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LOW FREQUENCY SECOND ORDER  
WAVE EXCITING FORCES ON  
FLOATING STRUCTURES

Dr. Ir. J. A. PINKSTER



PUBLICATION No. 650  
NETHERLANDS SHIP MODEL BASIN  
WAGENINGEN - NETHERLANDS

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## I. INTRODUCTION

Stationary vessels floating or submerged in irregular waves are subjected to large, so-called first order, wave forces and moments which are linearly proportional to the wave height and contain the same frequencies as the waves. They are also subjected to small, so-called second order, mean and low frequency wave forces and moments which are proportional to the square of the wave height. The frequencies of the second order low frequency components are associated with the frequencies of wave groups occurring in irregular waves.

The first order wave forces and moments are the cause of the well known first order motions with wave frequencies. Due to the importance of the first order wave forces and motions they have been subject to investigation for several decades. As a result of these investigations, methods have evolved by means of which these may be predicted with a reasonable degree of accuracy for many different vessel shapes.

This study deals with the mean and low frequency second order wave forces acting on stationary vessels in regular and irregular waves in general and, in particular, with a method to predict these forces on basis of computations. Knowledge concerning the nature and magnitude of these forces is of importance due to the effect they have been shown to have on the general behaviour of stationary structures in irregular waves.

The components of mean and low frequency second order wave forces can affect different structures in different ways and although of the same origin have even been called by different names. The horizontal components of the mean and low frequency second order wave forces are also known as wave drift forces since, under the influence of these forces, a floating vessel will carry out a steady slow drift motion in the general direction of wave propagation if it is not restrained.

The importance of the mean and low frequency wave drift forces from the point of view of motion behaviour and mooring

loads on vessels moored at sea has been recognized only within the last few years. Verhagen and Van Sluijs [I-1], Hsu and Blenkarn [I-2] and Remery and Hermans [I-3] showed that the low frequency components of the wave drift forces in irregular waves could, even though relatively small in magnitude, excite large amplitude low frequency horizontal motions in moored vessels. It was shown that in irregular waves the drift forces contain components with frequencies coinciding with the natural frequencies of the horizontal motions of moored vessels. Combined with the fact that the damping of low frequency horizontal motions of moored structures is generally very low, this leads to large amplitude resonant behaviour of the motions. See Figure I-1.

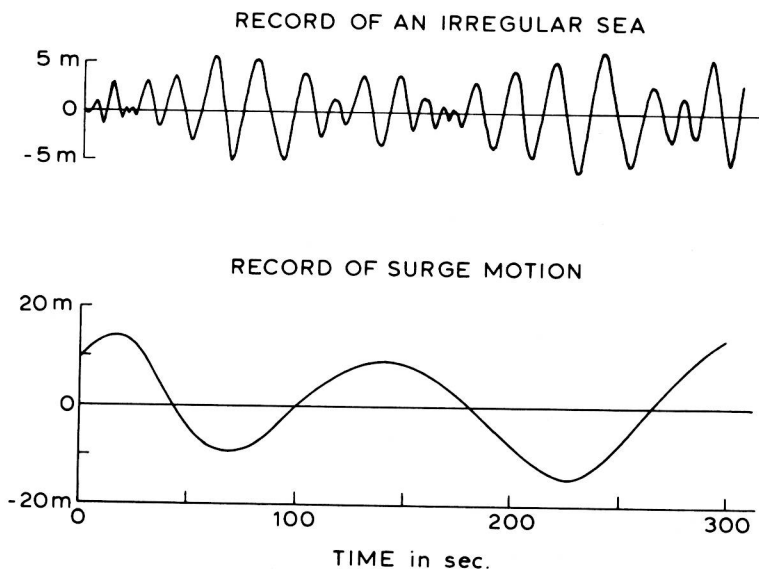


Fig. I-1 Low frequency surge motions of a moored LNG carrier in irregular head seas.

Remery and Hermans [I-3] established that the low frequency components in the drift forces are associated with the frequencies of groups of waves present in an irregular wave train. See Figure I-2.



