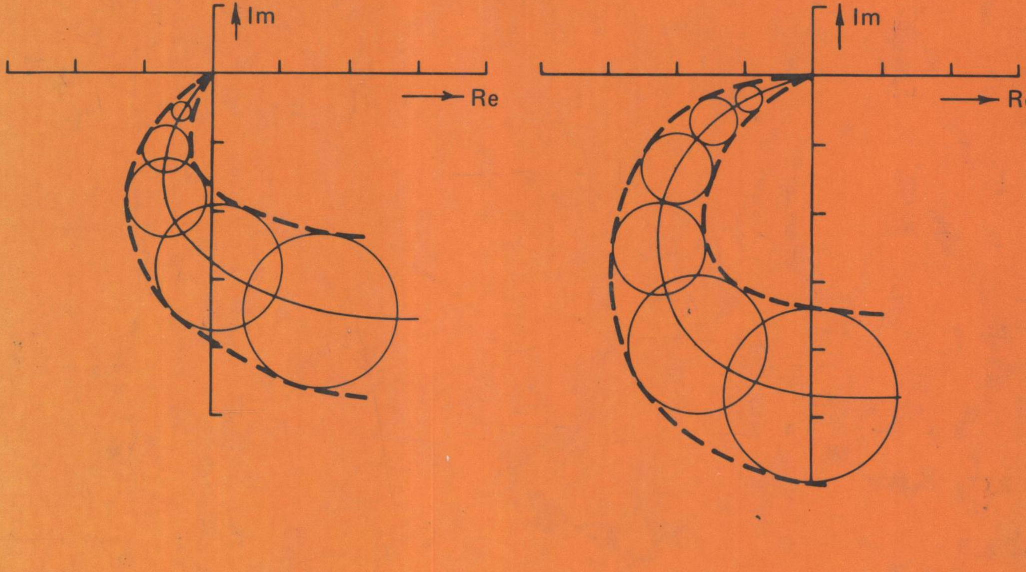
Electrical Engineering and Electronics/19

# Multivariable Control

## An Introduction

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# Multivariable Control

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To Alex and Jay

# Preface

This book is intended for use as a reference work for control engineers, systems engineers, systems analysts, and applied mathematicians, and as a graduate and senior undergraduate textbook. The material is arranged to form a logical progression from the elementary concepts in system theory to the more advanced synthesis and design techniques. Although some of the structural properties of multivariable systems are now being expressed within the framework of abstract geometric concepts, an algebraic approach has been adopted throughout the book to establish a direct correspondence between the analytical results and their applications. For the student, it presents an evolution of the basic concepts of some of the fundamental aspects of multivariable theory and their relevance in design. For the practicing engineer, the book provides a systematically arranged source of reference. Although many basic aspects of control have been included, the reader is assumed to have an acquaintance with the rudiments of feedback theory.*

P. K. Sinha

*Covered in the companion volume, An Introduction to Linear Control Systems by T. E. Fortmann and K. L. Hitz (Marcel Dekker, 1977), Control and Systems Theory series, Vol. 5.

# Acknowledgments

Although a certain amount of original material has been included in this book, a vast majority of material has been drawn from the literature. I have attempted to acknowledge my sources as much as possible. However, in view of the extensive research activities in this area of control theory, the included list must remain incomplete. I am deeply indebted to the authors of many books, published papers, and unpublished reports, and to the originators of many now-classical concepts associated with the analysis and design of multivariable systems. I am particularly grateful to the editors of Automatica, the IEE Proceedings, the IEEE Transactions, and the International Journal of Control for permitting me to use material from many papers published in these journals. I have derived a number of classical results from State Space Analysis by K. Ogata (Prentice-Hall), State Variables for Engineers by P. M. DeRusso, J. R. Roy, and C. M. Close (Wiley), and Introduction to Linear Systems by C. T. Chen (Holt, Rinehart and Winston). I would like to thank these publishers, and many others too numerous to mention individually, for their permission to use many mathematical examples.

I am grateful to Professor J. C. West for encouraging me to write this book, and to Professor J. L. Douce, and the late Professor J. A. Shercliff for their support during the preparation of this book. A one-term sabbatical from the University of Warwick to complete the final version of the book is gratefully acknowledged. I express my appreciation to Sandra Callanan, Nada Gvero, Terri Moss, Dinah Staples, and Linda Wooldridge for typing the various sections of the manuscript, and to Christine Allsopp for her skillful drawing of the line diagrams. I thank my wife for freeing me from the domestic chores during the writing of the book.

