Introductory Statistics

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Introductory Statistics

Foreword

Introductory Statistics, by James Twaite and Jane Monroe, is an ideal introduction to the subject, particularly for those students with little or no background in mathematics. Twaite and Monroe both have extensive experience teaching the introductory course. They have carefully designed their text to promote a thorough understanding of both basic statistical concepts and the statistical procedures through which these concepts are applied in specific research situations. The result is a book that combines clarity of presentation, skillful organization, and technical accuracy, with a wide coverage of topics.

A number of special features contribute to the exceptional effectiveness of this book in teaching statistical concepts and procedures:

1. A "Classification Matrix," appearing at the end of each chapter, summarizes the statistical problems covered in the chapter and classifies each problem according to the specific conditions and techniques involved. Particular emphasis is given in each matrix to the relationship between the type of measurement scale represented in the data and the statistical analysis to be carried out.
2. A Student Guide section, also following each chapter, provides students with both a review of important concepts and an opportunity to test their comprehension. In addition to chapter Summaries, Key Terms, and Review Questions, the Student Guide includes a variety of Problem Sets organized according to a systematic breakdown of experimental designs. Students will find this particularly useful in learning the application of statistics to real-life situations.
3. Finally, a Review of Basic Math, Symbols, and Formulas as well as an unusually comprehensive Glossary of Terms are included at the back of the book to facilitate students' computation and overall review.

In short, this is a book designed not merely to describe statistics but to teach it. Twaite and Monroe have made an important contribution to the instruction of statistics: their book successfully encourages enjoyment—through understanding—of this most stimulating field of study.

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1 Introduction

1

Introduction to Statistical Techniques in Empirical Research

A research project begins when an individual asks a question that cannot be answered on the basis of information immediately at hand. The type of question asked will determine where the researcher must seek the necessary information. For subjective questions like "What was the principal cause of the War of 1812?" or "What is the meaning of the swan image in the poetry of W.B. Yeats?," researchers would probably go to a library to find out what other scholars have said on the question. They would combine the subjective impressions of previous workers with their own impressions to reach an answer that they consider satisfactory. This is research, but it is not empirical research. For more objective questions like "Which of these two routes from New York to Boston is the quickest?" or "What is the average reading achievement level in the Newark, N.J. public high schools?," researchers may attempt to find an answer by observing actual events. For example, in determining the quickest route to Boston they might establish two groups of drivers and assign one route to each group. They would then record the time required to make the trip by each driver in each group. Whenever researchers directly or indirectly observe actual events in this manner, they are engaging in empirical research.

The focus of this book is on the elementary statistical techniques that are researchers' basic tools for organizing, presenting, and analyzing empirical data. In this chapter, we have two goals: (1) to introduce you to some of the essential principles and terminology of empirical research; and (2) to provide you with a logical framework that you may use to organize the statistical techniques that will be presented in the chapters that follow.

THE PRINCIPLES AND THE TERMINOLOGY OF EMPIRICAL RESEARCH

The Scientific Method

In their observations, researchers should be guided by the principles of the *scientific method.* These principles may be broken down into two groups. One group of principles aims at ensuring that researchers obtain a correct answer to their questions. The other group aims at making this answer available to other interested parties. The first set of principles may be summarized in the following way. All observations must be made in a careful and systematic manner and with an effort to control for the effect of irrelevant factors. This set of principles applies to any empirical research question. Consider the researchers attempting to determine which of two routes is the quickest route from New York to Boston. Of course, they would want to make sure that their observations were accurate. Rather than ask drivers how long their trips took, they would actually want to time them. They would want to be certain that their timing device was accurate. They would want to make sure that the drivers using the two routes had comparable weather conditions. They would want to know if there had been any unusual delays on one route or the other. These kinds of considerations are obvious. They apply to any empirical research question.

The other set of principles is particularly relevant to empirical research in psychology, education, or any of the natural or social sciences. If the researchers are scholars, they will be interested in sharing the results of their work with other members of their profession. This requires that all the procedures employed in their research be specified exactly. This is essential for two reasons. First, other members of the scientific community must know exactly what was done if they are to be able to evaluate the results critically and relate the results to their own work and the work of others in the field. Second, other researchers should be able to verify the results of a research project by reproducing, or *replicating,* the original research. In order to replicate a study, they must know exactly what was done. Thus, the exact specification of research procedures provides the basis for the growth of scientific knowledge.

