ALEXANDER
J. SMITS

A PHYSICAL INTRODUCTION TO
Fluid Mechanics

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ALEXANDER J. SMITS
Department of Mechanical Engineering
Princeton University

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A PHYSICAL INTRODUCTION TO FLUID MECHANICS

PREFACE

The purpose of this book is to summarize and illustrate basic concepts in the study of fluid mechanics. Although fluid mechanics is a challenging and complex field of study, it is based on a small number of principles, which in themselves are relatively straightforward. The challenge taken up here is to show how these principles can be used to arrive at satisfactory engineering answers to practical problems. The study of fluid mechanics is undoubtedly difficult, but it can also become a profound and satisfying pursuit for anyone with a technical inclination, and I hope the book conveys that message clearly.

The scope of this introductory material is rather broad, and many new ideas are introduced. It will require a reasonable mathematical background, and those students who are taking a differential equations course concurrently sometimes find the early going a little challenging. The underlying physical concepts are highlighted at every opportunity to try to illuminate the mathematics. For example, the equations of fluid motion are introduced through a reasonably complete treatment of one-dimensional, steady flows, including Bernoulli's equation, and then developed through progressively more complex examples. This approach gives the students a set of tools that can be used to solve a wide variety of problems, as early as possible in the course. In turn, by learning to solve problems, students can gain a physical understanding of the basic concepts before moving on to examine more complex flows. Dimensional reasoning is emphasized, as well as the interpretation of results (especially through limiting arguments). Throughout the text, worked examples are given to demonstrate problem-solving techniques. They are grouped at the end of major sections to avoid interrupting the text as much as possible. Historical references are given throughout, and some brief biographical sketches are collected near the end of the text. I hope they add to the fabric of the book, and that they will stimulate further reading in the history of fluid mechanics.

The book is intended to provide students with a broad introduction to the mechanics of fluids. The material is sufficient for two quarters of instruction. For a one-semester course only a selection of material should be used. A typical one-semester course, might consist of the material in Chapters 1 to 10, not including Chapter 7. If time permits, one of Chapters 11 to 14 may be included. For a course lasting two quarters, it is possible to cover Chapters 1 to 6, and 8 to 10, and select three or four of the other chapters, depending on the interests of the class. The sections marked with asterisks may be omitted without loss of continuity. Although some familiarity with thermodynamic concepts is assumed, it is not a strong prerequisite. Omitting the sections marked by a single asterisk, and the whole of Chapter 12, will leave a curriculum that does not require a prior background in thermodynamics.

A limited number of Web sites are suggested to help enrich the written material. In particular, a number of Java-based programs are available on the Web to solve specific fluid mechanics problems. They are especially useful in areas where traditional methods limit the number of cases that can be explored. For example, the programs designed to solve potential flow problems by superposition and the programs that handle compressible flow problems, greatly expand the scope of the examples that can be solved in a limited amount of time, while at the same time dramatically reducing the effort involved. A listing of current links to sites of interest to students and researchers in fluid dynamics may be found at *http://www.princeton.edu/~gasdyn/fluids.html.* In an effort to keep the text as current as possible, additional problems, illustrations and Web resources, as well as a Corrigendum and Errata may be found at *http://www.princeton.edu/~asmits/fluidmechanics.html.*

In preparing this book, I have had the benefit of a great deal of advice from my colleagues. One persistent influence that I am very glad to acknowledge is that of Professor Sau-Hai Lam of Princeton University. His influence on the contents and tone of the writing is profound. Also, my enthusiasm for fluid mechanics was fostered as a student by Professor Tony Perry of the University of Melbourne, and I hope this book will pass on some of my fascination with the subject.

Many other people have helped to shape the final product. Professor David Wood of Newcastle University in Australia provided the first impetus to start this project. Professor George Handelman of Rensselaer Polytechnic Institute, Professor Peter Bradshaw of Stanford University, and Professor Robert Moser of the University of Illinois Urbana-Champaign were very helpful in their careful reading of the manuscript and through the many suggestions they made for improvement. Professor Victor Yakhot of Boston University test-drove an early version of the book, and provided a great deal of feedback, especially for the chapter on dimensional analysis. My wife, Louise Handelman, gave me wonderfully generous support and encouragement, as well as advice on improving the quality and clarity of the writing. I would like to dedicate this work to the memory of my brother, Robert Smits (1946–1988), and to my children, Peter and James, who represent the future.

Alexander J. Smits
Princeton, New Jersey, USA

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CHAPTER 1

INTRODUCTION

Fluid mechanics is the study of the behavior of fluids under the action of applied forces. Typically, we are interested in finding the force required to move a solid body through a fluid, or the power necessary to move a fluid through a system. The speed of the resulting motion, and the pressure, density, and temperature variations in the fluid, are also of great interest. To find these quantities, we apply the principles of dynamics and thermodynamics to the motion of fluids, and develop equations to describe the conservation of mass, momentum, and energy.

