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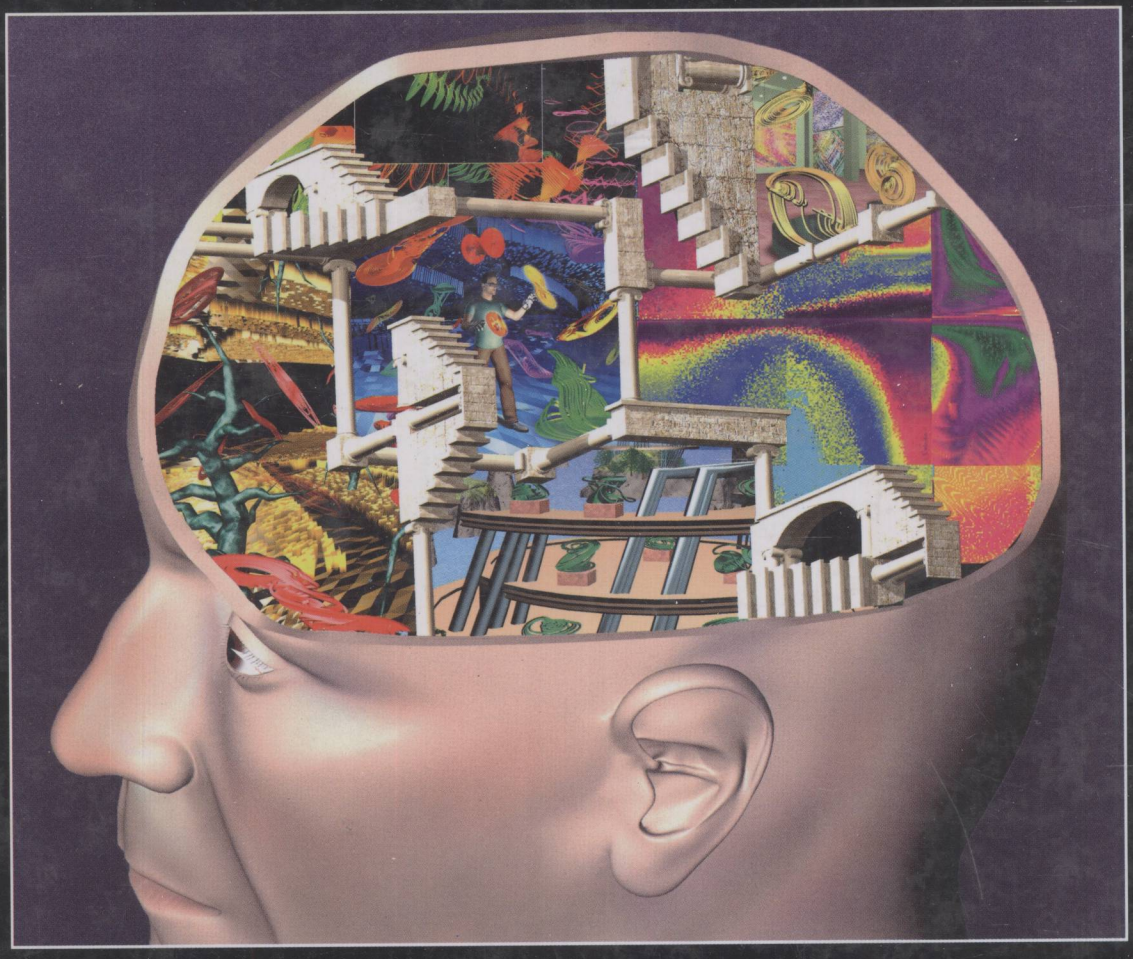
Series A Vol. 61
Series Editor: Leon O. Chua

A GALLERY OF CHUA ATTRACTORS

Eleonora Bilotta
Pietro Pantano



World Scientific

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To my beloved family
Piero, Francesca and Arrigo

To my charming daughters
Maria Fiorella and Eleonora

PREFACE

Chua's circuit — the central theme of this book — is one of the most important scientific discoveries of recent times and one of the most fertile fields of research in complexity science. The book comes twenty-five years after Chua's original work, and fifteen after Madan's celebration of the circuit as a *"paradigm of complexity"*. Without attempting to provide a complete picture of research in the field, it provides an up-to-date review of the relevant literature and a "gallery" of nearly a thousand chaotic attractors generated by the circuit and its generalizations.