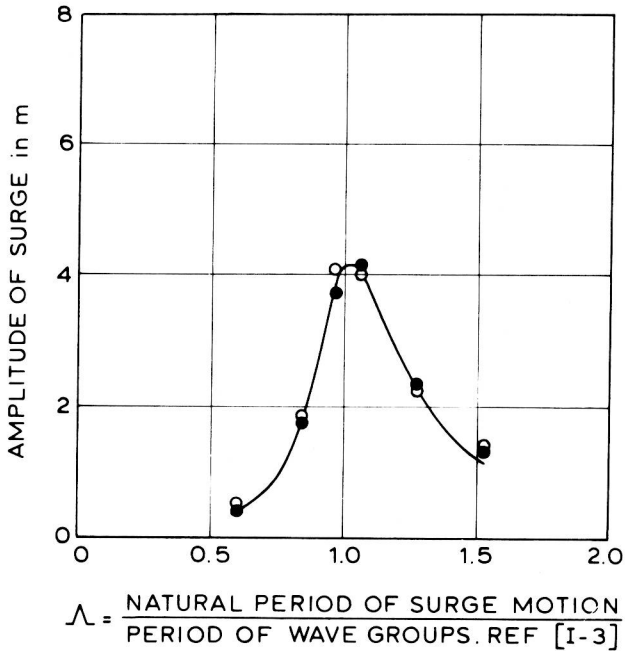


Fig. I-2 Surge motions of a moored barge in regular head wave groups. Ref. [I-3].

Dynamically positioned vessels such as drill ships which remain in a prescribed position in the horizontal plane through the controlled use of thrust generated by propulsion units are also influenced by mean and low frequency wave drift forces. The power to be installed in these vessels is dependent on the magnitude of these forces. The frequency response characteristics of the control systems must be chosen so that little or no power is expended to compensate the large oscillatory motions with wave frequencies, while the mean and low frequency horizontal motions caused by the mean and low frequency drift forces should be reduced to values commensurate with the task of the vessel. This has led to the development of sophisticated control systems. See Figure I-3.

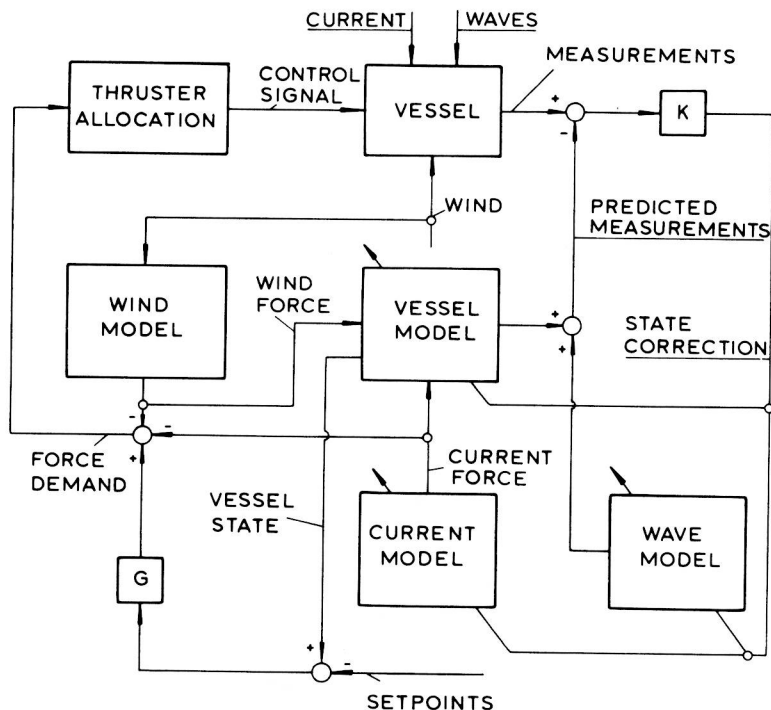


Fig. I-3 Block diagram for typical dynamic positioning system.

The vertical components of the second order forces are sometimes known as suction forces. This term is generally applied in connection with the mean wave induced vertical force and pitching moment acting on submarine vehicles when hovering or travelling near the free surface. It is shown by Bhattacharyya [I-4] that in extreme cases the upward acting suction force due to waves can cause a submarine vehicle to rise and broach the surface, thus posing a problem concerning the control of the vehicle in the vertical plane. See Figure I-4.

The vertical components of the second order wave forces have also been connected with the phenomena of the steady tilt of semi-submersibles with low initial static stability as indicated by Kuo et al [I-5]. Depending on the frequency of the waves it has been found that the difference in the suction forces on the floaters of a semi-submersible can result in a tilting moment,

which can cause the platform to tilt towards or away from the oncoming waves. Such effects are of importance in judging the minimum static stability requirements for such platforms. From observations in reality and from the results of model tests it has been found that large, deep floating storage vessels can carry out low frequency heave motions in irregular waves which are of the same magnitude as the heave motions with wave frequencies.

