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Michael J. Hirsch
Panos M. Pardalos
Robert Murphey
Don Grundel (Eds.)

Advances in Cooperative Control and Optimization

Proceedings of the 7th International Conference on Cooperative Control and Optimization

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Preface

Optimal control of cooperative systems continues to be at the forefront of research initiatives in the military sciences. Recently, cooperative system research has expanded from the military domain to other engineering disciplines, including drug design and disaster recovery. While there exist many powerful techniques for optimal cooperative control problems, this area is still considered one of the most difficult in the applied sciences. Thus, there must be continual improvements and new insight directed to the modeling and analysis of optimal cooperative control problems. This present volume, as well as volumes from previous years, clearly illustrate novel solutions from some of the best and brightest optimal cooperative control researchers.

This volume represents the most recent in a series of publications discussing recent research and challenges in the field of optimal cooperative control. Most of the chapters in this book were presented at the Seventh International Conference on Cooperative Control and Optimization, which took place in Gainesville, Florida, January 31 – February 2, 2007. It is our belief that this book will be an invaluable resource to faculty, researchers, and students in the fields of optimization, control theory, computer science, and applied mathematics.

We gratefully acknowledge the financial support of the Air Force Research Laboratory, The Center for Applied Optimization at The University of Florida, and Raytheon, Inc. We thank the contributing authors, the anonymous referees, and Springer Publishing for making the conference so successful and the publication of this book possible.

Michael J. Hirsch
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Locating RF Emitters with Large UAV Teams

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Abstract. This chapter describes a principled, yet computationally efficient way for a team of UAVs with Received Signal Strength Indicator (RSSI) sensors to locate radio frequency emitting ground vehicles in a large environment. Such a capability has a range of both civilian and military applications. RSSI sensor readings are noisy and multiple emitters will cause ambiguous, overlapping signals to be received by the sensor. Generating a probability distribution over emitter locations requires integrating multiple signals from different UAVs into a Bayesian filter, hence requiring cooperation between the UAVs. To build a coherent distributed picture given communication limitations, the UAVs share only those sensor readings that induce the largest changes in their local filter. Each UAV translates its probability distribution into a map of information *entropy* and then plans a path that will maximize the reduction in entropy (or conversely provides the highest information gain.) Planned paths are shared with a subset of other UAVs to minimize overlapping search. Experiments in a medium fidelity simulation environment show the approach to be lightweight and effective. Live flight results with lightweight Class I UAVs validate our approach.

1 Introduction

The rapidly improving availability of small, unmanned aerial vehicles (UAVs) and their ever decreasing cost is leading to considerable interest in multi-UAV applications. However, while UAVs have become smaller and cheaper, there is a lack of sensors that are light, small and power efficient enough to be used on a small UAV yet are capable of taking useful measurements of objects often several hundred meters below them. Static or video cameras are one option, however image processing normally requires human input or at least computationally intensive offboard processing, restricting their applicability to very small UAV teams. In this chapter, we look at how teams of UAVs can use very small Received Signal Strength Indicator (RSSI) sensors whose only capability is to detect the approximate strength of a Radio Frequency (RF) signal, to search for and

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accurately locate such sources. RSSI sensors give at most an approximate range to an RF emitter and will be misleading when signals overlap. Applications of such UAV teams range from finding lost hikers or skiers carrying small RF beacons to military reconnaissance operations. Moreover, the core techniques have a wider applicability to a range of robotic teams that rely on highly uncertain sensors, e.g., search and rescue in disaster environments.

Many of the key technologies required to build a UAV team for multi-UAV applications have been developed and are reasonably mature and effective [1,2]. However, for large UAV teams with very noisy sensors, key problems remain, specifically, much previous work is formally grounded but impractical [3]. Often the coordination and planning algorithms and the representations of the environment are not appropriate for more than two or three UAVs and targets. For example, some solutions require a UAV to know the planned paths of all other UAVs in order to plan its own path [8], but this is infeasible (both in terms of communication and computation) when the number of UAVs is large. Other approaches only solve part of the problem, e.g., estimating locations from sensor readings [12] or planning cooperative paths [11], but do not combine these elements in an integrated solution, although there are some exceptions [4]. Signal processing techniques for creating probability distributions from noisy signals have been extensively studied, but rarely have distributed filters versions been created and those that have been do not scale to larger teams [9].

