SIMULATION MODELING AND ANALYSIS

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To my wife, Steffi, and children, Heather and Adam, for their encouragement and understanding during the writing of this book.

AVERILL M. LAW

For Christie, who understood and helped more than I could tell her.

W. DAVID KELTON

PREFACE

The goal of *Simulation Modeling and Analysis* is to give an up-to-date treatment of all the important aspects of a simulation study, including modeling, simulation languages, validation, and output data analysis. In addition, we have tried to present the material in a manner understandable to a person having only a basic familiarity with probability, statistics, and computer programming. The book does not sacrifice statistical correctness for expository convenience, but contains virtually no theorems or proofs. Technically difficult topics are placed in starred (*) sections or in an appendix to an appropriate chapter, and left for the advanced reader. (More difficult problems are also starred.) The book strives to motivate intuition about difficult topics and contains a large number of examples, figures, problems, and references for further study. There is also a solutions manual for instructors.

We feel that two of the book's major strengths are its treatment of modeling and of output data analysis. Chapters 1 and 2 show in complete detail how to build simulation models in FORTRAN of a simple queueing system, an inventory system, a time-shared computer model, a multiteller bank with jockeying, and a job-shop model. Chapter 8 contains what we believe is a complete and practical treatment of statistical analysis of simulation output data. Since lack of definitive output data analyses appears to have been a major shortcoming of most simulation studies, we feel that this chapter should enhance the practice of simulation.

We believe that *Simulation Modeling and Analysis* could serve as a textbook for the following types of courses:

1. A beginning course in simulation at the junior, senior, or first-year graduate level for engineering, business, or computer science students (Chaps. 1 through 4 and parts of Chaps. 5 through 8, 10, and 11).
2. A second, advanced course in simulation (most of Chaps. 7 through 12).
3. An introduction to simulation as part of a general course on operations research or management science (Chaps. 1 through 3).

The book should also be of interest to simulation practitioners. As a matter of fact, a large number of such practitioners from industry, government, and the military have used preliminary drafts of the manuscript while attending a seminar on simulation which has been given by the first author for the last four years.

There are a number of people and organizations that have contributed considerably to the writing of this book. Foremost among them are Dr. Thomas Varley and the Office of Naval Research, without whose research support during the past five years this book simply would not have been possible. We would also like to thank the Army Research Office for its research funding to the Mathematics Research Center at the University of Wisconsin. This support in 1980 allowed for the expeditious completion of the book. Most of the development of the simulation language SIMLIB which is discussed in Chap. 2, and almost all of the research of the statistical methods in Chap. 5 was done by Stephen Vincent, a graduate student at Wisconsin. The organization and content of Chap. 7 benefitted greatly from our having in-depth discussions with Professor Bruce Schmeiser of Purdue University. In addition, conversations with the following people positively influenced our thinking on particular chapters of the book: William Biles (Penn State), Edward Dudewicz (Ohio State), James Henriksen (Wolverine Software), Stephen Lavenberg (IBM), Richard Nance (Virginia Tech), Alan Pritsker (Purdue), Edward Russell (CACI), Robert Sargent (Syracuse), Thomas Schriber (Michigan), Edward Silver (Waterloo), and Glenn Thomas (Kent State). Finally, we acknowledge the following graduate students at Wisconsin who read the entire manuscript and made many valuable suggestions: Steven Kimbrough, Lloyd Koenig, Insup Lee, and Muslim Yildiz.

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W. David Kelton

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CHAPTER

ONE

BASIC SIMULATION MODELING

1.1 THE NATURE OF SIMULATION

This is a book about techniques for using computers to imitate, or *simulate,* the operations of various kinds of real-world facilities or processes. The facility or process of interest is usually called a *system,* and in order to study it scientifically we often have to make a set of assumptions about how it works. These assumptions, which usually take the form of mathematical or logical relationships, constitute a *model* which is used to try and gain some understanding of how the corresponding system behaves.

If the relationships which compose the model are simple enough, it may be possible to use mathematical methods (such as algebra, calculus, or probability theory) to obtain *exact* information on questions of interest; this is called an *analytic* solution. However, most real-world systems are too complex to allow realistic models to be evaluated analytically, and these models must be studied by means of simulation. In a *simulation* we use a computer to evaluate a model *numerically* over a time period of interest, and data are gathered to *estimate* the desired true characteristics of the model.

As an example of the use of simulation, consider a manufacturing firm that is contemplating building a large extension onto one of its plants but is not sure whether the potential gain in productivity would justify the construction cost. It certainly would not be cost-effective to build the extension and then remove it later if it does not work out. However, a careful simulation study could shed some light on the question by simulating the operation of the plant as it currently exists and as it *would* be *if* the plant were expanded.