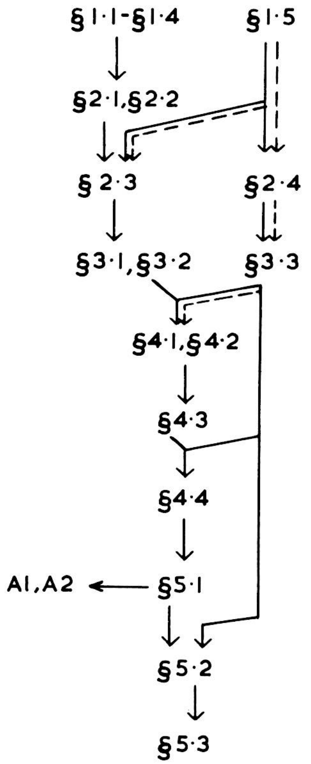
I owe a significant debt of gratitude to Dr. A. D. G. Hazlerigg for introducing me to the problems of designing interacting control systems and to Professor B. V. Jayawant for providing the opportunity to write this book. I dedicate this book to them for their moral support which often went beyond academic interests.

# Introduction

The development as well as the choice of topics in this volume have been the result of a compromise between the formulation of a sufficiently broad analytical framework and the presentation of a number of specific applicable design procedures. In this rather compact treatment of an extensive literature covering many significant contributions, many points of detail and some relevant topics had to be omitted. The book is divided into two parts: Part I deals with analysis by using the concept of state variables (time domain) and by extending the transform methods (frequency domain) to multi-input/multioutput systems; Part II presents a systematic development of a number of time- and frequency-domain synthesis methods which are conceptually simple and easier to implement rather than to present a state-of-the-art survey of multivariable synthesis methods. Wherever appropriate, illustrative examples have been included to outline the importance or usefulness of the analytical results. An overview of these two parts is given below.

Topics in the first part have been developed to establish a mathematical framework for the analysis of linear time-invariant multivariable systems. In view of the wide-ranging concepts necessary to introduce the many facets of multivariable control, topics with related themes have been grouped into five self-contained chapters. This approach, while creating a certain amount of compartmentalization, has been adopted primarily to present a (large) number of related concepts without too much reference to their implications in the broader (analytical) context. Each chapter, however, has been structured and cross-referenced to enable the reader to appreciate the interrelationships among the various analytical results. The interdependence of the different sections in this part is shown in the accompanying figure.

A brief review of a number of essential results of matrix theory is given in Chapter 1. The contents here have been organized into five sections covering the introductory concepts of matrices and vectors, followed by the definition and computation of matrix inverses and eigenvectors. The notions of transformation and function of matrices are then outlined. The final section contains some salient results associated with polynomial and rational function matrices. Material in this section forms the basis for algebraic control theory covered in subsequent chapters.




Development of sections in Part I (dashed line shows the initial progression for readers with introductory control background).

