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TIME-DELAY SYSTEMS

ANALYSIS, OPTIMIZATION AND APPLICATIONS

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NORTH-HOLLAND

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Analysis, Optimization and Applications

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To my Parents (MM-Z)

In Memory of my Parents and
Brother M. Hassan Jamshidi (MJ)

PREFACE

This book is intended to familiarize the reader with an important class of systems: time-delay systems. Such systems arise either as a result of inherent delays in components of the system, or because of deliberate introduction of delay into the system for control purposes.

Delays often occur in the transmission of information or material between different parts of a system. Transportation systems, communications systems, chemical processing systems, metallurgical processing systems, environmental systems and power systems are examples of time-delay systems.

Considerable research has been done on various aspects of time-delay systems in the last 25 years. However, unlike linear nondelay systems on which a host of books have been published, very few books exist on time-delay systems. The existing books are either limited in scope or are too theoretical, often to the point of obscurity. The present book is intended to fill this void. It covers the techniques of analysis and design of time-delay systems without dwelling deeply on mathematical rigor; but this, of course, is not at the expense of sacrificing accuracy. The book is concerned mainly with linear time-delay systems. However, nonlinear time-delay systems are also included whenever applicable. Large-scale time-delay systems are also presented. The book is useful for graduate students in engineering, science and mathematics. It may be used as a textbook for a graduate course on time-delay systems. It would also be useful for researchers in this field since up-to-date surveys on most topics are provided.

The authors have been doing research on time-delay systems since 1969 at various locations including University of Illinois (Urbana, IL), Shiraz University (formerly Pahlavi University, Shiraz, Iran), IBM Thomas J. Watson Research Center (Yorktown Heights, NY), Hughes Aircraft Co. (Canoga Park, CA), the University of New Mexico (Albuquerque, NM) and AT&T Bell Laboratories (Holmdel, NJ). The book includes the authors' course notes as well as the results of their research and other researchers in this field. It consists of four parts and nine chapters as follows:

Part I - Modeling (Chapter 2)

Part II - Analysis (Chapters 3-5)

Part III - Optimization (Chapters 6-8)

Part IV - Applications (Chapter 9)

Chapter 1 serves as an introduction to the book. Time-delay systems are introduced here and examples of their applications are presented. Part I consisting of Chapter 2 presents mathematical description of time-delay systems. System modeling, utilizing both transfer functions (frequency domain) and state-space representation (time-domain), as well as methods of linearization of nonlinear time-delay systems are presented here. Also, modeling of large-scale systems with time delays is considered in this chapter.

Part II consists of Chapters 3, 4 and 5. Chapter 3 is concerned with the analysis of linear time-delay systems. The solution of the first-order time-delay state equation with single delay or multiple delays in the unforced and forced cases is provided here. The concept of adjoint state equation is also presented in Chapter 3. The results of this chapter are used in subsequent chapters dealing with stability, controllability, observability and optimal control. Chapter 4 deals with the stability of time-delay systems. Methods in time domain and frequency domain are presented here. Controllability and observability of time-delay systems are discussed in Chapter 5 where criteria for controllability and observability of stationary and nonstationary linear time-delay systems are developed. The concept of duality is invoked to relate controllability and observability.

Part III of the book consists of Chapters 6, 7 and 8. Optimization of time-delay systems is the topic of Chapter 6. The maximum principle for time-delay systems is stated here and its proof is outlined. The computational difficulties in the application of the maximum principle to the optimal control of time-delay systems is discussed in this chapter and the dynamic programming method of optimal control of time-delay systems is presented. Methods of suboptimal control of time-delay systems are discussed in Chapter 7. Nonlinear and multiple delay systems are treated here as well. Chapter 8 deals with suboptimal control of large-scale time-delay systems. The optimal hierarchical control methods for such systems are considered here.

Part IV consisting of Chapter 9 is concerned with the applications of time-delay systems in different disciplines. Illustrative examples for different applications such as cold rolling mills, traffic control and water resources control systems are presented in this chapter.

To make the book self-sufficient, the required mathematical background is provided in Appendices A and B. These appendices present a review of linear algebra and the transform (Laplace, z and modified z) theories.

