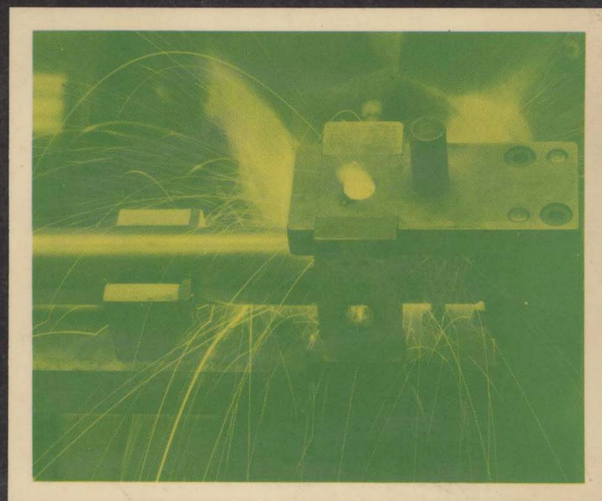
Mechanical Science III



P. R. Lancaster D. Mitchell

Macmillan Technician Series

Mechanical Science III

P. R. Lancaster

D. Mitchell

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Preface

This book follows the Standard TEC Unit *Mechanical Science III* which is a *third-level* Unit in the certificate programmes in mechanical and production engineering.

The *specification* of the course is as follows.

Unit Title	Mechanical Science
Unit Level	III
Unit Value	One
Design Length	60 hours
Prerequisite Units	TEC U75/036 Engineering Science II
Credits for Units	None
Aims of Unit	To develop the student's analytical techniques in the application of scientific principles to mechanical engineering situations
Special Note	The Unit is designed to be studied *concurrently* with or after TEC U75/037 Engineering Science III.

To fulfil the aims of the Unit, the book is written with reference to Engineering Science II and recognises the fact that Engineering Science III may be studied concurrently. However, it is assumed that only the statics and dynamics sections of Engineering Science II are needed as far as Mechanical Science III is concerned, although some small overlap between the two is inevitable and consequently forms some revision of essential information.

The combination of Physical Science I and Engineering Science II collectively covers forces on materials, dynamics, static equilibrium of forces and pressure in fluids in sufficient detail to be able to cover the contents of Mechanical Science III (and the relevant section of mechanics in Engineering Science III if this is studied concurrently).

The contents of Mechanical Science III are

(1) Stress, Strain and Elasticity of Materials
(2) Simple Theory of Bending of Symmetrical Beams
(3) Simple Theory of Torsion of Circular Section Bars
(4) Laws of Angular Motion

(5) Simple Harmonic Motion
(6) Linear and Angular Kinetic Energy
(7) The Application of Bernoulli's Equation to Fluids in Motion.

These topics are covered essentially from first principles using these to show where any formula, which is to be applied to problems, comes from. The range of problems covered goes slightly beyond the bare minimum required, so as to allow the student an opportunity of extending his mechanical engineering knowledge for the purpose of progression in the fields of mechanical and production engineering.

Our thanks go to Mrs P. R. Lancaster for the careful typing of the script and to Dr P. Gallagher of Bradford College for his help in the early stages of preparation.

March 1977

P. R. LANCASTER
D. MITCHELL

Contents

Corresponding Section in TEC Standard Unit U75/058

A. DEFORMATION OF MATERIALS

1. Uses stress, strain, elasticity in problems

Solves from first principles problems involving composite bars under uni-axial loads only, at uniform temperature.

Solves similar problems but including the effect of uniform temperature change.

States that total direct strain is the sum of the strain due to loading and temperature change.

Defines (a) shear stress, (b) shear strain and (c) modulus of rigidity (shear modulus) and solves associated problems.

Defines Poisson's ratio.

Applies the Poisson's ratio effect in stress – strain relationships to solve associated problems in two dimensions (excluding shear stress action).

2. Develops, and uses, the simple theory of bending of symmetrical beams

Identifies, with the aid of a sketch, the position of a neutral plane in a symmetrical beam under the influence of a bending force system.

Defines bending moment.

Shows, by use of the equations of equilibrium, that under equilibrium conditions (a) the neutral plane (axis) passes through the centroid of cross-section, (b) the bending stress is given by $\sigma = My/I$

Solves problems, with section characteristics restricted to rectangular, circular or idealised I-section beams, involving: (a) maximum allowable stresses, (b) bending moments or (c) loading conditions.