As we look around, we can see that fluid flow is a pervasive influence on all parts of our daily life. To the ancient Greeks, the four fundamental elements were Earth, Air, Fire, and Water; and three of them, Air, Fire, and Water, involve fluids. The air around us, the wind that blows, the water we drink, the rivers that flow, and the oceans that surround us, affect us daily in the most basic sense. In engineering applications understanding fluid flow is necessary for the design of aircraft, ships, cars, propulsion devices, pipe lines, air conditioning systems, heat exchangers, clean rooms, pumps, artificial hearts and valves, spillways, dams, and irrigation systems. It is essential to the prediction of weather, ocean currents, pollution levels, and greenhouse effects. Not least, all life-sustaining bodily functions involve fluid flow since the transport of oxygen and nutrients throughout the body is governed by the flow of air and blood. Fluid flow is, therefore, crucially important in shaping the world around us, and its full understanding remains one of the great challenges in physics and engineering.

What makes fluid mechanics challenging is that it is often very difficult to predict the motion of fluids. In fact, even to observe fluid motion can be difficult. Most fluids are highly transparent, like air and water, or they are of a uniform color, like oil, and their motion only becomes visible when they contain some type of particle. Snowflakes swirling in the wind, dust kicked up by a car along a dirt road, smoke from a fire, or clouds scudding in a stiff breeze, help to mark the underlying fluid motion (Figure 1.1). It is clear that this motion can be very complicated. By following a single snowflake in a snowstorm, for example, we see that it traces out a complex path, and that each flake follows a different path. Eventually, all the flakes end up on the ground, but it is difficult to predict where and when a particular snowflake ends up. The fluid that carries the snowflake on its path experiences similar contortions, and generally the velocity and acceleration of a particular mass of fluid vary with time and location. This is true for all fluids in motion: the position, velocity, and acceleration of a fluid is, in general, a function of time and space.

To describe the dynamics of fluid motion, we need to relate the fluid acceleration to the resultant force acting on it. For a rigid body in motion, such as a satellite in orbit, we can follow a fixed mass, and only one equation (Newton's second law of motion, $F = ma$) is required. Fluids can also move in rigid-body motion, but more commonly one part of the fluid is moving with respect to another part (there

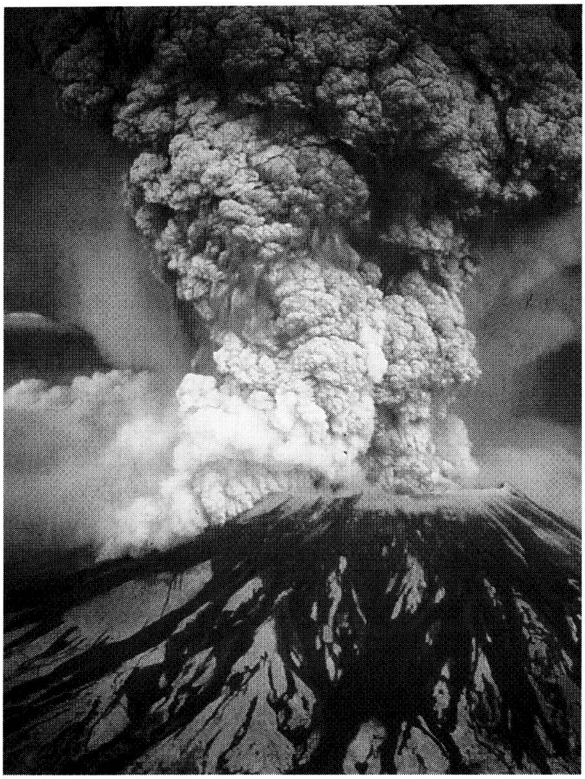

FIGURE 1.1 The eruption of Mt. St. Helens, May 18, 1980. Austin Post/U.S. Department of the Interior, U.S. Geological Survey, David A. Johnston, Cascades Volcano Observatory, Vancouver, WA.

is *relative* motion), and then the fluid behaves more like a huge collection of particles. Each snowflake, for example, marks one small mass of fluid (a fluid *particle*) and to describe the dynamics of the entire flow requires a separate equation for each fluid particle. The solution of any one equation will depend on every other equation because the motion of one fluid particle depends on its neighbors, and solving this set of simultaneous equations is obviously a daunting task. It is such a difficult task, in fact, that for almost all practical problems the exact solution cannot be found, even with the aid of the most advanced computers. It seems likely that this situation will continue for many years to come, despite the projected developments in computer hardware and software capabilities.

To make any progress in the understanding of fluid mechanics and the solution of engineering problems, we usually need to make approximations and use simplified flow models. But how do we make these approximations? Physical insight is often necessary. We must determine the crucial factors that govern a given flow, and to identify the factors that can safely be neglected. This is what sometimes makes fluid mechanics difficult to learn and understand: physical insight takes time and familiarity to develop, and the reasons for adopting certain assumptions or approximations are not always immediately obvious.

To help develop this kind of intuition, this book starts with the simplest types of problems and progressively introduces higher levels of complexity, while at the same time stressing the underlying principles. We begin by considering fluids that are in rigid-body motion, then fluids where relative motions exist under the action of simple forces, and finally more complex flows where viscosity and compressibility are important. At each stage, the simplifying assumptions will be discussed, although the full justification is sometimes postponed until after the later material is understood. By the end of the book, the reader should be able to solve basic problems in fluid mechanics, while understanding the limitations of the tools used in their solution.