Our approach to this problem has three key elements that enable locating RF emitters with large teams of lightweight UAVs. The first key element is a distributed filter to localize RF emitters in the environment. Each UAV has a Binary Bayesian Grid Filter [7] where a value of a cell in the grid represents the probability that there is an emitter in the corresponding location on the ground. Due to limitations on available communication bandwidth, it is infeasible for UAVs to share their entire distribution, instead they share a small subset of their sensor readings with others in the team. Hence, departing from previous approaches that elicited a model of what teammates know in order to choose what to send [9], we started from the assumption that if some information leads to large local information gain, it will probably do so for much of the team. We investigated two information gain based heuristics for choosing which readings to share with teammates. The first heuristic is to send sensor readings that have the greatest impact on the UAV's local probability distribution. The second heuristic is to create a parallel probability distribution based purely on readings received from teammates and send sensor readings that have the biggest impact on that distribution. Intuitively, the first heuristic sends readings that were most important for the local UAV, while the second sends sensor readings that are most important to the team, given a local model of what the team knows. Experiments show that the first heuristic results in better team behavior than sending random messages, but the second heuristic performs worse than random.

The second element of the approach is to tightly couple estimates of the current locations of the emitters to the UAV path planning process. Specifically, a probability distribution over emitter locations is translated into a map of the information *entropy* in the environment. UAVs plan paths through areas of maximum entropy, hence maximizing expected information gain. The UAVs plan only a relatively short distance ahead in each planning cycle. This approach allows the UAVs to be reactive to new information, which is critical when sensors are highly uncertain and the domain is dynamic. For example, if a UAV traverses an area, but the sensor readings do not provide an accurate picture of that area, the entropy will remain high and the UAV will consider re-traversing the area. Notice that the entropy map coupled with the path planner looking to maximize information gain provides an integrated way for trading off between going to the locations where there will be most information gain and locations that can be quickly reached.

The third key element of the approach is a very lightweight, computationally inexpensive method for cooperative path planning. The important application feature underlying the approach is that due to the high uncertainty and dynamicism in the environment, some overlap of paths is acceptable (or even desirable), provided that the UAVs mainly spread out and search areas of maximum entropy. Our approach is for each UAV to share its planned path with some other members of the team. When planning, each UAV estimates the change in entropy that would be induced by those paths being flown by others and plans on the resulting entropy map. If the most current path of a particular UAV is not known the most recent location is used to roughly estimate where that UAV might be searching.

2 Problem

This chapter presents a method for localizing an unknown number of RF emitters using a team of UAVs. UAVs are outfitted wth RSSI sensors which detect the power of an RF signal at a position in space. The UAVs must maintain a belief over the state of all emitters in the environment in a decentralized manner.

The emitters are represented by the set: $E = \{e_1 \dots e_n\}$ where n is not known to the team of UAVs. Emitters are all assumed to be emitting at a single known frequency.[1] Emitters are mobile and emit intermittently. The homogeneous UAVs are represented by the set: $U = \{u_1 \dots u_m\}$. Each u_i flies a path given by $\boldsymbol{u}^i(t)$. During flight a UAV takes sensor readings, $z_t(\boldsymbol{loc})$ which are the received signal power at a location $\boldsymbol{loc} = \{x, y, z\}$ where $\{x, y, z\}$ gives the Euclidean coordinates of a point in space relative to a fixed origin. The power of the signal received is a result of three components. The first component, $\Gamma(\boldsymbol{loc}, e_i) = \frac{e_{const}}{dist(\boldsymbol{loc}, e_i)^2}$, where $dist(loc, e_i)$ is the Euclidean distance between loc and e_i and e_{const} is a constant that gives the power at $dist(loc_{e_1}, e_i) = 0$, is due to the sources themselves. The second component, $EN(\boldsymbol{loc}, E)$, is due to multi-path and attenuation of the signal due to environmental factors. Multi-path occurs when a reflected component of the signal arrives at a receiver and in combination with an attenuated direct signal results in a perturbed source

[1] This will be relaxed in future work.

location estimate. Finally ϵ gives typical zero-mean normally distributed sensor noise. The total power received at a location (loc) in space is then given by:

$$z_t(\boldsymbol{loc}) = \sum_{e_i \in E} \Gamma(\boldsymbol{loc}, e_i) + EN(\boldsymbol{loc}, E) + \epsilon \sim \mathcal{N}(0, \sigma)$$

Figure 1 shows some signals that will be received at different distances from a single emitter (i.e., no overlap). This is the basic signal model used in the simulation results below and closely represents real data collected from RSSI sensors on a physical UAV. The x-axis shows the distance and the y-axis shows the signal strength in dB (which is a log scale.) There are two important things to notice about this signal. First, it is very noisy, with high variation at all distances from the emitter, with some background noise high enough to represent being close to the emitter. Second, it has a very long "tail", i.e., at a reasonable distance from the emitter there is still useful information in the signal. Figure 2 shows the sensor readings when the UAV flies near one emitter and then another. Notice the overlap in the signals between the emitters, which are about 350m apart.