The book brings together six tutorial review papers, originally published in 2007, by the *International Journal of Bifurcation and Chaos*:

1. Bilotta, E., Pantano, P. & Stranges, S. [2007a] "A gallery of Chua attractors. Part I," Vol. **17**, 1–60.
2. Bilotta, E., Pantano, P. & Stranges, S. [2007b] "A gallery of Chua attractors. Part II," Vol. **17**, 293–380.
3. Bilotta, E., Pantano, P. & Stranges, S. [2007c] "A gallery of Chua attractors. Part III," Vol. **17**, 657–734.
4. Bilotta, E., Di Blasi, G., Pantano, P. & Stranges, S. [2007d] "A gallery of Chua attractors. Part IV," Vol. **17**, 1017–1078.
5. Bilotta, E., Di Blasi, G., Pantano, P. & Stranges, S. [2007e] "A gallery of Chua attractors. Part V," Vol. **17**, 1383–1511.
6. Bilotta, E., Di Blasi, G., Pantano, P. & Stranges, S. [2007f] "A gallery of Chua attractors. Part VI," Vol. **17**, 1801–1910.

For this edition, we have rechecked the original papers and removed misprints. Section and equation numbers remain unchanged. All references to sections or equations contain a reference to the chapter in which they appear.

The main text is preceded by a broad-ranging "Prologue". This provides detailed "recipes" for readers who wish to build Chua's circuit for themselves, links to software simulating the circuit with dimensionless equations, and a description of how it can be used to produce music. This last part uses findings originally reported in a review-tutorial of 2005 and further elaborated in two lectures at Berkeley, in March 2007. For the original paper see:

Bilotta, E., Gervasi, S. & Pantano, P. [2005] "Reading complexity in Chua's oscillator through music. Part I: A new way of understanding chaos," *Int. J. Bifurcation and Chaos* **15**, 253–382.

Finally we present covers from the six issues of the *International Journal of Bifurcation and Chaos* in which our work originally appeared and the covers of two additional issues of the journal, also based on our work.

For additional references to the multimedia materials and software referred to in the book please visit:

http://galileo.cincom.unical.it/chua

The inside covers show pictures of attractors generated by the Chua circuit and are based on a poster originally created in 2005. It was this poster that inspired us to create our Gallery.

Together with the book, readers will find an audio CD with thirteen pieces, obtained by editing the original music generated by Chua's circuit. The first ten were obtained by applying musical styles to the sequences produced by the circuit. The last two are closer to the originals; here the only edits are minor changes designed to enhance the differences with respect to the earlier pieces and to bring out the expressive power of the music. In what follows, we provide additional details on the edited pieces.

Track 1

This piece was produced from a selection of tunes produced by the Chua attractor. The tunes are organized to provide both a basic structure for the piece and improvised solos. The overall form resembles "fusion" music.

Track 2

Here a single time series is subjected to a sequence of transformations, each of which doubles the previous tempo. The same sequences of notes are transposed into different keys and composed into a longer sequence — creating the arpeggio in the cello part. Different notes in the time series are played by different instruments. In the second part of the piece, the original time series is filtered and a delay effect applied.

Track 3

This piece explores the possibility of creating traditional musical forms, using material produced by the Chua attractor. The piece is based on selected melody fragments, transposed into different keys with different tempos. The voices are organized in the form of a canon.

Track 4

This piece is based on two different time series representing similar melodies which have both been produced in the same way. The first can be recognized by the initial beat and the arpeggio on the harp, the second from the sound of the oboe and the violins. Parts of the piece are transposed, creating harmonic passages and modulations into other keys.