Given a specified section modulus (Z) uses standard section handbooks to select appropriate beams.

3. Develops and uses the simple theory of torsion of circular-section bars

Describes the assumptions necessary to develop the simple theory of torsion of circular-section bars.

Derives from first principles the relationship between shear strain and twist per unit length, i.e.

$$\gamma = \frac{r\theta}{l} = \frac{\tau}{G}$$

Uses the equation of equilibrium to derive the further relationship

$$T = \frac{GJ\theta}{l}$$

Solves problems involving torsion in solid and hollow shafts.

B. DYNAMICS

4. States the laws of, and solves problems on, angular motion

States the equations of motion, for angular motion with constant angular acceleration.

Solves problems involving constant angular acceleration.

Derives from first principles the relationship between applied torque, angular acceleration and moment of inertia.

Defines radius of gyration, k, by reference to the expression $I = mk^2$.

Solves problems relating to the angular motion of discs and flywheels.

Derives from first principles the expression $r\omega^2$ for centripetal acceleration of a body moving in a circle with uniform angular velocity.

Solves problems involving motion of bodies in a circle including considerations of banking.

5. Describes, and solves problems involving, simple harmonic motion
   Describes the relationships between (a) restoring force and displacement (b) displacement and time (c) velocity and time (d) acceleration and displacement.
   Defines simple harmonic motion.
   Derives the interrelationships between the quantities specified in (a), (b), (c) and (d) above.
   Relates simple harmonic motion to circular motion of a phasor and circular frequency.
   Solves problems involving simple harmonic motion including the simple pendulum and body supported by a spring.
   Describes resonance as occuring when the applied frequency equals the natural frequency.
   Discuss the problems that can arise when resonance occurs, e.g. in the use of tools.

6. Describes, and solves problems, involving linear and angular kinetic energy
   Derives from first principles the expressions for (a) linear kinetic energy and (b) angular kinetic energy of a body.
   Solves problems involving linear and/or angular kinetic energy, including flywheels and lift systems.

C. FLUIDS IN MOTION

7. States and uses Bernoulli's equation
   States Bernoulli's equation.
   States the equation of continuity for steady flow through tapered pipes.
   Applies Bernoulli's equation to solve problems involving

flow of liquids through pipes, including tapered and inclined pipes and orifices.
Uses the momentum principle to calculate the force produced by the impact of a liquid jet on a normal flat plate.

PART ONE
DEFORMATION OF MATERIALS

1 Stress, Strain and Elasticity

1.1 COMPOUND BARS

An example of the class of problems called 'statically indeterminate' is that of compound bars subjected to axial forces, that is, bars made of two or more different materials with forces applied along the lengths of the bars. In this context, 'statically indeterminate' simply means that the equations of static equilibrium are not sufficient to find the separate forces in each component of the compound bar.

Consider the example of a cylinder of material A with a core of material B subjected to a compressive load W, as shown in figure 1.1. The platform through which the load is applied is assumed to be rigid, that is, it can transmit forces, but is not deformed by them.

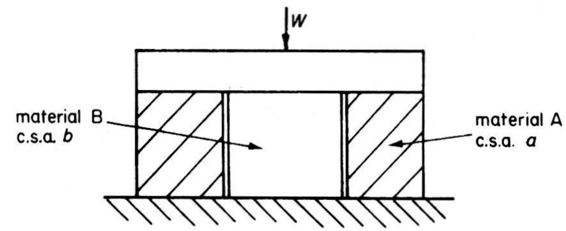


Figure 1.1

Let the force carried by material A be F_A and the force carried by material B be F_B. Then, by consideration of equilibrium of the platform through which the load is applied

$$F_A + F_B + W = 0 \tag{1.1}$$

(In figure 1.2, F_A and F_B are the forces exerted *by* the assembly *on* the platform, that is, it is assumed that the stresses in the materials A and B are *tensile* stresses, until proved otherwise. The reader will appreciate that the opposite is true and expect, therefore, to obtain a negative numerical answer for F_A and F_B.)

Equation 1.1 is the *only* equation that can be derived by considering the equilibrium of the system. The problem is sym-

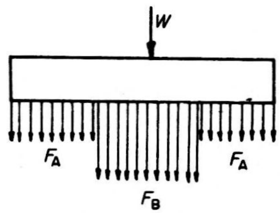


Figure 1.2

metrical about an axis so that the moment equation yields nothing, and there are no horizontal forces. Another equation is required to enable the two unknowns F_A and F_B to be found, and this can only come from consideration of the *deformation* of materials A and B.

