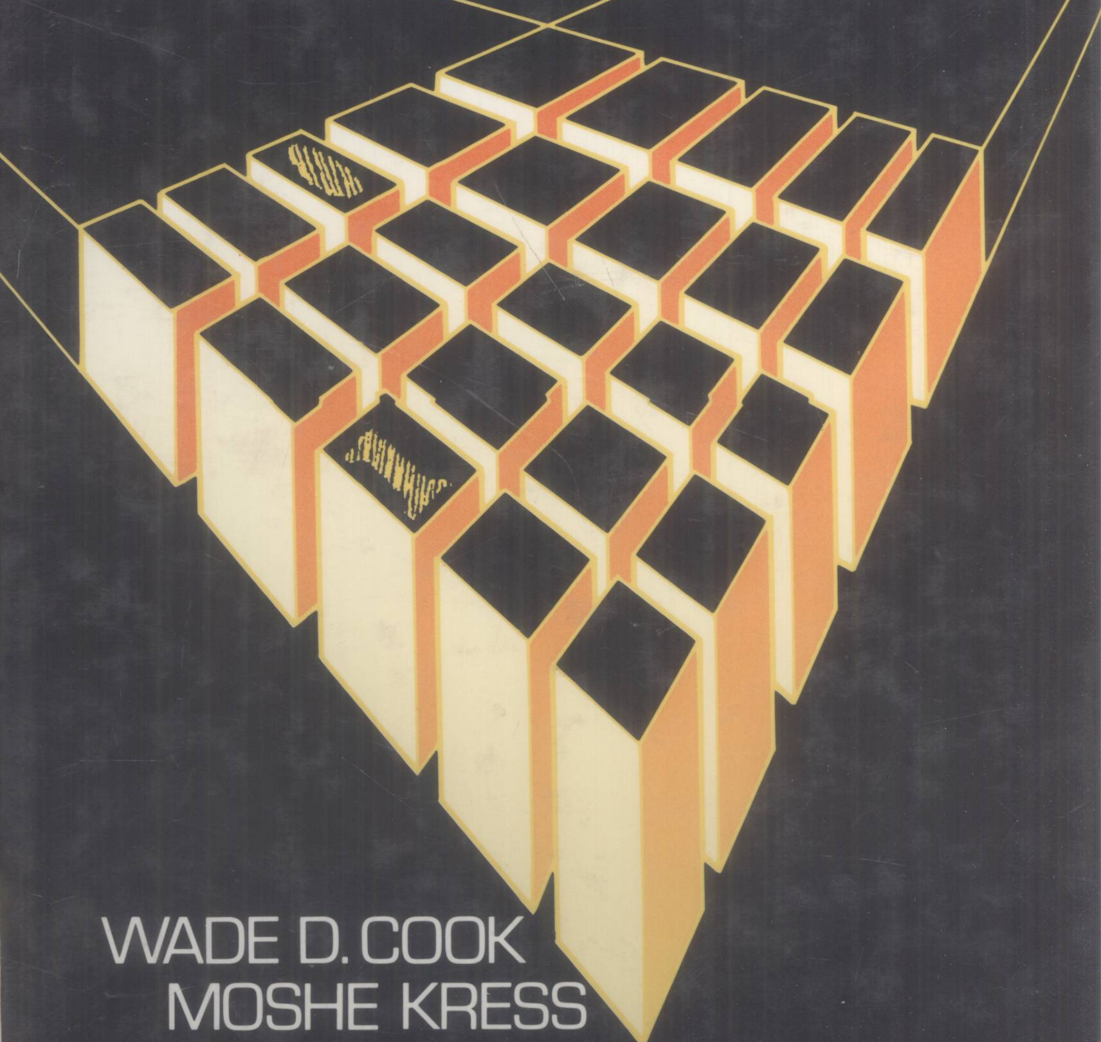
# ORDINAL INFORMATION & PREFERENCE STRUCTURES

## DECISION MODELS AND APPLICATIONS



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# Ordinal Information and Preference Structures
## Decision Models and Applications

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To our preferences, both ordinal and cardinal:
Jeanne, Marsha and Lisa
Nurit, Hadas, Naama, and Noa

# *Preface*

Information or data in the form of *ordinal preferences* are common in a wide variety of real-world comparisons and choice situations. When a consumer is asked, for example, to rate or compare several flavors of pudding, it is natural that an ordinal response be given: "I prefer the flavor of chocolate to that of vanilla, the flavor of strawberry to that of chocolate." It can be unreasonable in such situations to expect an individual to be able to quantify his or her responses to these stimuli (flavors) on an absolute cardinal scale (chocolate rates at 6 while vanilla rates at 5). Another example is the fitting of lenses by an optometrist. At each trial, the optometrist presents to the patient two lenses from which the patient chooses the one through which he or she sees the clearest. In this case, the patient is not even required to provide an ordinal ranking of the lenses, but simply to express ordinal preferences between two samples at a time. The result of this set of pairwise comparisons is the identification of the most suitable lens. The analysis of stimuli impacts, therefore, constitutes an important area where the data are inherently ordinal.