Track 5

Here, the three coordinates of the Chua attractor are used to code the pitch, volume and duration of the notes. This raw material is then used as inspiration for the final arrangement. As previously, the transformations applied to the raw material produce arpeggios which are used both in the accompanying and the solo parts. Sometimes we can hear the melody modulated from the minor to the major. The way the Chua attractor controls the dynamics, gives the piece a natural, lively feel.

Track 6

This piece attempts to recreate classical feelings of "lightness". A single time series is subjected to various time transformations, with each version being played on a different instrument. Thus the slowest version is played on the double bass; the harp plays an arpeggio — a transposition of the original melody — at an intermediate tempo. The original melody — without modification — is played by the violins. Individual musical phrases are brought out by selecting specific notes from the time series and deleting others.

Track 7

The raw material for this piece was produced by filtering the data from the attractor to produce scales in different keys. The filtering was performed in real time, allowing the composer to perceive passages in the music as they emerged and to control the volume. This raw material was then subjected to a selection procedure to identify the most interesting parts of the work. Finally, a completely new bass line was added, giving meaning to the music.

Track 8

Different parts were obtained by transforming note durations in a single original time series. For example, the synthesizer we hear in the background, plays its notes at twice the speed of the notes played by the bass. The guitar plays two times faster than the synthesizer. The second part of the piece is marked by the return of the original time series, played on a second synthesizer which applies a series of sound effects.

Track 9

In this piece, the aim was to create musical structures similar to those we find in serious music from the beginning of the XX century. What is special here, is the way the time series are used to control the percussion instruments. In the orchestration, the different coordinates are used to control different families of instruments: (X: Woodwinds, Y: Percussion , Z: Strings).

Track 10

This piece uses three distinct time series to create a complex orchestrated structure.

Whenever the values lie within a specific range they are assigned to specific instruments. This creates the impression of a musical dialog. The durations of the notes are subjected to a series of transformations, creating a contrast between moments of relative ecstasy and calm.

Track 11

This piece presents the chaotic expressiveness of a Chua attractor, without any arrangement required by the user.

The three time-series of the system play three different musical instruments. The tempo of the notes is organized using a rhythmic sampling process, that is able to create melodic themes and variations. Moreover, the values provided from the three time-series are sampled with different durations in order to make the generated melodic sequences more recognizable.

It is possible to notice some perceptive structures due to the chaotic system. These structures change over time even if the main feeling seems repetitive, since the shape of the attractor remains fixed. The musification process uses a chromatic scale.

Track 12

This fragment shows the expressive capabilities of the circuit without any external arrangement. Sample times are very narrow, causing a perceptive cohesion and the creation of new timbres. The formation of these phenomena are possible since there are many repeated melodic expressions that capture the listener's attention on single frequency or groups of frequencies.

The auditory representation allows the listener to grasp the dynamic nature of Chua's attractor, which we have translated into music. Three different instruments follow the evolution of the note values, sometimes giving us organized melodic patterns, and sometimes chaotic ones.

Track 13

This is a collection of many pieces obtained from different Chua attractors and using different codification processes. In each composition, the three time-series of the system play three different musical instruments.

Some compositions are realized using an instrument to provide melodic sequences, while others use the time-series to produce varying timbres. The tempo of the notes is organized by using a rhythmic sampling process, that is able to create some melodic and non-melodic themes and variations.

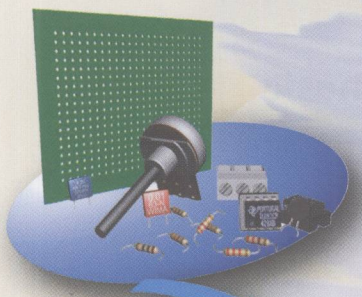
We wish to thank Enrico Cupellini and Costantino Rizzuti who took our original material and created the undoubtedly suggestive pieces we present here. We also wish to thank Richard Walker for his precious help in translating the original Italian text and editing the English edition.