In the present example, materials A and B have the same original length (unloaded length) and have the same deformed length at all stages of loading, *because the load-applying platform is rigid*, that is, the *change in length* and the *original length* are the same for materials A and B. Thus

the strain in A = the strain in B

$$e_A = e_B$$

or

$$\frac{\sigma_A}{E_A} = \frac{\sigma_B}{E_B} \tag{1.2}$$

This gives the necessary second equation, because equation 1.1 can be rewritten in terms of stress, as follows

$$\sigma_A a + \sigma_B b + W = 0 \tag{1.3}$$

and equations 1.2 and 1.3 can be solved for σ_A and σ_B.

The essential point in this type of problem is that some information about the deformation of the component materials must be avilable (or be deduced) before the problem can be solved. A numerical example should help to make this clear.

Example 1.1

A cylinder of steel, outside diameter 50 mm, inside diameter 30 mm encloses a concentric core of aluminium of diameter 25 mm. The original length of the steel is 50.01 mm and that of the aluminium 50 mm; the assembly is subjected to a compressive load of 10 kN.

Calculate what proportion of the load is carried by each material. Take E_s (steel) = 205×10^9 N/m²; E_a (aluminium) = 90×10^9 N/m².

Solution First calculate the areas of each component.

$$A_s = \frac{\pi}{4}(50^2 - 30^2) = 400\pi \text{ mm}^2$$

$$A_a = \frac{\pi}{4}(25)^2 = 156.25\pi \text{ mm}^2$$

Then equation 1.3 gives

$$400\pi \times \sigma_s + 156.25\pi \times \sigma_a + 10\,000 = 0 \tag{1.4}$$

In this example, the final length of the components is the same, but the initial length is not. Thus if Δ is the change in length of the aluminium, the change in length of the steel is $(\Delta + 0.01)$ mm, that is

$$e_a = \frac{\Delta}{50}$$

$$e_s = \frac{\Delta + 0.01}{50.01} \approx \frac{\Delta + 0.01}{50}$$

(While 0.01 can be neglected in the denominator, it cannot be neglected in the numerator because Δ is of about the same magnitude.) Therefore

$$e_s = e_a + \frac{0.01}{50}$$

or in terms of stress

$$\frac{\sigma_s}{E_s} = \frac{\sigma_a}{E_a} + 0.0002 \qquad (1.5)$$

Equation 1.5 is the equivalent of equation 1.2. 0.0002 is the extra strain carried by the steel before it is compressed to the same length as the aluminium. Thereafter the strain in both components is the same. Equation 1.4 becomes

$$\sigma_s + 0.391\sigma_a + 7.958 = 0$$

and equation 1.5 becomes

$$\sigma_s = 2.278\sigma_a + 41$$

hence

$$2.278\sigma_a + 0.391\sigma_a + 41 + 7.958 = 0$$

$$\sigma_a = -\frac{48.958}{2.669}$$

$$= -18.343\ \text{N/mm}^2$$

and

$$\sigma_s = -7.957\ \text{N/mm}^2$$

and

$$\Delta = 50e_A = -\frac{50 \times 18.343}{90 \times 10^3} = -0.010\ \text{mm}$$

Example 1.2

A light rigid bar is suspended horizontally from two wires 1 m apart. Wire A is of steel, 6 mm diameter, $E = 205\ \text{GN/m}^2$ and wire B is of duralumin, 15 mm diameter, $E = 70\ \text{GN/m}^2$. Where must the load be applied if the bar remains parallel to its original position?

Solution For equilibrium of the bar (see figure 1.3) resolving vertically

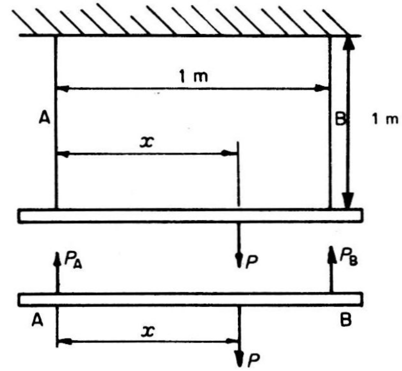


Figure 1.3

$$P_A + P_B = P \qquad \text{(a)}$$

and

$$P \times x = P_B \times 1 \qquad \text{(b)}$$

where P_A and P_B are the tensile loads in the steel and duralumin wires respectively.