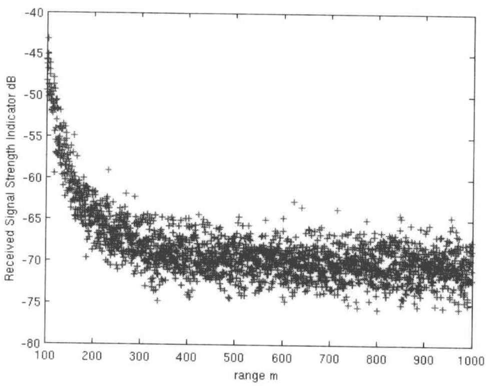

Fig. 1. Sensor readings taken from different distances from an RF emitter

The sensor readings taken by the ith UAV, up until time t are $z^i_{t_0} \dots z^i_t$. Each UAV maintains a posterior distribution P over emitter locations given by $P^i_t(e_1 \dots e_n | z^i_{t_0} \dots z^i_t)$. The UAVs proactively share sensor readings to improve each other's posterior distribution. At time t each u_i can send some subset of locally sensed readings: $\boldsymbol{z}^i_t \subset z^i_{t_0} \dots z^i_t$.

The true configuration of the emitters in the environment at time t is represented as a distribution Q such that

$$Q_t(e_1 \dots e_n) = 1$$

when $e_1 \dots e_n$ gives the true configuration of the emitters at t. The objective is to minimize the divergence between the team belief and the true state of the emitters, while minimizing the cost of UAV flight path, and minimizing the total

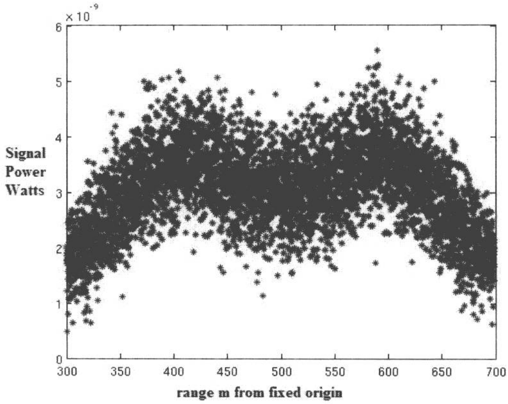

Fig. 2. Sensor readings taken when flying between two emitters, first near one, then near the other

number of messages shared between UAVs. The following function expresses this mathematically:

$$\min_{\boldsymbol{u}^i} \sum_{t} \sum_{u_i \in U} \beta_1 Cost(\boldsymbol{u}^i(t)) + \beta_2 D_{\mathrm{KL}}(P_t^i \| Q) + \beta_3 |\boldsymbol{z}_t^i|$$

where D_{KL} denotes the Kullback Leibler divergence and $\beta_{1\ldots3}$ are weights which control the importance of the individual factors in the optimization process.

3 Algorithm

The most important feature of the overall algorithm is the tight integration of all the key elements to maximize performance at a reasonable computational and communication cost. A Binary, Bayesian Grid Filter (BBGF) maintains an estimate of the current locations of any RF emitters in the environment. This distribution is translated into a map of the entropy in the environment. The entropy is captured in a *cost map*. UAVs plan paths with a modified Rapidly-expanding Randomized Tree (RRT) planner that maximize the expected change in entropy that will occur due to flying a particular path. The most important incoming sensor readings, as computed by the KL information gain they cause, are forwarded to other members of the team for integration into the BBGFs of other UAVs. Planned paths are also shared so that other UAVs can take into account the expected entropy gain of other UAVs when planning their own paths. The paths of other UAVs are also captured in a *cost map*. Additional cost maps, perhaps capturing results of terrain analysis or no-fly zones, can be easily added to the planner.

3.1 Implementation

The overall, integrated process aims to balance the desire to have a principled, formally grounded approach, yet be lightweight and robust enough to be practical for a team of UAVs. The hardware independent components (planners, filters, etc.) are isolated from the hardware specific components (sensor drivers, autopilot) to allow the approach to be quickly integrated with different UAVs or moved from simulation to physical UAVs. The hardware independent components are encapsulated in a *proxy* which will either be on the physical UAV or on a UAV ground station, depending on the vehicle. In the experiments below, the simulations use *exactly* the same proxy code as the live flight experiments with physical UAVs. Figure 3 shows the main components and information flows from the perspective of one UAV-proxy.