December 2007

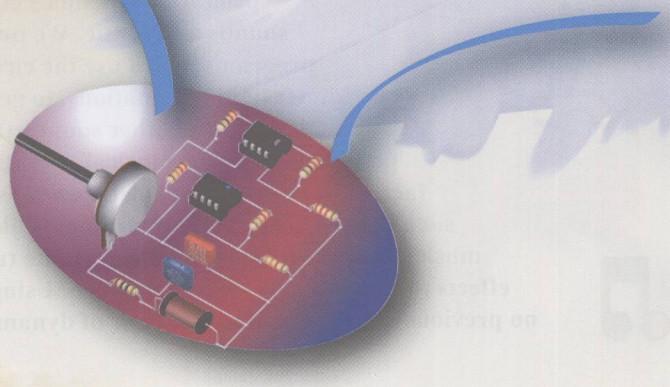
Prologue

Recent years have seen the development of new education and design tools, making it easier for scientists to visualize, modify and personalize their creations. This Prologue provides a guide for readers who wish to build Chua's circuit for themselves and view (or listen to) the enormous variety of patterns the circuit can produce. The prologue is organized in three parts. Each part corresponds to an essential step in understanding the functioning of the Chua circuit. First we show how to build the physical circuit itself. Then we simulate its working, using sounds and music to represent the results of our explorations. Finally we present images of eight navigable environments that the International Journal of Bifurcation and Chaos has used on the covers of some of its issues.

Building the circuit (Building Chua's circuit in twenty two simple steps; coupling the circuit to an oscilloscope)



When trying to understand a physical structure such as Chua's circuit, one of the most effective strategies is "learning by doing". Therefore, in this first part of the prologue, we describe how to build the physical circuit. The "recipe" presented here is based on [Kennedy, 1992] and uses a small number of components, all easily available at low cost. The list includes two operational amplifiers (Op Amps), seven resistors, two capacitors, an inductor and a potentiometer. By connecting the circuit to an oscilloscope users can visualize the values of relevant physical variables. Particularly important is the part of the circuit used to implement Chua's diode, the component that actually produces the chaotic behavior.



