Dynamical Systems II

edited by

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Proceedings of a University of Florida
International Symposium

Edited by

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Preface

This volume contains the lectures and contributed papers presented at the Second International Symposium on Dynamical Systems held at the University of Florida in Gainesville, Florida, February 25–27, 1981. The first symposium in this series was held at the University of Florida, March 24–26, 1976.

The central theme of the symposium was the relation of dynamical systems to current research on ordinary and functional differential equations, partial differential equations, stability theory, and optimal control. In addition, two lectures in memory of Professor Karl Pohlhausen (17 May 1892–18 November 1980) were delivered by R. A. Dandl and Michael Longuet-Higgins at a special session highlighting the engineering applications of the dynamics of wave theory and plasma theory. These lectures are published elsewhere.

As in 1976 this international symposium was sponsored by the Center for Applied Mathematics of the University of Florida and was made possible by funding under the State of Florida's Quality Improvement Program.

The program committee for the symposium was composed of A. R. Bednarek and Knox Millsaps, Codirectors of the Center for Applied Mathematics at the University of Florida, L. Cesari of the University of Michigan, Jack Hale of Brown University, and T. T. Bowman and Z. R. Pop-Stojanovic of the University of Florida.

The editors wish to thank the members of the program committee, the local arrangements committee and others who assisted in the organization of this symposium. We are particularly grateful to Paula Barrett for her intelligent and energetic handling of the myriad details of the symposium and her careful editorial assistance in the preparation of these proceedings.

We also thank Sharon Bullivant for her patience and skillful typing of this volume.

Lectures

Contents

Lectures

Contributed Papers

VOLTERRA SERIES AND THE ALTERNATIVE METHOD OF CESARI

Thomas T. Bowman

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The recent book [7], Nonlinear System Theory: The Volterra Wiener Approach by Wilson J. Rugh, indicates the increasing popularity of Volterra's methods among systems and control engineers. The author was introduced to this area in 1976 by R. W. Brockett who gave a series of seminars at the University of Florida related to his papers [2] and [3]. The question was raised during the seminar whether the method could be applied to nonlinear boundary value problems. The answer at that time was negative, but the question was raised in the author's mind about the connection of the Volterra series and the alternative method of Cesari. The author had attended a series of lectures on the alternative method by Professor Lamberto Cesari at the University of Florida in 1975. Since the mid-sixties Professor Cesari and his school [4] had successfully applied the alternative method to nonlinear boundary value problems in ordinary and partial differential equations.

It is the author's intent in this paper to point out that the Volterra series is a special case of the alternative method where the kernel of a certain

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operator is zero; it is precisely this kernel being nonzero that makes solving boundary value problems difficult. The second goal is to give a more concrete description of the operators involved in the alternative method than those found in the literature, which should be of interest to those employing the Volterra series.

1. THE ALTERNATIVE PROBLEM

We will consider boundary value problems for the system of ordinary differential equations

$$(1)\qquad \begin{aligned} X' &= A(t)X + f(t,X,X',\beta,\varepsilon) \quad t \in (a,b) \\ B_1X(a) &+ B_2X(b) = 0 \end{aligned}$$

where $X(t) = \text{col }(x_1,\ldots,x_n)$ is composed of n variables, $A(t) = (a_{ij}(t))$ is an $n \times n$ matrix with real essentially bounded measurable entries, β a parameter on a compact region of E^v, ε a "small" real parameter, and B_1, B_2 are constant $m \times n$ matrices with the $m \times 2n$ matrix (B_1B_2) of rank m. In (1), $f(t,X,X',\beta,\varepsilon) = \text{col }(f_1,\ldots,f_n)$ is an $n \times 1$ vector valued function defined on $[a,b] \times E^{2n+v+2}$ which is Borel measurable in t for every (X,X',β,ε) and continuous in (X,X',β,ε) for every t. Moreover, we assume that for any given pair of real nonnegative constants R_1, R_2 and compact region $B \subset E^v$ there are real continuous functions $L(\varepsilon)$, $M(\varepsilon)$ such that $M(0) = 0$, $L(0) = 0$

$$|f(t,X,X',\beta,\varepsilon)| \le M(\varepsilon)$$

and f satisfies the Lipschitz property

$$|f(t,X,X',\beta,\varepsilon) - f(t,Y,Y',\beta,\varepsilon)| \le L(\varepsilon)\{|X-Y| + |X'-Y'|\}$$