The authors are indebted to many people for their various contributions. M. Malek-Zavarei especially thanks Jack Appel, Mo Iwama, John O'Rourke and John Williams for their encouragement and support. M. Jamshidi would like to thank Dean Jerry May of the University of New Mexico's College of Engineering and Russ Seacat, Chairperson of the Electrical and Computer Engineering Department for their leadership and continuous support. The authors would like to thank Professor Madan Singh of the University of Manchester Institute of Science and Technology for reviewing the manuscript and offering many helpful suggestions. They would also like to thank Professor Singh as the editor of North-Holland Systems and Control Series for his suggestion to write the book. We are indebted to many of our colleagues and former students who have contributed to the book in various forms. Special thanks are due Mr. Soo Ryong Lee for a thorough proofreading of the final manuscript.

Finally, the manuscript was prepared using the text processing tools of the UNIX™ operating system. Without these tools, the typing and proofreading of this 196,225-word manuscript would have been indeed an enormous task. We greatly appreciate the efforts of the staff of AT&T Bell Laboratories Text Processing Center and Art Center in preparing the manuscript. Last but not least, we are grateful to our families for their patience and support during the preparation of the manuscript.

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CHAPTER 1

PRELIMINARIES

1.1 INTRODUCTION

The purpose of this chapter is to serve as a general introduction to the book. It is intended to provide motivation for the study of time-delay (TD) systems as well as to build a framework within which the properties of such systems can be explained. As such, it may refer to some concepts which are not defined yet but will be defined in later chapters.

Time-delay systems* are those systems in which time delays exist between the application of input or control to the system and their resulting effect on it. They arise either as a result of inherent delays in the components of the system or as a deliberate introduction of time delay into the system for control purposes. Time delays occur often in electronic, mechanical, biological, metallurgical, and chemical systems. They correspond to transport time (as in shock waves in the earth, hormones in the blood stream, fluids in a chemical process, or electromagnetic radiation in space) or to computation times (as in cortical processing of a visual image, analyzing a TV picture by a robot, evaluating the output of a digital control algorithm, or performing a chemical composition analysis) [1.1]. The mathematical formulation of a TD system results in a system of delay-differential equations. A particular class of these equations, the integro-differential equations, was first studied by Volterra [1.2-1.3] who developed a theory for them and investigated time delay phenomena in different systems. Others [1.4-1.20] have made significant contributions to the development of the general theory of functional differential equations of Volterra type.

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* Sometimes also referred as *time-lag* or *retarded systems*. We will consistently use *time delay systems* in this book.

Another class of delay-differential equations is the differential-difference equations. Major contributors in this area are Bellman, *et al.* [1.21-1.24], Elsgolts [1.25], Hahn [1.26], Halanay [1.27], Krasovskii [1.28], Wright [1.29-1.31] and Zubov [1.32].

Control processes with time delays were first studied by Callender, *et al.* [1.33]. A historical account of control theory and the relevance of time delays to it is given by Bateman [1.34]. Two excellent bibliographies by Weiss [1.35] and Chosky [1.36] list the contributions to different aspects of TD systems until 1959. A large number of papers have appeared in the literature since then reporting research performed on TD systems.

Despite the considerable amount of research done on different aspects of TD systems, the results of this research is scattered mainly in the form of technical papers published in various journals. The existing books [1.27, 1.37, 1.38] treat TD systems essentially from the mathematical viewpoint with scant reference to the applications of such systems. The book by Marshall [1.39] treats some practical aspects of TD control systems. However, in addition to other shortcomings [1.40], it treats TD systems only in the frequency domain.

In the following sections, first different system classifications will be discussed; then several examples of TD systems will be presented for the purposes of motivation. The notation to be used in the book will be explained in another section and a final section on the scope of the book will conclude this chapter.

1.2 TYPES OF SYSTEMS

From this point on by the word "system" we mean a system model, *i.e.*, a mathematical representation of a physical system. (See Chapter 3.) Systems can be classified according to the type of equations describing them. One important classification is as follows.

1.2.1 Lumped-Parameter and Distributed-Parameter Systems

Lumped-parameter systems are those which can be described by ordinary differential (or difference) equations. In contrast, distributed-parameter systems are