Neil R. Ullman

Elementary Statistics

An Applied Approach

ELEMENTARY STATISTICS

AN APPLIED APPROACH

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ELEMENTARY STATISTICS: AN APPLIED APPROACH

PREFACE

I have pursued this book with one major goal in mind: to present and clarify the purposes and concepts of statistics to people who have little mathematical background.

In my teaching and dealings with students and workers in diverse fields I have observed a widespread awe and fear of statistics. The first encounter with statistical reasoning is often traumatic, even for those with fairly good mathematical preparation. However, a statistical awareness and understanding is becoming more and more necessary to a well-rounded education.

This textbook should be able to be used with a minimum of mathematics. The statistical work will mainly involve arithmetic. Some algebra is used to simplify the presentations. A separate chapter on probability is also provided, but the main thrust of that chapter is the relationship of probability to statistical reasoning.

I have observed that a major problem expressed by students in a first statistics course is their realization that it is not an exact study. They anticipate a mathematics course that is precise; they end up with a course in "uncertainty," with an emphasis on reasoning and decision making. I have presented these ideas of statistics in a way that the average nonmathematical student can understand. I have found that this manuscript and the textbook from which it was largely derived, *Statistics: An Applied Approach*, have been easily understood and used by a wide range of people, including students in areas from mathematics to law enforcement and humanities, faculty members studying for graduate degrees, and secretaries.

The organization of the textbook is designed to be flexible. Although the first three chapters should be covered in that order, most of the remaining chapters can be studied without other previous reading. Thus, Chapters Four, Five, Six, Eight, Eleven, Thirteen, and parts of Chapters Seven and Twelve can be read in any sequence. Although I have placed the discussion of the normal distribution (Chapter Five) before the binomial distribution (Chapter Six), there

is no need to follow this order (except to defer the normal approximation until completing the study of the normal distribution or to rely on the empirical rule). The only other major sequential chapters are that Chapter Ten (tests involving the binomial distribution) assumes completion of Chapter Six, and that Chapters Five, Seven, and Eight, should be completed before Chapter Nine (on tests involving the normal distribution) is attempted.

Most of the problems require calculations, and I have provided a sufficient number of problems in all chapters that can be performed without calculators. Additional problems are also provided with more realistic numbers for which a calculator becomes important. Several special topics and some more difficult problems are also presented and noted with an asterisk (*).

I thank the many students, faculty members, and reviewers who have contributed numerous corrections, suggestions, and ideas for the organization and details of this textbook.

I am grateful to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S. and to Longman Group Ltd., London, for permission to reprint Tables III and VII from their book *Statistical Tables for Biological, Agricultural and Medical Research* (sixth edition, 1974).

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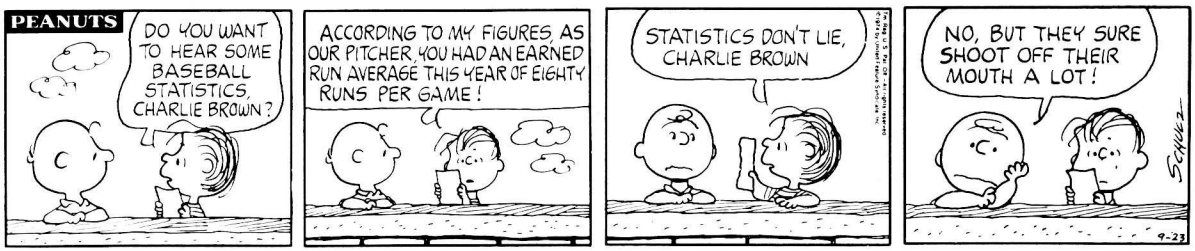
CONTENTS

INTRODUCTION



The use of statistics has become so common today that even poor Charlie Brown cannot escape from them. Look how just one single sentence can summarize so much so simply. A whole summer of baseball is lumped into one number, and yet all of us can feel the terrible frustrations that number represents.

And so it often is that a single "statistic" can tell us so much.

But Linus still has more disturbing information to report.

Statisticians like to think that they can assist others in finding solutions to questions and problems. Apparently Lucy isn't interested. How easy it is to find excuses or to read the results of a study the way you want it in the first place. Scientists, researchers, all inquiring people, must keep their minds open.

INTRODUCTION

You must have a willingness and a desire to understand and to use the tools available to you. Statistical principles and methods will be one such set of valuable and useful tools. Use them wisely and they will serve you. Ignore them and you will miss opportunities and waste time and effort. Unfortunately, you must also be conscious of how not to use them poorly—because then you may come to wrong decisions.