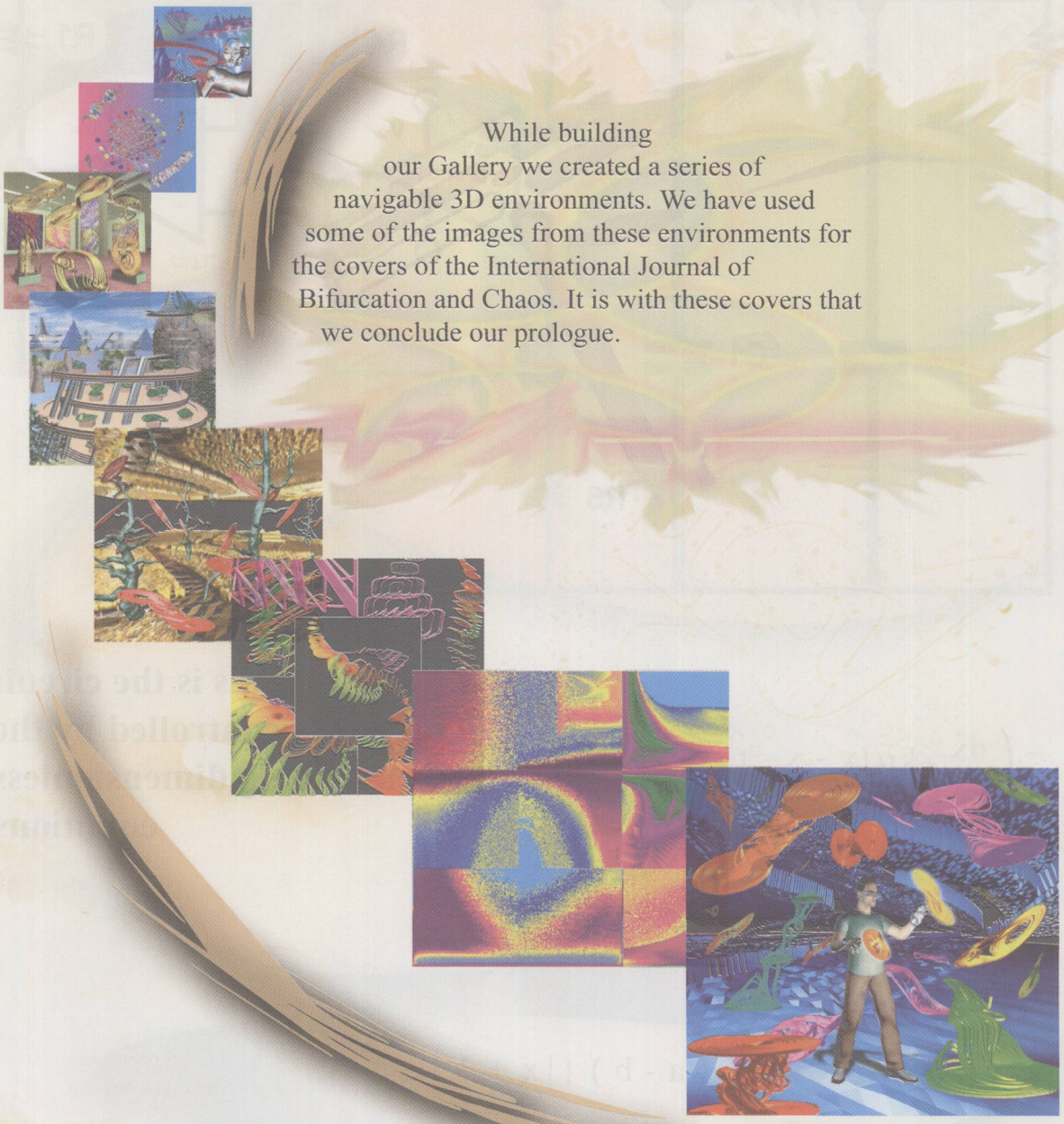
Simulation:
Simulating Chua's circuit with dimensionless equations

In this section we present a computer program implementing Chua's circuit with dimensionless equations. The software reproduces the mechanisms and processes present in Chua's physical circuit. Attractors are represented using 3D images. The Gallery produced in this way contains 150 patterns. The software allows users to manipulate them in various ways. Possible manipulations include changes in the values of the control parameters, changes in coloring and lighting, moving objects, and changing the size of their basic components. Users can explore different routes to chaos in real time. In this way they gain insights into the way changes in parameter values influence the patterns produced by chaotic behavior. Some of the patterns produced in this way are extremely beautiful.