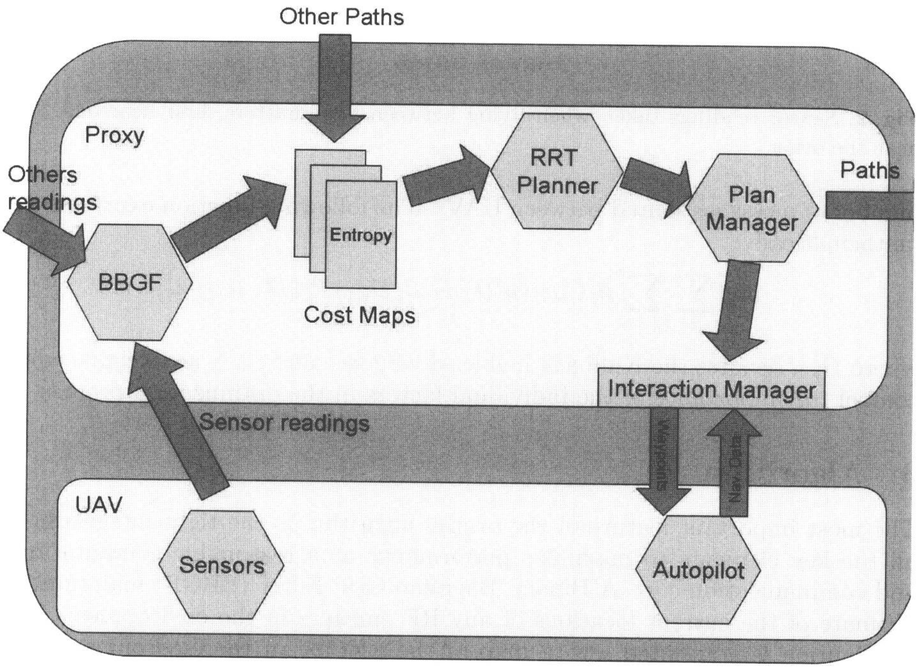

Fig. 3. Block diagram of architecture

4 Distributed State Estimation

In this section, we describe the filter used to estimate the locations of the emitters and the decisions individual UAVs make about sending information to one another.

4.1 Binary, Bayesian Grid Filter

The filter uses a grid representation, where each cell in the grid represents the probability that there is an emitter in the area on the ground corresponding to that location.[2] For a grid cell c the probability that it contains an emitter is written $P(c)$. The grid as a whole acts as the posterior $P_t^i(e_1 \dots e_n | z_{t_0}^i \dots z_t^i)$.

To make calculations efficient, we represent probabilities in *log odds* form, i.e., $l_t = logP(i)$. Updates on grid cells are done in a straightforward Bayesian manner.

$$l_t = l_{t-1} + log\frac{P(e_i|z_t)}{1 - P(e_i|z_t)} - log\frac{P(e_i)}{1 - P(e_i)}$$

where $P(e_i|z_t)$ is a inversion of the signal model, with the standard deviation extended for higher powered signals, i.e.,

$$P(e_i|z_t) = \begin{cases} \frac{1}{\sqrt{2\pi(\sigma_1^2)}} e^{-\frac{1}{2}(z_t - \Gamma)^2} & \text{if } z_t \geq \Gamma \\ \frac{1}{\sqrt{2\pi(\sigma_2^2)}} e^{-\frac{1}{2}(z_t - \Gamma)^2} & \text{otherwise} \end{cases}$$

where $\sigma_1 > \sigma_2$ scales the standard deviation on the noise to take into account structural environmental noise and overlapping signals. Intuitively, overlapping and other effects might make the signal stronger than expected, but they are less likely to make the signal weaker than expected. Figure 4 shows a plot of the (log) probability (y-axis) of a signal of a particular strength (x-axis) when the emitter is 500 m from the sensor.

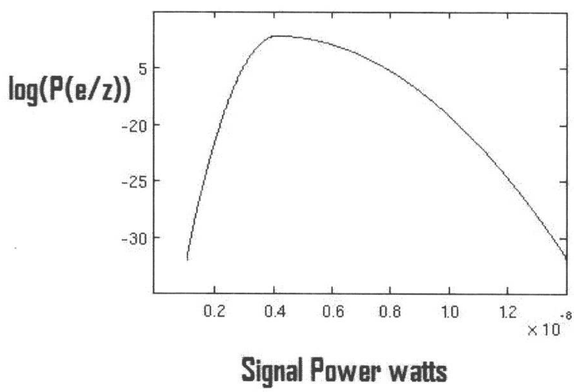

Fig. 4. Mapping between probability and signal strength

Notice that there is no normalization process across the grid because the number of emitters is not known. If the number of emitters were known, a normalization process might be able to change the probability of emitters even in areas where no sensor readings had been taken. Initial values of grid cells are set to values reflecting any prior knowledge or some small uniform value if no knowledge is available.

[2] A quad-tree or other representation might reduce memory and computational requirements in very large environments, but the algorithmic complexity is not justified for reasonable sized domains.