In brief, the specification of research procedures may be broken down into three critical steps: (1) Researchers must clearly specify what it is that they are observing; (2) they must indicate precisely how they go about observing it; and (3) they must describe the group on which these observations are made. In the jargon of empirical research these three steps have special names. When researchers specify what it is that they are observing, they are defining the *variable* (or variables) *of interest.* When they indicate exactly how they will be observing this variable, they are defining the *measurement process.* And when they describe the group on which their observations are made, they are defining the *population of interest.* We now proceed to a more complete explanation of these terms.

Variables

In empirical research, we constantly refer to variables. It is essential that you have a clear understanding of this term. A variable is a characteristic that may have different values from individual to individual or from observation to observation. A variable has a name and associated with that name is a *set of values.* It may be the name of a physical characteristic, a psychological trait, an ability, a type of achievement, or any number of other attributes. For example, the name "eye color" is the name of a variable that is a physical characteristic. In a particular study we might establish three values for the variable "eye color": (A) brown, (B) blue, and (C) other. Notice that these values are not numbers. There is no requirement that the values of a variable be numerical scores. In order for a variable to be employed in empirical research, the only requirement is that we be able to *differentiate* between the various values of the variable. Thus, if we were using the variable "eye color" in a study of human subjects, we would need to be able to assign each subject in the study to *one and only one* of the three values of that variable. The value "other" enables us to assign individuals with unusual or hard-to-describe eye colors to one specific value.

There are several different types of variables that may be employed in empirical research. It is important that we be able to distinguish among the different types of variables, for different statistical techniques are appropriate for use with different types of variables. There are four broad classes of variables. These are the categorical variable, the rank-order variable, the interval scale variable, and the ratio scale variable. Moreover, one of these four broad classes has an important subdivision. Categorical variables may be subdivided into one of two types, the pure nominal scale variable and the variable with ordered categories. In the paragraphs that follow, we will describe each of these different types of variables.

Categorical Variables. A variable is categorical when the values of the variable are not numbers but simply categories into which observations may be classified. As already noted, there are two different types of categorical variables, the *nominal scale variable* and the *ordered categorical variable* with ordered categories. The variable "eye color" just mentioned is an example of a nominal scale variable. The variable is categorical because the values of the variable are the categories (A) brown, (B) blue, and (C) other. The variable is a nominal scale variable because these three categories are not related to each other in any quantitative or qualitative sense. That is, brown is not bigger than blue, and blue is not better than other. The three categories are simply different from each other.

Let us consider a set of nominal scale data. If we observe the 25 children in Mr. White's first-grade class and record their values on the variable "eye color," our results might appear as in Table 1.1. We see from Table 1.1 that John and Helen have the same eye color, brown. They have a different eye color from Bill and Jane, who both have blue eyes. They also have a different eye color

TABLE 1.1
Eye Color of 25 Children in Mr. White's First-Grade Class

Subject	Name	Eye color	Subject	Name	Eye color
1	John	brown	14	Tom	blue
2	Helen	brown	15	Barry	brown
3	Bill	blue	16	Joan	brown
4	Fred	other	17	Shawn	brown
5	Jane	blue	18	Toby	other
6	Willy	brown	19	Larry	brown
7	Dave	blue	20	Dick	blue
8	Joe	other	21	Lisa	brown
9	Nancy	brown	22	Mark	blue
10	Peggy	brown	23	Cathy	brown
11	Janet	blue	24	Jody	brown
12	Chris	blue	25	Brian	brown
13	Owen	other			

from Fred, whose eyes were neither brown nor blue but some other color. Because the values of the variable "eye color" are not related to each other in any quantitative sense, we cannot say that John's eye color is bigger than Bill's eye color or that John's eye color is better than Bill's eye color. Certainly, we cannot say that John's eye color is one unit greater than Bill's eye color. We have no units to work with. We can only say that they have different eye colors. We use the word nominal to refer to this type of variable because nominal is derived from the Latin word for name. The values of such a variable are simply the names of the different categories. Other examples of nominal scale variables include sex, with the values (A) male and (B) female; and marital status, with the values (A) single, (B) married, (C) divorced or separated, or (D) widowed.

The other type of categorical variable is that in which the categories are related to each other in some quantitative sense. For example, we often find items in questionnaires that ask the respondents to indicate the extent to which they agree with a particular statement by circling or checking one of several response categories like the following: strongly agree, agree, neutral, disagree, or strongly disagree. An example of this type of questionnaire follows.

Please indicate your attitude toward each of the following statements by checking the response option that is closest to your own feeling.

