneous equation can be expressed as

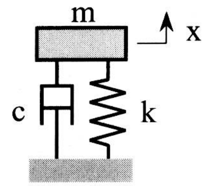



**Figure 1.6** Single-degree-of-freedom mass-dashpot system.

$$x(t) = [x_0\cos(\omega_d t) + (\dot{x}_0 + \xi\omega x_0)/\omega_d \sin(\omega_d t)]e^{-\xi\omega t} \tag{1.8}$$

where

$$\omega_d = \sqrt{1-\xi^2}\,\omega_r \tag{1.9}$$

is the damped frequency of the system and the damping ratio $\xi$ is defined as

$$\xi = c/c_c = c/\left(2\sqrt{mk}\right) = c/\left(2m\omega_r\right) \tag{1.10}$$

by means of the critical damping factor $c_c$. The solution to Eq. (1.8) describes the natural response of the vibratory system in the absence of the external forcing.

Depending on whether the critical damping factor is less than, equal to, or larger than 1, the vibrations are called, respectively, underdamped, critically damped, or overdamped.

### 1.2.2 Forced response — the resonance

When a force defined as

$$f(t) = F\sin(\omega t) \tag{1.11}$$

acts on the mass shown in Fig. 1.6, then Eq. (1.3) changes to

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{1.12}$$

The general solution of Eq. (1.12) is the sum of a complementary solution (which describes the system's vibration at the natural frequency) and a particular solution (which is vibration-generated at the driving frequency). The latter part of the solution is also called the steady-state solution and is generally analyzed in the frequency domain by studying its amplitude and phase angle.

Often Eq. (1.12) is written in the alternate form:

$$\ddot{x} + 2\xi\omega_r \dot{x} + \omega_r^2 x = \frac{F}{m}\sin(\omega t) \tag{1.13}$$

The solution to Eqs. (1.12) and (1.13), as shown by Timoshenko,[1] Thomson,[2] or Rao,[3] is the sum of the homogeneous solution—Eq. (1.8)—and a particular solution which is of the form:

$$x_p(t) = X\sin(\omega t - \varphi) \tag{1.14}$$

where the amplitude $X$ is

$$X = \frac{X_{st}}{\sqrt{(1 - m\omega^2/k)^2 + (c\omega/k)^2}} = \frac{X_{st}}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} \tag{1.15}$$

with the frequency ratio $\beta$ being defined as

$$\beta = \frac{\omega}{\omega_r} \tag{1.16}$$

and the phase angle between excitation and response $\varphi$ as

$$\varphi = \arctan\frac{2\xi\beta}{1-\beta^2} \tag{1.17}$$

The particular solution of Eq. (1.14) is of special importance as it describes the forced response of a vibratory system. In Eq. (1.15) the static displacement is $X_{st}$ and is defined as $F/k$. Figures 1.7 and 1.8 are plots of the amounts $X/X_{st}$ and $\varphi$ as functions of $\beta$ for various values of $\xi$.

As Fig. 1.7 indicates, when the driving frequency equals the resonant frequency ($\beta = 1$), the amplitude ratio reaches a maximum, which, for very small damping ratios, goes to infinity. Even in the presence of moderate damping, the amplitude at resonance is large, and this feature is utilized as a working principle in mechanical microresonators.

At resonance, when $\beta = 1$, the amplitude ratio of Eq. (1.15) becomes

$$\frac{X_r}{X_{st}} = \frac{1}{2\xi} \tag{1.18}$$

which gives an amplitude of

$$X = \frac{F_0}{2k\xi} \tag{1.19}$$