Chua's Circuit:
Exploring chaos through sounds and music

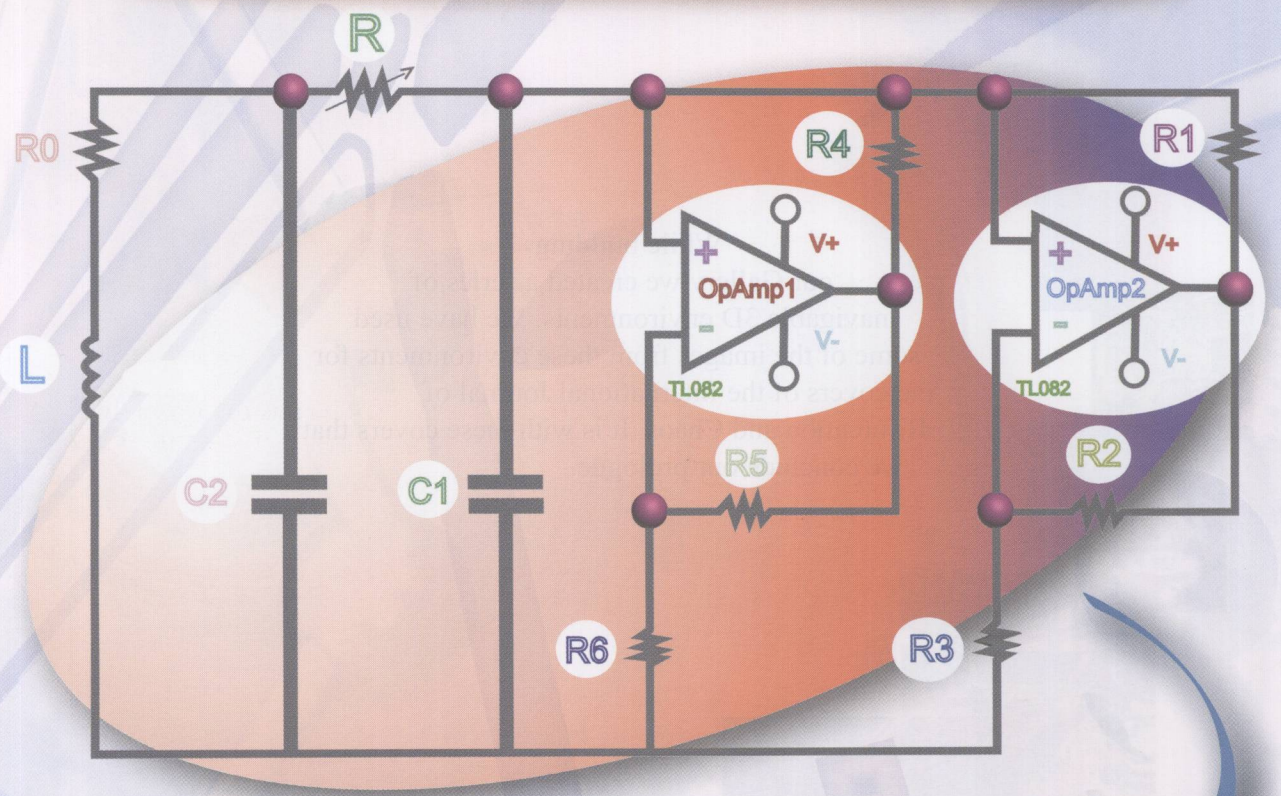
Here we describe a new way of understanding chaos and the forms produced by Chua's circuit - through sounds and music. We present computer programs which use the circuit with dimensionless equations, to generate tonal and atonal music and other sound effects. The programs show the musical potential of chaos. In fact we have even developed new musical instruments, such as the Chua Synth. Instruments like this allow musicians and ordinary people to create tunes and sound effects they could never have imagined. Using them requires no previous knowledge of software or of dynamic systems.

Navigable 3D environments



While building
our Gallery we created a series of
navigable 3D environments. We have used
some of the images from these environments for
the covers of the International Journal of
Bifurcation and Chaos. It is with these covers that
we conclude our prologue.

Building the Circuit

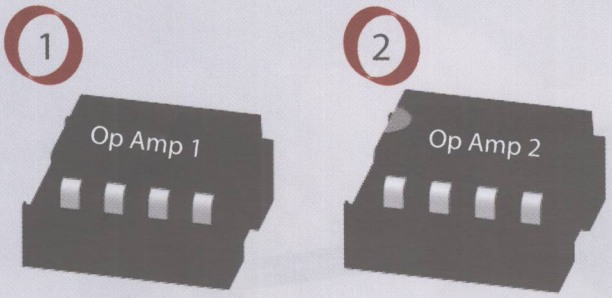


This is the circuit diagram controlled by the following dimensionless equations

$$\begin{cases} \dfrac{dx}{d\tau} = k\alpha(y - x - f(x)) \\ \dfrac{dx}{d\tau} = k(x - y + z) \\ \dfrac{dx}{d\tau} = -k(\beta y + \gamma z) \end{cases}$$