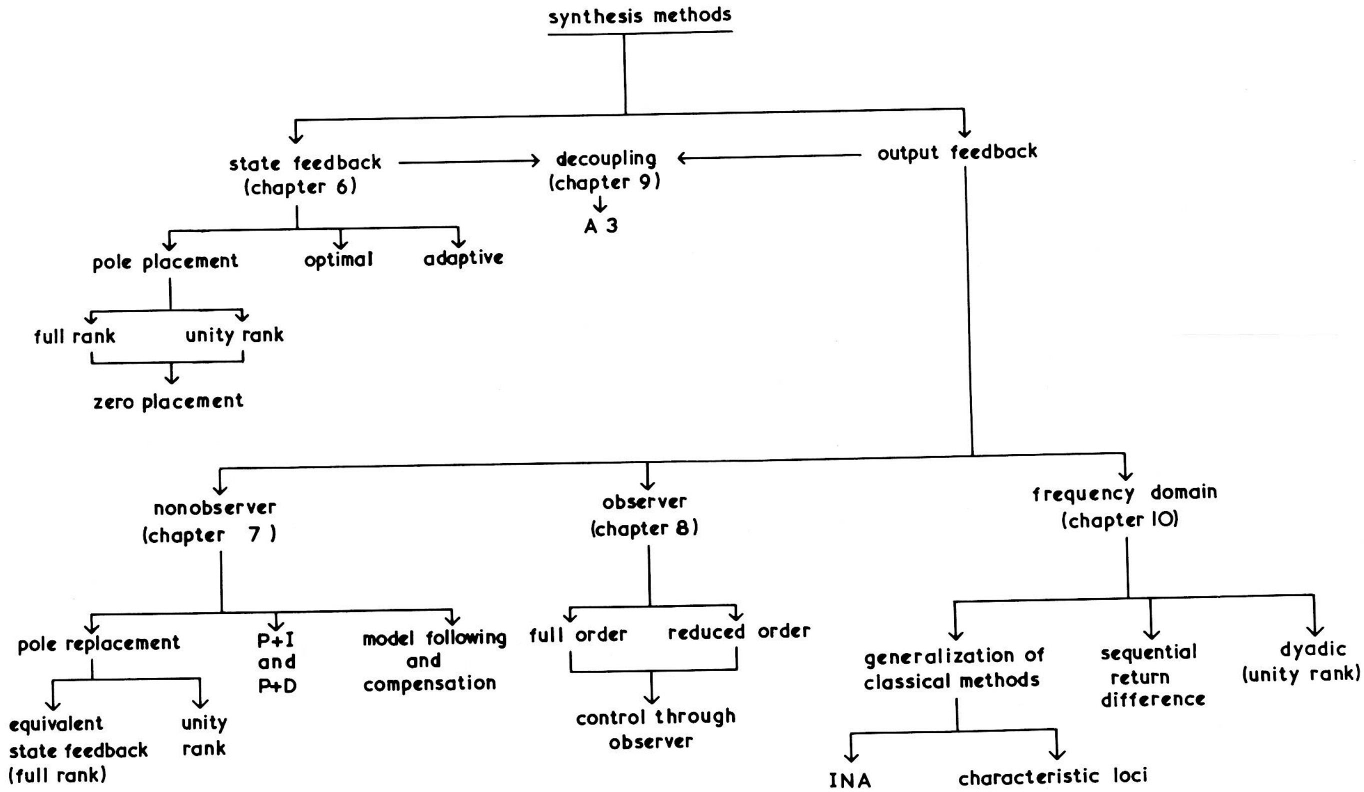
The basic theory of system description is developed in Chapter 2. The first three sections cover the fundamentals of system representations in the time and frequency domains, and the final section introduces the polynomial-function descriptions of linear multivariable systems. A significant part of the results in this section has been developed during the past fifteen years and forms the basis of algebraic control theory now being extensively used in the generalization of classical stability analyses.

Chapter 3 begins with an introductory section covering the basic concepts and definitions of controllability and observability and their relationship to pole-zero cancellation. A short account of controllable and observable forms is given in the second section. Since many of these concepts and the related transformations are covered in most introductory books on control, treatment here is brief, aimed primarily at presenting the main results within the framework of the book. The last section presents a fairly extensive algebraic interpretation of controllable and observable modes through decoupling zeros. The material in this section has been structured to establish, as far as possible, a one-to-one correspondence between the time- and frequency-domain results and to relate them to the reduction of system order through the removal of modes (zeros) which do not appear in the input/output description.

In view of the growing use of a combination of state-space and transfer-function representations in synthesis, transformation of one form to another plays an important role in many computer-aided-design techniques. This is likely to remain so as more user-oriented software support of distributed computer networks becomes available. The criterion for the selection of material in Chapter 4 has been not merely to cover the principles, but also to help the reader appreciate the mechanisms of realization and develop some expertise in using "abstract" analytical results. The first two sections present the basic definitions and direct (nonminimal) realization methods, and the last two sections present a selection of irreducible realization methods. The section of realization from matrix-fraction description has been included to complement previous results on matrix polynomials.