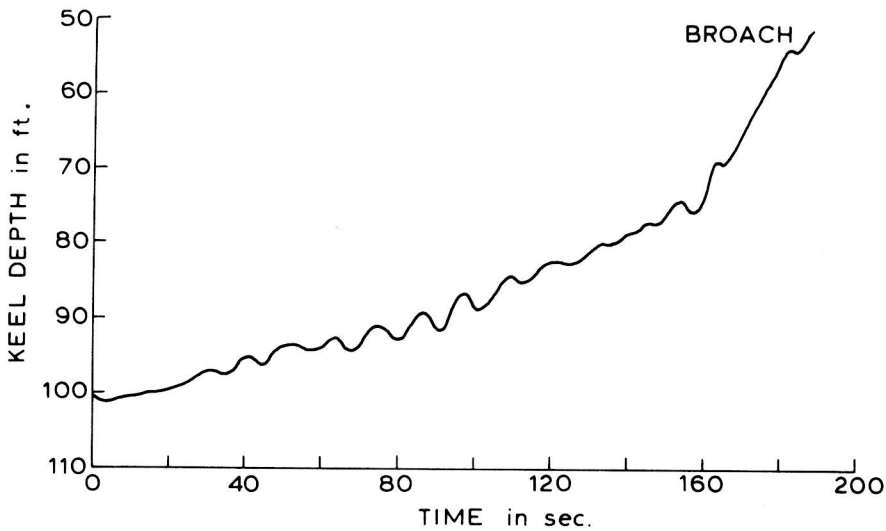


Fig. I-4 Depth record showing effect of suction force on submarine under waves. Quartering sea, wave height = 18 ft., vessel speed = 0 knot. Ref. [I-4].

From the foregoing it can be seen that, depending on the kind of structure or vessel considered, one or more of the six components of the mean and low frequency second order wave forces in irregular waves can be of importance. In order to be able to evaluate the influence of such forces on the performance or behaviour of a structure the most reliable method available, which can take into account in a relatively straightforward way those factors which are deemed of importance for the behaviour of a

system, is by means of model tests. In many practical cases sufficient insight in the complex behaviour of, for instance, a large tanker moored to a single point mooring system is still lacking for reliable prediction of the motion behaviour and forces in the mooring system to be made by means other than physical model testing.

Simulation techniques based on numerical computations are becoming of increasing importance in the design phase of many floating structures however. For instance, in order to evaluate the effectiveness of control systems for dynamically positioned vessels, time domain simulations, which take into account the equations of motion of the vessel and the behaviour of external loads such as the mean and low frequency wave drift forces, are carried out. In such cases, due to the complexity of the control system and the objectives of the study, it is more practical to make use of simplified equations describing the environmental forces and the reaction of the structure or vessel to external forces than to simulate the characteristics of the control systems during a model test. See for instance Sjouke and Lagers [I-6], Sugiura et al [I-7] and Tamehiro et al [I-8]. For such simulation studies accurate numerical data on the behaviour of the mean and low frequency wave forces are desirable, so that meaningful results can be given regarding the systems under investigation. See for instance Van Oortmerssen [I-9] and Arai et al [I-10]. In order to produce numerical results, however, a theory must be available on which calculations can be based. In this study such a theory is developed based on potential theory. The final expressions are valid for all six degrees of freedom and are obtained through direct integration of the fluid pressures acting on the instantaneous wetted surface of the body. The final expressions are evaluated using an existing computer program based on three-dimensional linear potential theory. Numerical results are compared with analytical results obtained for a simple shaped body using a different theory. Experimental results for different, more practical shapes of vessels and structures are compared with results of computations. It is shown that the expressions obtained for the mean and low frequency second order wave forces can be used to gain more insight in the mechanism by which waves and

structure interact to produce the forces. It is also shown that the insight gained using the method of direct integration can be used to enhance the positioning accuracy of dynamically positioned vessels in irregular waves. This is effected through the use of a wave-feed-forward control signal based on the instantaneous relative wave height measured around the vessel.

## II. PAST DEVELOPMENTS CONCERNING THE COMPUTATION OF MEAN AND LOW FREQUENCY WAVE FORCES

### II.1. Introduction

In this section, in which a review is given of developments in the past concerning theories which may be used to predict the second order wave forces, theories concerning the prediction of the added resistance of ships travelling in waves will also be taken into account, since the physical aspects are the same in both cases. In fact the added resistance is simply the longitudinal component of the mean second order wave forces for the case of non-zero forward speed. Indeed, initially emphasis was placed on obtaining good estimates of the added resistance in waves of vessels with forward speed. Only in recent years, due to the enormous increase in the number of vessels being moored at sea, have theories been developed which did not have to take into account the effect of forward speed which is of great importance for the added resistance. Most of the work carried out in the past has been concerned solely with the mean second order wave forces on a vessel or structure travelling or stationary in regular waves.

Maruo [II-1] and Gerritsma [II-2] show that on basis of this information the mean component of the second order wave force can be determined in irregular waves. As shown by Dalzell [II-3] the low frequency component of the second order wave forces on bodies in irregular waves can, strictly, only be determined from knowledge of the low frequency excitation in regular wave groups consisting of combinations of two regular waves with different frequencies. The low frequency wave force will then have the frequency corresponding to the difference frequency of the component regular waves. As will be seen in this section only in recent times have attempts been made to determine these components of the second order forces.