Another typical example of a situation where ordinal data are to be considered is tournaments. In a chess tournament, for example, each match ends either in a win by one of the players or in a draw. The problem is how to rank order the players at the end of the tournament such that the ranking reflects their performances in the matches played.

Group decision methods constitute another major area where ordinal models are utilized. Each member in a committee that is formed to select a candidate for a certain position may rank order the potential candidates according to his or her preferences. To obtain a decision, these rankings are aggregated into a consensus or group ranking. The result of this aggregation is an overall ranking of the set of candidates and, in particular, the selection of an ultimate winner. Preferential voting and election models are also examples where ordinal group decision situations arise.

In decision analysis, and in particular in multicriteria decision problems, the data may often be ordinal. If a criterion is qualitative, it may not be possible or meaningful to *measure* or to assign a *numerical value* to an alternative when it is evaluated with respect to that criterion. In this case, we can only supply qualitative judgments that can usually be expressed in terms of ordinal prioritizations among the various alternatives that are considered.

Consider, for example, the problem of selecting energy research and development projects. The R&D projects may be classified according to possible energy sources, such as (1) oil and gas, (2) coal, (3) nuclear, (4) solar, and so on. Possible criteria according to which these R&D options are evaluated are (a) existing scientific activity, (b) energy efficiency of the resource, (c) availability of that resource, and (d) social acceptability. The first stage of analysis would normally involve a rank ordering of the criteria according to their importance. Suppose availability ranks first, scientific activity second, efficiency third, and social acceptability last. That is, $c > a > b > d$. The R&D options are next evaluated with respect to these criteria, and a rank ordering may be obtained. For example, with the scientific activity criterion, nuclear research may be ranked first, coal second, solar third, and oil and gas last. After obtaining the rank ordering of the alternatives with respect to each criterion and taking into account the criteria importance, as well as other factors such as the fuzziness of criteria, the problem is to agglomerate these ordinal evaluations into an overall rank ordering of the R&D options.

Although it is evident that such ordinal preference information is prevalent in many environments and decision situations, models that require the more specific *cardinal* rating scale data are far more common than are ordinal models. Typically, market research models fall into this category, often requiring that the consumer provide such cardinal values, even when ordinal preferences are more sensible responses. It must be emphasized that, while the models discussed in this book are designed to handle ordinal data, many of the techniques deal with the data in a cardinal fashion. This is the case, for example, in the rating methods for ranking ordinal preference structures, where the number of preferences is given a cardinal meaning.

While a significant amount of research effort has been directed toward ordinal problems, no attempt has, as yet, been made to present a unified approach to decision making in this environment. It is the purpose of this book to fill the gap in this important area. The various components of ordinal decisions are examined, specifically (1) appropropriate formats for collecting and presenting ordinal information, (2) obtaining a ranking from possibly inconsistent ordinal preference structures, (3) deriving a consensus of (ordinal) options among respondents, and (4) aggregating ordinal data in the context of multicriteria decision problems. Solution methods are presented and illustrated with numerical examples, and properties of ordinal models are discussed. Finally, the techniques and models are demonstrated using a variety of actual case applications.

# Acknowledgments

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Finally, we are indebted to our families for their patience and understanding during the writing of this book.

# Contents

# 1

# *Overview*

## 1.1 INTRODUCTION

In assessing the worth of each member of a collection of alternatives (products, projects, candidates in an election), it is often the case that the decision maker's set of subjective *ordinal preferences* or priorities is the only information available. Specifically, the most that can be achieved by the decision maker is to *rank order* the alternatives: $a > b > c$ (that is, $a$ is preferred to $b$ which is preferred to $c$).

Ordinal data appear in many situations in our everyday world. The choice among brands of supermarket products is commonly based on subjective ordinal preferences. The selection of a candidate or subset of candidates from a ballot in an election involves ordinal ranking of the candidates.

When faced with the choice among a set of alternative plans for influencing the public's demand for electric power, a hydroelectric company must take into account various attributes that these alternative plans possess. It is necessary to account for social impacts, damage to the environment, economic implications, and the like. Very often the various plans can only be ranked in an ordinal sense, and subjective preferences or beliefs of the decision maker must be considered. The final selection will depend on the way the various attributes are weighted and on the preferences specified by the decision maker.