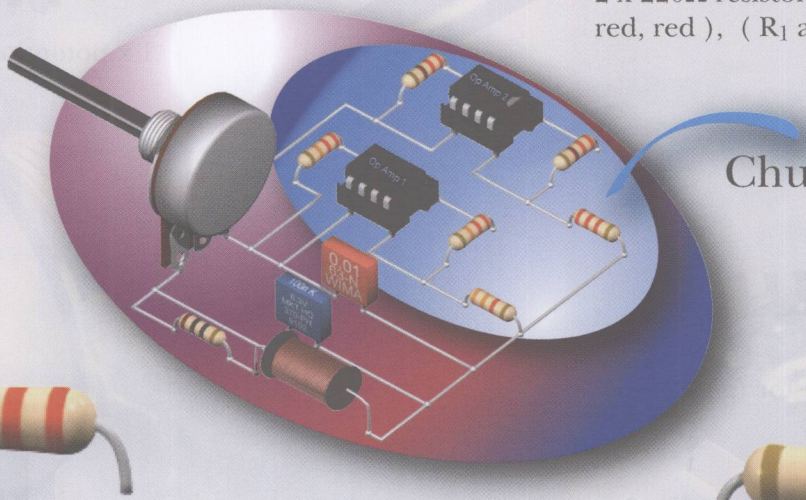
$$f(x) = bx + \frac{1}{2}(a - b)\{|x + 1| - |x - 1|\}$$

Components for Chua's diode



2 x TL082 operational amplifiers (Op Amp 1, Op Amp 2)

2 x 220Ω resistors (gold, brown, red, red), (R_1 and R_2)

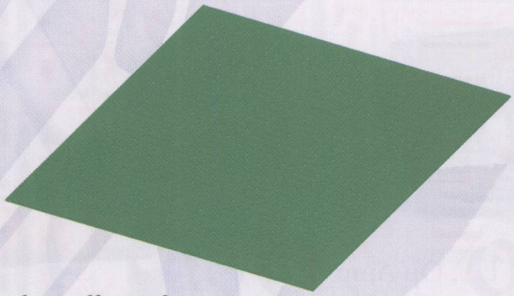


2 x 22kΩ resistors (gold, orange, red, red), (R_4 and R_5)

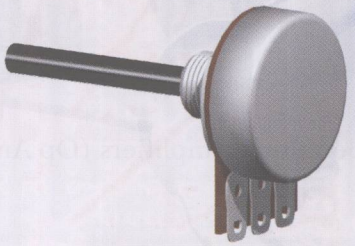
1 x 2.2kΩ resistors (gold, red, red, red), (R_3)

1 x 3.3kΩ resistors (gold, red, orange, orange), (R_6)

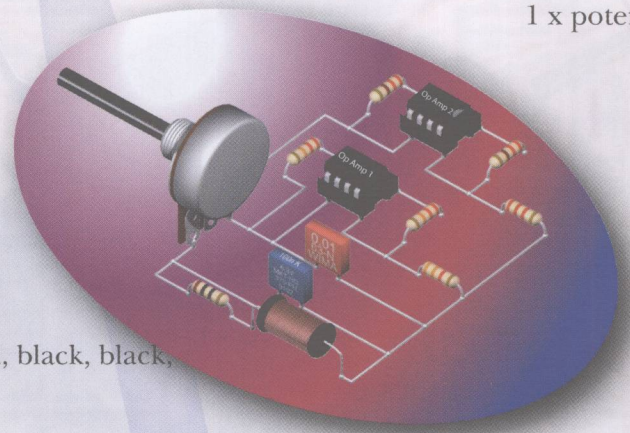
Building Chua's Circuit: Components



1 x breadboard

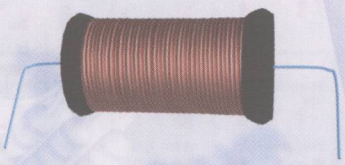


1 x potentiometer (R)



1 x 10Ω resistor (gold, black, black, brown) (R_0)