Obviously, we can devise situations for all types of questions. But we must be careful that our information is accurate and complete. If Linus had overlooked the bubble gum he might have missed the most important item that Snoopy has been consuming.

I already said that the ultimate goal of all the work of our statistician is usually to help us in arriving at decisions. Charlie Brown is attempting to understand his problem. Unfortunately, he is pretty much alone.

I sometimes wish it were so easy. Just remember, though, that changing statisticians doesn't eliminate the problem of the team.

Most people are engaged in serious work. One part of many of their jobs is the gathering of data, or at least the studying of some data. But we do not gather data for fun. We hope to gain as much meaningful information as possible from our work.

As I have already said, the use of statistical methods is one important way that we can hopefully gain such meaningful information. I hope that you will embark on this subject with enthusiasm and interest.

CHAPTER ONE
AN OVERVIEW

Pick up a copy of a large daily newspaper and carefully look through it. You will usually find a wide array of articles that present the results of different studies. A typical assortment might include:

A survey that reports 60% of the population of the United States is dissatisfied with the way the government is handling the economy.

An advertiser claiming miles per gallon increased because of a new additive in the gasoline.

One article presenting arguments that "increases in a liver disease may be related to marijuana smoking," while another report describes the results of birth control efforts in a developing country.

We also find a continual deluge of statistics about unemployment, consumer prices, athletics, and so on. Very little of our everyday lives is not touched on, at least partly, or examined somehow with statistics.

Even more important is the fact that a large part of somebody's job is to gather information or data, evaluate or interpret it, and then present it for others to read and use. More and more people in virtually every field are involved in these activities. Even if you will never prepare such material, you will inevitably need to read, understand, and evaluate some amount of statistical information. Therefore, you should try to gain a clear understanding of the *concepts* of statistics.

1.1 NON-MATHEMATICAL DESCRIPTIONS

All too often the first question that comes to mind as you open this textbook and begin to study statistics is, "why bother?" This chapter provides some initial understanding of when we need statistical methods.

Every one of us has a need to provide descriptions. For example, I ask you to describe an egg. Let us try by using words, comparisons, and pictures.

Words

You might start by saying that an egg is a smooth, oval, or spheroidal-shaped object. Here, we are describing the object with a collection of words that are merely symbolic representations or condensations of the actual object. *Provided you and I both understand the words in the same way*, we can convey to each other an accurate image of the real thing. The set of

words thus serves as a *substitute* for the object. The substitute never fully replaces the object, but often it is the only adequate way of transmitting information regarding the object.

You may claim that the above description is not complete, and you would be correct. Words alone can never fully *replace* the object. We can expand the definition from four words to 40 or even 400 words by enlarging on details and by including descriptions of additional aspects of an egg. Perhaps we would introduce the fact that it consists of an outer layer or shell and that inside this shell is contained a fluid of varying composition. Of course, this description could still be elaborated on and it would still be inadequate as a *complete* description. We will, however, have a very useful substitute, one that can be carried around completely independent of the real object, one that will not break and make a mess, one that can fit in my head as well as in a book.

Although words are useful tools, the immediate problem with them is in the area of interpretation of the words. Different people derive different meaning from the same words. We must be aware not only of the incompleteness of the description, but also of the *different ways in which different people interpret individual words.* The complete set of words can combine to give quite a diverse set of images different for each person. Remember, you just have the description "a smooth, oval, or spheroidal-shaped object, having an outer layer or shell and containing a fluid of varying composition." From this description you have been asked to construct an image of an object. Will it be an egg? Try to improve the description.

Comparisons

From the preceding we see that the direct use of words to replace an object is limited. Try to keep these limitations in mind as you read. They will recur later in a different setting—with numbers.

We can add another dimension to our description if we establish some standards, some item that we will mutually agree on as a *basis for comparison.* We can then compare our object to the chosen standard. For instance, we may select as standards two objects with which both of us are equally familiar. The two objects selected may be a marble and a 100-watt light bulb. We could further state that we will select a marble of a particular size and a light bulb of a particular brand name and catalog number, thereby establishing more exact standards.

In our future work we will find that we will need standards for comparison and that they will serve as important aids in describing many problems. We will also encounter the notion of exactness in setting these standards. We will state at times that the standard is a theoretical value.

We may say, for example, that the marble is much smaller than the egg. The light bulb is much larger. We observe that a marble is a perfect sphere. What about the egg? It is generally longer in one direction than in another, so it is not nearly as spherical as the marble. The light bulb is even more oddly shaped, with a concave section.