1. A strategic arms limitation agreement would increase the prestige of the United States overseas.

   _____ strongly agree
   _____ agree
   _____ neutral
   _____ disagree
   _____ strongly disagree

2. A strategic arms limitation agreement would jeopardize the military readiness of the United States.

_____ strongly agree
_____ agree
_____ neutral
_____ disagree
_____ strongly disagree

Alternatively, a questionnaire might require respondents to indicate the frequency with which they perform certain behaviors by checking one of these response options: always, usually, sometimes, rarely, never. In each of these cases, the variables are categorical. Each subject will have a score that is simply one of the response option categories. However, these categorical variables are not nominal scale variables because the categories provided are related to each other. The categories are arranged in a clear order. The order may be from strongly agree to strongly disagree, it may be from always to never, or it may be from the smallest category to the largest. When the categories of a categorical variable are ordered in this manner, we refer to the variable as an ordered categorical variable.

Observations on an ordered categorical variable provide us with more information than observations on a nominal scale variable. For example, let us assume that a hypothetical subject 1 responded to question 1 on strategic arms limitation with the response option "agree," and the hypothetical subject 2 responded to the same item with the response option "strongly disagree." Given this information, we know that the two subjects have *different* values on the variable, just as we would know if the variable were a nominal scale variable. However, we know more. We also know that subject 1 indicated a *greater* degree of agreement with the statement than subject 2. It is this additional element of comparison along a dimension that distinguishes an ordered categorical variable from a nominal scale variable. Note that the ordered categorical variable does not enable us to say *how much more* subject 1 agrees with the statement than does subject 2. We only know that subject 1 does agree more than subject 2.

Ordered categorical variables are frequently employed in survey questionnaires. However, they often appear in other research situations as well. For example, suppose a winemaster were interested in determining which of four types of wine was most popular. He might conduct an experiment in which subjects tasted each wine and indicated their preference. If the four wines could be placed in a clear order along some dimension, such as from driest to sweetest, then the categorical variable "wine preferred" would be an ordered categorical variable.

In order to illustrate the properties of ordered categorical variables, let us suppose that our winemaster ascertained the preference of 25 wine drinkers for four wines labeled "very dry," "dry," "sweet," and "very sweet." Let us further suppose that, of the 25 subjects, 14 preferred the very dry wine; 9 preferred the dry wine; 2 preferred the sweet wine, and none preferred the very sweet wine. Because the categories are clearly ordered along a dimension from very dry to very sweet, we know that all 14 of the subjects in the very dry category prefer a wine that is drier than the wine preferred by the 9 subjects in the dry category or the 2 subjects in the sweet category. However, we cannot differentiate between the subjects falling into any one category. There may very well be differences between the 14 subjects choosing the very dry wine in terms of the precise amount of dryness they consider ideal. However, our method of measuring the dryness they prefer is rather crude in the sense that it cannot uncover these differences.

We can think of the data from this study as it is represented in Figure 1.1. There we depict our 25 subjects as they might be located along a dimension or continuum of preference ranging from most dry to most sweet. Each subject's actual preference is represented by an X on this continuum. As we have represented these 25 subjects, no two are exactly the same in terms of their actual preferred level of dryness. Above this continuum of actual preferred level of dryness we have shown the four wines used in the experiment to measure preferred dryness. Each of these four wines has an amount of dryness corresponding to a point on the continuum. This is indicated by the dotted line connecting each glass to the underlying continuum. We do not know whether the four wines are equally distant from each other on the continuum. They may well not be. We only know that they are definitely in order along the dryness dimension.

Given this situation, we assume that each subject will select the wine that is closest to his or her actual preferred level of dryness. We have indicated this by the arrows directed from each X to one of the four possible choices. When a subject makes a selection, the winemaster has measured that subject on preferred dryness, using an ordered categorical scale with four values. In practice, the winemaster would never see the underlying continuum of preferred dryness. Only the ordered categorical measurement of the variable would be seen.

However, it is important that we keep in mind the existence of this underlying dimension. It is important when we consider the statistical techniques appropriate for use with ordered categorical data. In general, the statistical techniques that are appropriate to nominal scale variables are also appropriate for use with ordered categorical variables. However, because of the additional character of order associated with the ordered categorical variable, there are certain techniques that are not applicable to nominal scale data but that are applicable to ordered categorical data. These techniques will be pointed out at the appropriate points in this text.

Rank-Order Variables. A second general class of variables is the *rank-order* or *ordinal scale variable*. With rank-order variables, there are no fixed categories into which observations may fall. Rather, observations are compared to each other and put in order, perhaps from best to worst or from biggest to smallest. An example of a rank-order variable is rank in class. If we were considering the rank in class of the 15 students in Ms. Jones' tenth-grade English class, our data might be presented as in Table 1.2. Note that each student, that is, each observa-