Before starting along that path, we need to consider some fundamental aspects of fluids and fluid flow. In this chapter, we discuss the differences between solids and fluids, and introduce some of the distinctive properties of fluids such as density, viscosity, and surface tension. We will also consider the type of forces that can act on a fluid, and its deformation by stretching, shearing, and rotation. We begin by describing how fluids differ from solids.

1.1 THE NATURE OF FLUIDS

Almost all the materials we see around us can be described as solids, liquids, or gases. Many substances, depending on the pressure and temperature, can exist in all three states. For example, H_2O can exist as ice, water, or vapor. Liquids and gases are both called fluid states, or simply fluids.

Fluids behave differently from solids in two respects. The most obvious property of fluids that is not shared by solids is the ability of fluids to flow and change shape; fluids do not hold their shape independent of their surroundings, and they will flow spontaneously within their containers. In this respect, liquids and gases respond somewhat differently in that gases fill a container fully, whereas liquids occupy a definite volume. When a gas and a liquid are both present, an interface forms between the liquid and the gas called a *free surface* (Figure 1.2). At a free surface, surface tension may be important, and waves can form. Gases can also be dissolved in the liquid, and when the pressure changes bubbles can form, as when a soda bottle is suddenly opened.

The most distinctive property of fluids, however, is its response to an applied force or an applied stress (stress is force per unit area). For example, when a shear stress is applied to a fluid, it experiences a continuing and permanent distortion. Drag your hand through a basin of water and you will see the distortion of the fluid (that is, the flow that occurs in response to the applied force) by the swirls and eddies that are formed in the free surface. This distortion is permanent in that the fluid does not return to its original state after your hand is removed from the

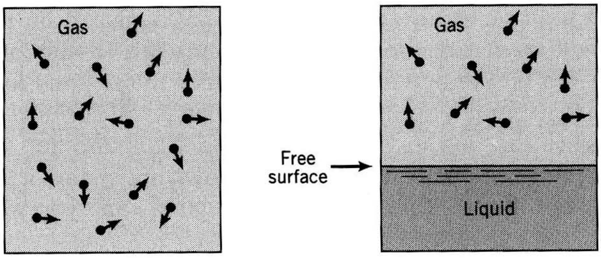

FIGURE 1.2 Gases fill a container fully (left), whereas liquids occupy a definite volume, and a free surface can form (right).

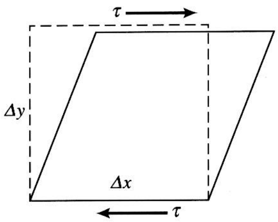

FIGURE 1.3 When a shear stress τ is applied to a fluid element, the element distorts. It will continue to distort as long as the stress acts.

fluid. Also, when a fluid is squeezed in one direction (that is, a normal stress is applied), it will flow in the other two directions. Squeeze a hose in the middle and water will issue from its ends. If such stresses persist, the fluid continues to flow. Fluids cannot offer permanent resistance to these kinds of loads. This is not true for a solid; when a force is applied to a solid it will deform only as much as it takes to accommodate the load, and then the deformation stops.

> *Thus, a fluid can be defined unambiguously as a material that deforms continuously and permanently under the application of a shearing stress, no matter how small. This definition does not address the issue of how fast the deformation occurs and as we shall see later this rate is dependent on many factors including the properties of the fluid itself. The inability of fluids to resist shearing stress gives them their characteristic ability to change shape or to flow; their inability to support tension stress is an engineering assumption, but it is a well-justified assumption because such stresses, which depend on intermolecular cohesion, are usually extremely small. . . .*
>
> *Because fluids cannot "support" shearing stresses, it does not follow that such stresses are nonexistent in fluids. During the flow of real fluids, the shearing stresses assume an important role, and their prediction is a vital part of engineering work. Without flow, however, shearing stresses cannot exist, and compression stress or pressure is the only stress to be considered."*[1]

So we see that the most obvious property of fluids, their ability to flow and change their shape, is precisely a result of their inability to support shearing stresses (Figure 1.3). Flow is possible without a shear stress, since differences in pressure will cause a fluid to experience a resultant force and an acceleration, but when the shape of the fluid is changing, shearing stresses must be present.

With this definition of a fluid, we can recognize that certain materials that look like solids are actually fluids. Tar, for example, is sold in barrel-sized chunks, which appear at first sight to be the solid phase of the liquid that forms when the tar is heated. However, cold tar is also a fluid. If a brick is placed on top of an open barrel of tar, we will see it very slowly settle into the tar. It will continue to settle as time goes by—the tar continues to deform under the applied load—and eventually the brick will be completely engulfed. Even then it will continue to move downwards until it reaches the bottom of the barrel. Glass is another substance that appears to be solid, but is actually a fluid. Glass flows under the action of its own weight. If you measure the thickness of a very old glass pane you would find

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[1] *Elementary Fluid Mechanics,* 7th edition, by R.L. Street, G.Z. Watters, and J.K. Vennard, John Wiley & Sons, 1996.