A further equation is found from the fact that the *extension* of each wire must be the same if the bar remains horizontal after P has been applied.

$$\text{Stress in steel wire} = \frac{4P_A}{\pi \times 6^2 \times 10^{-6}}$$

$$= 3.537 \times 10^4 P_A\ \text{N/m}^2$$

$$\text{Strain in steel wire} = \frac{\sigma_A}{E} = \frac{3.537 \times 10^4 P_A}{205 \times 10^9}$$

$$= 1.725 \times 10^{-7} P_A$$

$$\text{Extension of steel wire} = \text{strain} \times \text{original length}$$
$$= 1.725 \times 10^{-7} P_A \text{ m}$$

Similarly

$$\text{Extension of duralumin wire} = \frac{4P_B}{\pi \times 15^2 \times 10^{-6} \times 70 \times 10^9}$$
$$= 8.084 \times 10^{-8} P_B \text{ m}$$

Thus

$$8.084 P_B = 17.25 P_A$$

and

$$P_B = 2.134 P_A \quad \text{(c)}$$

Using this in equation (a) gives

$$P_A + 2.134 P_A = P$$

therefore

$$P_A = 0.319P \quad \text{(d)}$$

and

$$P_B = 0.681P \quad \text{(e)}$$

and from equation (b)

$$x = \frac{P_B}{P} = 0.681 \text{ m}$$

If the stress in the steel is 80 MN/m^2, calculate the value of P, the stress in the duralumin, and the extension of each bar.

$$P_A = \text{stress in steel} \times \text{area of steel}$$
$$= 80 \times 10^6 \times \frac{\pi}{4} \times (6 \times 10^{-3})^2$$
$$= 2262 \text{ N}$$

from equation (d)

$$P = 7091 \text{ N}$$

and from equation (e)

$$P_B = 4829 \text{ N}$$

hence

$$\text{stress in duralumin} = \frac{P_B}{\text{area of duralumin}}$$
$$= \frac{4829 \times 4}{\pi(15 \times 10^{-3})^2}$$
$$= 27\,330\,000 \text{ N/m}^2$$

that is

$$\sigma_B = 27.33 \text{ MN/m}^2$$

$$\text{Extension of each bar} = 1.725 \times 10^{-7} P_A$$
$$= 8.084 \times 10^{-8} P_B$$
$$= 3.902 \times 10^{-4} \text{ m}$$
$$= 0.39 \text{ mm}$$

1.2 THE EFFECT OF TEMPERATURE CHANGE

Owing to temperature rise, the linear dimensions of an engineering component will change. If l_0 is the original length of a bar and l is the length after a change in temperature, then

$$l = l_0(1 + \alpha T) \quad (1.6)$$

where α is the coefficient of linear expansion of the material, and T is the temperature change (which may be positive or negative).

Transposing equation 1.6 gives

$$\frac{l-l_0}{l_0} = \alpha T \tag{1.7}$$

or

$$\frac{\text{change in length}}{\text{original length}} = \alpha T$$

The quantity αT has the units and form of 'strain' and is sometimes called the 'temperature strain'.

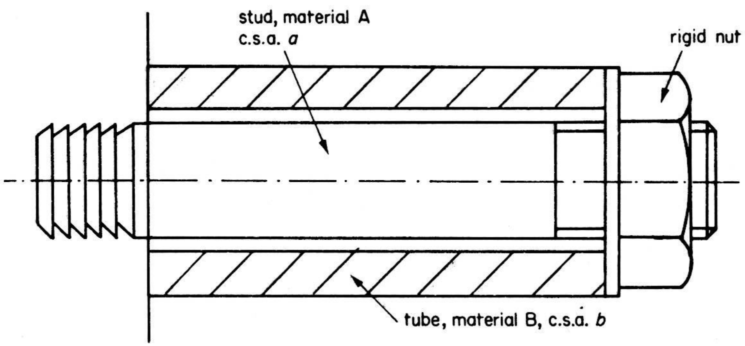


Figure 1.4

Consider the example shown in figure 1.4. The nut at the end of a stud of material A is screwed down finger tight on a tube of material B. The nut is assumed to be rigid and the whole assembly has its temperature uniformly raised by an amount T. What stresses are induced in the stud and the tube?

Assume that the coefficient of linear expansion of the tube is greater than that for the stud. *If the components were allowed to expand freely*, the result would be that the increase in length of the tube would be greater than that for the stud by an amount

$$l_0(\alpha_B - \alpha_A)T$$

see figure 1.5.