Let us stop and look at what I have attempted to do. I have attempted to make some direct comparison of the shape of several objects by using words only. It has been a rather poor attempt. I ask you to improve on it. Remember that the person you are talking to does not know what an egg looks like. I think you will experience the same kind of difficulty that I have encountered. Once again, we have found words limited in their ability to perform certain operations. We have observed that word descriptions may not be sufficient for complete comparisons.

Pictures There is an old adage that you have undoubtedly heard that goes, "A picture is worth a thousand words." So now let us resort to a picture to gain some further insight or description of our egg.

Figure 1.1*a* is a drawing of an egg; with no additional information the general shape of the egg is fairly obvious. The shape is more easily conveyed with a picture than with the use of the words. The picture, however, is pretty much limited to showing us the general shape or outline and outer surface characteristics. It does not tell us much about the structure or the variation of composition that we know exists.

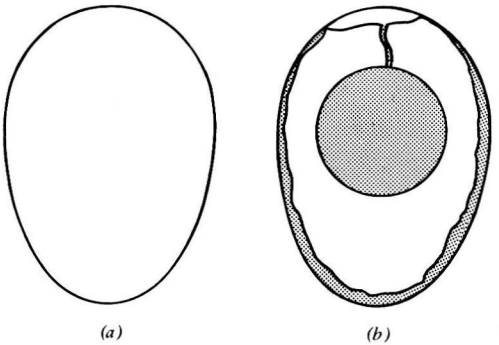


FIGURE 1.1 (*a*) A picture of an egg. (*b*) A cross section of an egg showing the shell and some internal structure.

If we refine our drawing or change the type of drawing, we may find that we can describe even more than we did before. If we make a cross-sectional drawing, we can illustrate the shell thickness and some of the internal structure. In Figure 1.1*b*, we have a better image of how the various compositions of liquid are distributed inside the egg, but we have lost some of the information about surface.

So we now see that *no picture is perfect* and *different pictures can be made for the same object*. The different pictures may give different kinds of insight, answer different questions, or provide different descriptions for the object. Thus, we might want a whole collection of pictures, each of which may serve a different purpose or elaborate on a different facet of the object.

What about comparisons? Can pictures be used to make comparisons? Of course. In fact some of the difficulty we ran into before with an egg and a light bulb is almost nonexistent. In Figure 1.2 we have a picture of a marble, an egg, and a light bulb. With this picture the general size and shape relations are easily communicated. You can quickly see certain similarities and differences among these three objects. But again, the picture is limited both in the total scope of information it can possibly convey and in the nature of the questions it can answer.

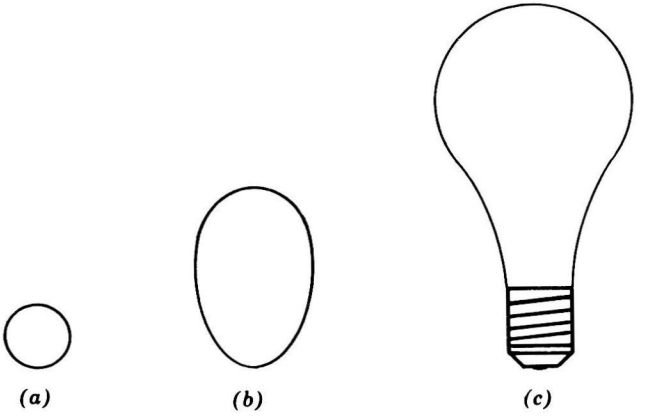


FIGURE 1.2 Pictorial comparison of (*a*) a marble, (*b*) an egg, and (*c*) a light bulb (100 watt).

AN OBSERVATION

At this point we may be able to stop. We have been asked to provide a description of an object. An attempt was made to explain the characteristics of that object by providing first a verbal or word description and, second, a pictorial description. It *is* possible that these methods of communication are sufficient. If so, then the subject of statistics is not applicable to the problem and we need not go any further. The key word is *possible*. A certain amount of description, comparison, and even inference can be made through the use of the technique we have been discussing. However, our capacity for comprehensive examination of even just one egg is extremely limited. The more detailed and more exact descriptions involved in most of today's world generally requires measurements of some type. We use numerical and mathematical quantities as better ways of characterizing or describing an object.

1.2 DATA

What if I wanted further information and more detailed descriptions about some particular characteristics of the egg. A chemist could provide the chemical composition or the amount of cholesterol in it. A dietician can tell me its protein value. An economist will tell me what it costs. A packaging engineer could tell me its breaking strength. And a market