Ordinal data are, therefore, a natural phenomenon in many real-world environments. Clearly, however, such data restrict the decision maker's capability to perform certain types of analyses. Consider the situation in which consumer preferences are being elicited regarding the *flavor* rating of different formulations of a pudding. If it were possible to obtain reliable cardinal data as to the actual perceived flavor *level* of each formulation, a marginal analysis could be performed. Specifically, by altering the flavor slightly, the improvement in overall attractiveness of the product could be ascertained, and management would then possess an important monitoring device to aid in developing product specifications. With ordinal data, such an analysis is not possible, hence restricting their usefulness.

To deal with those problems, however, where the only reliable information available is of the preference order variety (which arguably is the case in a consumer survey setting such as the above), there is a very real need for appropriate tools. It is the purpose of this book to present a comprehensive examination of the various components of ordinal data problems, thereby providing a structured framework within which to approach such problems in a step by step manner. Again, it is emphasized that some of the tools used to handle ordinal data tend to attribute cardinal properties to that data. The book also discusses several application areas where ordinal data models apply.

The following section describes briefly a number of problem settings that will serve as a useful backdrop for the topics of the ensuing chapters.

## 1.2 PROBLEMS INVOLVING ORDINAL DATA: SOME REAL-WORLD EXAMPLES

### Using Consumer Preferences to Evaluate Products

Opinions of consumers play a dominant role in the development and marketing of new products. Private enterprise relies on such opinions to aid in targeting their products toward particular segments of society according to age, sex, economic status, and so on. Because of the enormous financial implications associated with new product development and changes to product formulations, the obtaining and processing of consumer perceptions are crucial elements in the product's success or failure.

To have a particular problem setting as a backdrop, consider the situation faced by a researcher who is collecting consumer responses pertaining to preferences among five formulations of a pudding mix, which a company is considering for production. Denote the alternative formulations as $a, b, c, d,$ and $e$. The formulations vary in texture, flavor, smoothness, and the like. The company is attempting to determine which formulation would be most favorable with the public and which segments of the public should be targeted as the primary market for the product in question. In simple terms, therefore, the ultimate purpose of the survey is to arrive at a *preferred* product from among the five options, or more generally to obtain a ranking of the products.

This problem has been the focus of extensive research in the field of marketing for decades. Hundreds of papers have been written on the subject, and scores of models for characterizing and evaluating consumer preferences have been advanced. To gain a better understanding of the components of this *product selection* problem, let us examine four major issues that the researcher must deal with. These issues arise in many other problems of a similar structure (for example, project ranking) and provide a focal point for the chapters to follow.

1. *Data format:* Most existing consumer preference models require that cardinal responses be supplied regarding the choices (products) at hand. Specifically, the respondent must *rate* each product on say a 9-point scale. Such a scale is desirable in that it lends itself to easy analysis in additive models (for example, conjoint analysis models). These models generally rely on the data being cardinal in nature.

   From the point of view of the respondent, such a cardinal scale is often undesirable. It compels one to provide a specific evaluation of worth or importance in terms of attributes such as flavor, which cannot easily be quantified. Having to indicate that the flavor rates a 5 or 6 on a 9-point scale forces the consumer to give answers that may carry little conviction.

   A particularly convenient and arguably more reliable format for eliciting preferences in such a case is that of *paired compar-*

*isons*. Simply, the respondent is asked, "Given a choice between products $i$ and $j$, which is preferred?" With reference to an attribute such as flavor, the respondent can easily supply such preference data, in that *no* specification of *degree* or intensity of preferences is needed.

The example referred to in the preface pertaining to the evaluation of lenses necessitates such a pairwise format. For situations involving a high degree of subjectivity or where small differences exist from one option to another, this structure is particularly appropriate. Such pairwise responses can be summarized in the form of a binary matrix $A = (a_{ij})$, where $a_{ij} = 1$ if $i$ is preferred to $j$. Otherwise, $a_{ij} = 0$. A typical response matrix might be

$A =$

| | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | | 1 | 1 | 1 | 1 |
| $b$ | | | 1 | | 1 |
| $c$ | | | | 1 | 1 |
| $d$ | | 1 | | | |
| $e$ | | | | 1 | |

In some situations it is possible for a respondent to provide directly a rank ordering of the products, for example

$$\begin{bmatrix} a \\ b \\ d \\ c \\ e \end{bmatrix}$$

In this case, product $a$ is ranked in first place, $b$ in second, and so on. When a small number of choices (3,4, or 5) is under consideration, a ranking of this type can frequently be supplied. When larger numbers are involved, however, and fine distinctions must be made, pairwise responses are often the only practical framework within which to operate.

**2.** *Response inconsistencies:* In the event that pairwise responses are supplied by the consumer, inconsistencies or *intransitivities* can and very often do arise. Consider, for example, the consumer response matrix $A$ given above. Product $a > b$, $b > c$, $c > d$, yet $d > b$. As a result of this *cycle*, it is not obvious how the products should be ranked in the best way, that is, to be in closest agree-