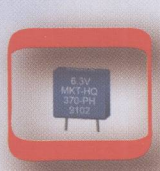
1 x 100 nF capacitor (C_2)



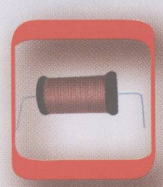
1 x 18 mH inductor (L)

1 x 10 nF capacitor (C_1)

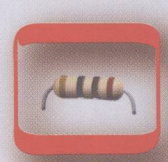
Chua's circuit construction step by step



10 nF Capacitor (C_2)



18 mH inductor (L)

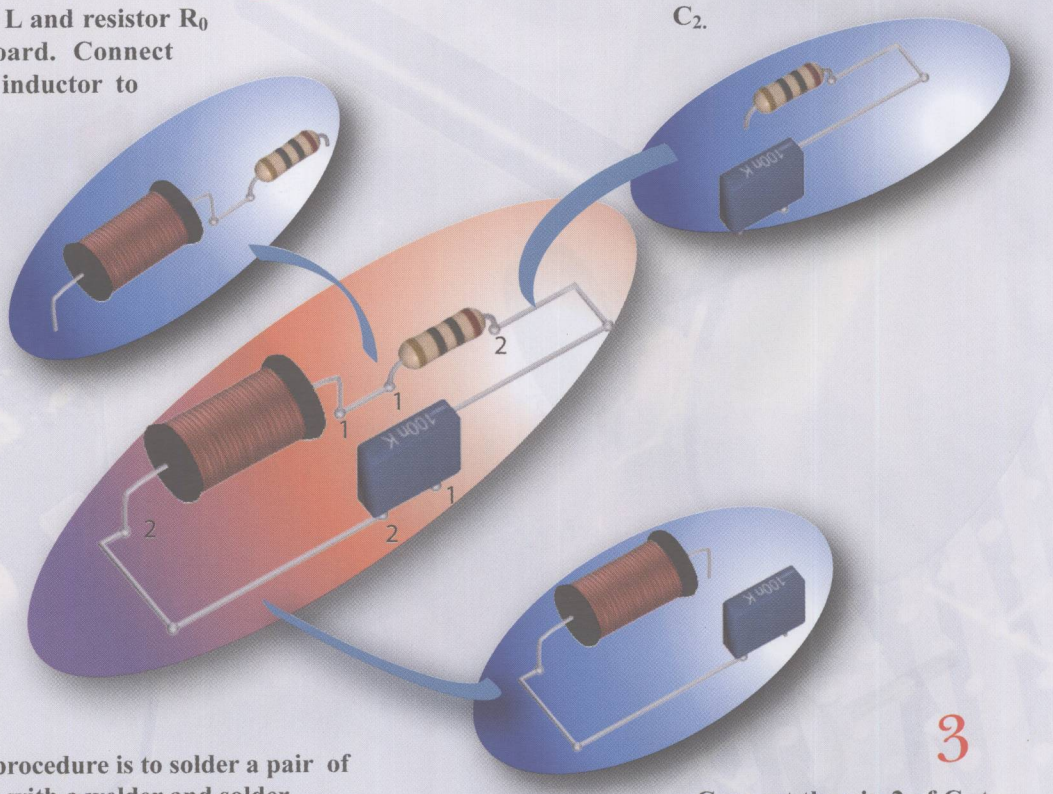


1 x 10Ω Resistor (gold, black, black, brown) R_0

1

Connect inductor L and resistor R_0 into the breadboard. Connect the pin 1 of the inductor to the pin 1 of the resistor. *

2 **Connect capacitor C_2 into the breadboard. Connect the pin 2 of R_0 to the pin 1 of C_2.**



3

Connect the pin 2 of C_2 to the pin 2 of L.

*** The simplest procedure is to solder a pair of terminals (pins) with a welder and solder, It is even possible to make the connection by twisting the wires together. In this case, it is necessary to ensure that the contact works effectively.**