Simulation is one of the most widely used techniques in operations research and management science, and by all indications its popularity is on the increase. There have been several impediments to its even wider acceptance and usefulness, however. First, models used to study large-scale systems tend to be very complex, and writing computer programs to execute them can be an arduous task indeed. This task has been eased in recent years by the development of several special-purpose computer languages that automatically provide many of the features needed to code a simulation model. A second problem with simulation of complex systems is that a large amount of computer time is often required. We anticipate, however, that this difficulty will become less severe as the cost of computing continues to fall. Finally, there appears to be an unfortunate impression that simulation is just an exercise in computer programming, albeit a complicated one. Consequently, many simulation "studies" have been composed of heuristic model building, coding, and a single run of the program to obtain "the answer." We fear that this attitude, which neglects the important issue of how a properly coded model should be used to draw inferences about the system of interest, has led to erroneous conclusions being drawn from many simulation studies. These questions of simulation *methodology*, which are largely independent of the programming language and computer hardware used, form an integral part of the latter chapters of this book.

In the remainder of this chapter (as well as in Chap. 2) we discuss systems and models in considerably more detail and then show how to write computer programs to simulate systems of varying degrees of complexity.

1.2 SYSTEMS, MODELS, AND SIMULATION

A *system* is defined to be a collection of entities, e.g., people or machines, which act and interact together toward the accomplishment of some logical end. (This definition was proposed by Schmidt and Taylor [14].†) In practice, what is meant by the system depends on the objectives of a particular study. The collection of entities which compose a system for one study might only be a subset of the overall system for another. For example, if one wants to study a bank to determine the number of tellers needed to provide adequate service for customers who only want to cash a check or make a savings deposit, the system can be defined to be that portion of the bank consisting of the tellers and the customers waiting in line or being served. If, on the other hand, the loan officer and the safety deposit boxes are to be included, the definition of the system must be expanded in an obvious way. We define the *state* of a system to be that collection of variables necessary to describe a system at a particular time, relative to the objectives of a study. In a study of a bank, examples of possible state variables are the number of busy tellers, the number of customers in the bank, and the time of arrival of each customer in the bank. We categorize systems to be of two types, discrete and continuous. A *discrete system* is one for which the state variables change only at a countable (or finite) number of points in time. A bank is an example of a discrete system since state variables, e.g., the number

†Numbers in brackets correspond to references at the end of the chapter.

of customers in the bank, change only when a customer arrives or when a customer finishes being served and departs. A *continuous system* is one for which the state variables change continuously with respect to time. An airplane moving through the air is an example of a continuous system since such state variables as position or velocity change continuously with respect to time. Few systems in practice are wholly discrete or continuous, but since one type of change predominates for most systems, it will usually be possible to classify a system as being either discrete or continuous.

Sometimes it is desired to study a system to understand the relationships between its various components or to predict its performance under a new operating policy. However, actual experimentation with the system may be infeasible, cost-ineffective, or disruptive of the present system's operation. This is particularly true when, as is often the case in practice, the system of interest does not yet exist. For example, suppose that it is desired to study (as a possible cost-saving measure) the effect of reducing the number of tellers in a bank. If the number of tellers in the bank were actually reduced temporarily, it might cause a significant increase in customers' delays and alienate them from doing future business with the bank. Because of the infeasibility of experimenting with many systems, a systems analyst often uses a model of a system to draw inferences about the operations of the actual system. We define a *model* to be a representation of a system developed for the purpose of studying that system. The model should be sufficiently detailed or "valid" to permit an analyst or decision maker to use it to make the same decisions about the system that would be made if it were feasible to experiment with the system itself.

In this book we restrict our attention to a particular type of *mathematical model* of a system which we call a *simulation model*. (Some models of systems are *physical* rather than mathematical, e.g., a scale model of an airplane tested in a wind tunnel.) Although we shall not explicitly define a simulation model in general, we distinguish between simulation models which are static or dynamic, deterministic or stochastic, and discrete or continuous. A *static* simulation model is a representation of a system at a particular time. Monte Carlo simulation models (Sec. 1.7.3) are typically of this type. A *dynamic* simulation model is a representation of a system as it evolves over time, e.g., a simulation model of a bank's activities over an 8-hour day. A simulation model is said to be *deterministic* if it contains no random variables. For a deterministic model, there is a unique set of model output data for a given set of inputs. On the other hand, a simulation model is *stochastic* if it contains one or more random variables. The output data for a stochastic model are themselves random and thus only estimates of the true characteristics of the model. A simulation model of a bank would normally treat the interarrival times and the service times of customers as random variables, each with their own probability distribution. Loosely speaking, we define *discrete* and *continuous* simulation models analogously to the way discrete and continuous systems were defined above. More precise definitions of discrete (event) simulation and continuous simulation are given in Secs. 1.3 and 1.7, respectively. It should be mentioned that a discrete model is not always used to model a discrete system and vice versa. The decision whether to use a discrete or continuous model for a particular system depends on the specific objectives of the study. For example, a model of traffic flow on a freeway would be discrete if the characteristics and movement of individual cars were important. Alternatively, if the cars can be

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