### II.2. Historical review

The existence of non-zero mean components in the total wave force acting on a floating vessel was first noted by Suyehiro

[II-4] who, from experiments, found that a vessel rolling in regular beam waves was subjected to a mean sway force. Suyehiro contributed this force to the capability of the vessel to reflect part of the incoming wave.

Watanabe [II-5] gave an expression for the mean sway force in regular waves based on the product of the first order roll motion and the Froude-Kryloff component of the roll moment, which indicated that the phenomenon involved was of second order. Results of Watanabe's calculations accounted for about half of the mean forces measured by Suyehiro.

Havelock [II-6] gave a similar second order expression for the mean longitudinal component of the second order wave force or added resistance on vessels in head seas involving the Froude-Kryloff parts of the heave force and pitching moment and the heave and pitch motions. This expression was used to estimate the increase in resistance experienced by a vessel travelling into head waves. The results obtained using Havelock's expression generally overestimate the added resistance at pitch resonance and underestimate the added resistance in the range of short wave lengths, where diffraction effects become more important. Watanabe's and Havelock's expressions for the mean second order wave forces in regular waves neglected diffraction effects.

Maruo [II-7] presented expressions for the longitudinal and transverse components of the mean horizontal second order wave force on stationary vessels in regular waves. The theory is valid for two and three-dimensions and is exact to second order within potential theory. It is based on the application of the laws of conservation of momentum and energy to the body of fluid surrounding the vessel. The final expressions derived are evaluated based on knowledge of the behaviour of the potential describing the fluid motions at great distance from the body. Numerical results given by Maruo are, however, limited and do not give satisfactory verification of the applicability of the theory since no correlation is given with experimental results.

Kudou [II-8] has given analytical results on the mean horizontal wave force on a floating sphere in regular waves using Maruo's [II-7] theory and shows reasonable correlation between computed and measured data.

Newman [II-9] rederived Maruo's three-dimensional expressions for the horizontal force components and extended the theory by including an expression for the mean yaw moment. The expressions were evaluated using slender body assumptions and results of computations compared with experimental results given by Spens and Lalangas [II-10]. Through lack of sufficient experimental data no final conclusions could be drawn regarding the validity of the theory.

Faltinsen and Michelsen [II-11] modified Newman's expression and evaluated their result by using a computer program based on three-dimensional potential theory using a distribution of singularities over the surface of the body. Results of computations compared with experimental results of the mean horizontal force on a box shaped barge in regular waves showed good agreement.

Recently Molin [II-12] modified Maruo's expression for the horizontal force and evaluated it using a numerical fluid finite elements method of computing the potential describing the fluid motion. The modification to the original formulation lies in the change of the surface of integration. Molin used the mean surface of the vessel while Maruo applied asymptotic expansions valid at great distance from the vessel. Molin's results compare well with experimental results on the mean longitudinal and transverse force and yawing moment on a stationary tanker in head, beam and bow quartering regular waves.

Kim and Chou [II-13] have made use of Maruo's [II-7] expression for the two-dimensional case of a vessel in beam seas to derive the mean sway force on stationary vessels in oblique waves. Comparisons made by Faltinsen and Løken [II-14] with results obtained by other methods and from experimental results with the method of Kim and Chou indicate that the method can show large



deviations.

Joosen [II-15] has determined, by application of slender body theory, the added resistance of ships using Maruo's [II-7] expression. The final result is similar to that found by Havelock [II-6]. In Joosen's case the added resistance is independent of speed.

Lee and Newman [II-16] have given expressions to determine the mean vertical force and pitching moment acting on deeply submerged slender cylinders. The method is based on momentum considerations. No computed results are given.

Karppinen [II-17] has developed a method to determine the mean second order wave force and moment on semi-submersible structures based on three-dimensional potential theory. Karppinen assumes that the structure may be subdivided into slender elements which do not interact. The total mean forces and moments are found by summation of the contributions of the elements. The mean force on each element is determined from momentum considerations in a manner similar to that given by Lee and Newman [II-16]. Karppinen gives computed results for a semi-submersible. No comparisons are made with experimental results. Mean forces on simple elements are compared with results obtained by others.

Lin and Reed [II-18] have presented a method, based on momentum consideration and through the use of an asymptotic form of the Green's function valid at a large distance, for the mean horizontal second order force and yaw moment on ships travelling at a constant speed in oblique regular waves. No results of computations are given.

An approximative theory for the added resistance in regular waves is given by Gerritsma and Beukelman [II-19]. In this method the mean force is derived by equating the energy radiated by the oscillating vessel to work done by the incoming waves. The expression obtained has been applied to the case of ships travelling in head seas and the correlation between the computed and measured added resistance is good. Strip theory methods are used to evalu-