The single-input/single-output frequency domain approaches of Nyquist and Bode are used extensively by control engineers in the analysis of a system's ability to achieve the desired operating requirements, due mainly to their conceptual simplicity. Because of this significant advantage, a considerable amount of research has been undertaken over the past fifteen years to extend these "classical" methods to multivariable systems. This has resulted in the establishment of a number of practically useful methods of analyzing the behavior of multi-input/multi-output systems under a wide range of operating conditions. The selection of material in the three sections in Chapter 5 reflects the relevance of algebraic control theory in the development of generalized stability theory. The first section covers the basic stability definitions and some associated properties. This is followed by a brief review of some "more established" generalized stability criteria reflecting the relevance of the classical concepts in the analysis of multivariable systems. (The analytical results presented in this chapter form the basis of the frequency-domain synthesis methods discussed in Chapter 10.) The final section presents an outline description of three principal design criteria usually needed in synthesis.

The second part of the book complements the first through the development of a number of directly applicable methods of designing linear time-invariant multivariable systems. The subdivision of synthesis methods considered in this half of the book is shown schematically in the accompanying figure. Most of the material in Chapters 6 to 10 has been collected from research papers published over the past decade, and structured to present self-contained derivations of the analytical results and their application. The illustrative examples have been worked out to help the reader develop adequate skill to select the most appropriate method for a given design




Classification of synthesis methods in Part II.

problem. A number of problems have been included at the end of each part rather than at the end of the individual chapters to provide a collection of numerical exercises which may be used in the application of results derived in more than one chapter.

Many of the analytical results developed here have numerous computational implications, and despite the importance of numerical analysis in modern control theory, these have not been covered in this volume. This is due mainly to the fact that this introductory book is aimed at providing a framework for the appreciation of the mechanisms of using "abstract" results. Literature covering the relevant numerical routines has been referenced wherever appropriate, and a selection of references has been included in the bibliography* for each part. It is hoped that the reader will find the illustrative examples, many worked out in algorithmic form, suitable bases for the development of software for computer-aided analysis/synthesis.

*An introductory treatment of numerical methods may be found in Mathematics of Finite-Dimensional Control Systems by D. L. Russell (New York: Marcel Dekker, 1979). A number of computer programs for basic control system design are contained in Computer Programs for Computational Assistance in the Study of Linear Control Theory by J. L. Melsa and S. K. Jones (New York: McGraw-Hill, 1970).

# Short Table of Laplace Transform Pairs

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

| | Function f(t) | Transform F(s) | |
|---|---|---|---|
| 1 | $u_i(t)$ | $1$ | Unit impulse |
| 2 | $u_s(t)$ | $\frac{1}{s}$ | Unit step |
| 3 | $u_r(t)$ | $\frac{1}{s^2}$ | Unit ramp |
| 4 | $e^{-at}$ | $\frac{1}{s+a}$ | Exponential |
| 5 | $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ | Sine |
| 6 | $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ | Cosine |
| 7 | $\frac{1}{(n-1)!} t^{n-1} e^{-at}$ | $\frac{1}{(s+a)^n}$; $n$ = positive integer | Repeated roots |
| 8 | $\frac{e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}} \sin \omega_n\sqrt{1-\zeta^2}\, t$ | $\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$ | Damped sine |
| 9 | $\frac{df(t)}{dt}$ | $sF(s) - f(0^-)$; $F(s) = \mathcal{L}\{f(t)\}$ | First derivative |
| 10 | $\frac{d^n f(t)}{dt^n}$ | $s^n F(s) - s^{n-1} f(0^-) - s^{n-2}\frac{df(0^-)}{dt} \cdots - \frac{d^{n-1} f(0^-)}{dt^{n-1}}$ | $n$th derivative |
| 11 | $\int_0^t f(t)\, dt$ | $\frac{1}{s} F(s)$ | Integration |
| 12 | $\int_0^t \cdots \int_0^t f(t)\, dt^n$ | $\frac{1}{s^n} F(s)$ | $n$ integrations |
| 13 | $\int_0^t f_1(\tau) f_2(t-\tau)\, d\tau$ | $F_1(s)F_2(s)$; $F_1(s) = \mathcal{L}\{f_1(t)\}$, $F_2(s) = \mathcal{L}\{f_2(t)\}$ | Convolution |
| 14 | $f(t-\tau) u_s(t-\tau)$ | $e^{-\tau s} F(s)$ | Delay |
| 15 | $f(at)$ | $\frac{1}{a} F\left(\frac{s}{a}\right)$ | Scale change |
| 16 | $e^{-at} f(t)$ | $F(s+a)$ | Exponential attenuation |
| 17 | $t f(t)$ | $-\frac{d}{ds}\{F(s)\}$ | Time multiplication |

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