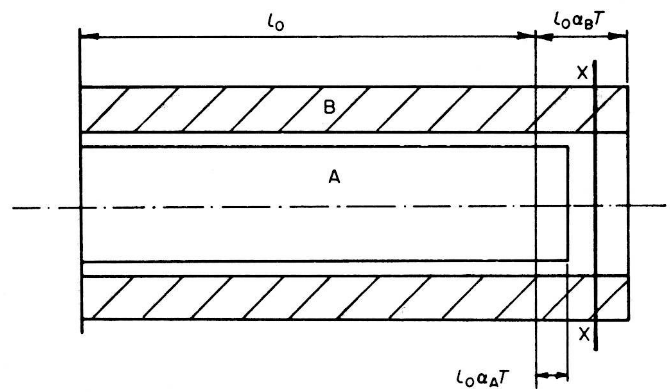


Figure 1.5

However, the effect of the rigid nut is to ensure that the lengths of tube and stud remain the same. This means that the nut pushes back the tube to some level XX, say, and also pulls out the stud to the same level. Thus the effect of the temperature rise is to induce a tensile stress in the stud and a compressive stress in the tube.

Consideration of figure 1.5 reveals that

$$\begin{matrix}\textit{increase} \text{ in length} \\ \text{of stud}\end{matrix} + \begin{matrix}\textit{decrease} \text{ in length} \\ \text{of tube}\end{matrix} = l_0(\alpha_B - \alpha_A)T$$

Thus if σ_A is the stress induced in the stud, the increase in length must be

$$l_0 e_A = l_0 \frac{\sigma_A}{E_A}$$

and the *decrease* in length of the tube must be

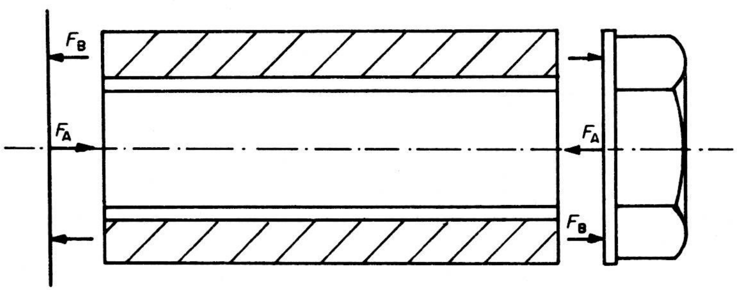
$$-l_0\frac{\sigma_B}{E_B}$$

The minus sign is included because all stresses are initially assumed tensile, that is, positive. Therefore

$$l_0\frac{\sigma_A}{E_A}-l_0\frac{\sigma_B}{E_B}=l_0(\alpha_B-\alpha_A)T$$

or, since the original lengths were equal

$$\frac{\sigma_A}{E_A}-\frac{\sigma_B}{E_B}=(\alpha_B-\alpha_A)T \tag{1.8}$$



F_A is force exerted *by* the stud *on* the nut (or wall)
F_B is force exerted *by* the tube *on* the nut (or wall)

Figure 1.6

Considerations of the static equilibrium of the assembly (figure 1.6) reveal that

$$F_A+F_B=0$$

or

$$\sigma_A a+\sigma_B b=0 \tag{1.9}$$

Compare this with equation 1.1 noting that in this example the externally applied load is zero. Equations 1.8 and 1.9 can now be solved for σ_A and σ_B. It should be noted that σ_A and σ_B found from equations 1.8 and 1.9 are stresses due to the rise in temperature only. In this example the nut was initially only finger tight so that *initial stresses* were zero. If the nut had been screwed down to give an initial compressive stress in the tube and a tensile stress in the stud, these stresses would simply have been added algebraically to those due to temperature rise.

1.3 ENGINEERING AND TEMPERATURE STRAIN

Equation 1.8 of the previous section may be transposed to read

$$\frac{\sigma_A}{E_A}+\alpha_A T=\frac{\sigma_B}{E_B}+\alpha_B T \tag{1.10}$$

Each component of this equation has the units of strain, and indeed reexamination of figure 1.5 will show that

$$l_0\left(\frac{\sigma_A}{E_A}+\alpha_A T\right)$$

is the *total change in length* of component A; similarly for component B. Thus

$$\frac{\sigma_A}{E_A}+\alpha_A T$$

is called the *total strain* of component A and is made up of two parts

the engineering strain $\dfrac{\sigma_A}{E_A}$

and