INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

COURSES AND LECTURES - No. 103

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ROBERT E. ROBERSON
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JENS WITTENBURG
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DYNAMICS OF FLEXIBLE SPACECRAFT

DEPARTMENT OF GENERAL MECHANICS
COURSE HELD IN DUBROVNIK
SEPTEMBER 1971

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P R E F A C E

At the suggestion of Professor Sobrero of CISM, I organized a series of lectures to be presented by me and several colleagues at Dubrovnik in September 1971, under the joint auspice of CISM and the University of Zagreb. For his encouragement and support, I wish to immediately express my thanks.

The lectures were organized in two series, and three hours of lectures were presented in each series each day during 13 - 17 September. This book contains the lectures of the second series, given by Undersigned, by Professor P.W. Likins, and by Dr. W. Wittenburg. Each series was devoted to one aspect of special current importance relating to the rotational behaviour of spacecraft.

The subject of this second series was the dynamics of flexible rotating spacecraft. This is a topic of considerable current interest to rotational dynamics as a science, as well as to its technological application area of rotating spacecraft. We have attempted to describe here the two major approaches to the problem: first, the approach through linearly elastic dynamical equations, generalized from the traditional structural dynamical equations by reference to rotating bases; second, the approach through the dynamics of a discrete set of interconnected, individually rigid bodies. Each *has its own domain of applicability.*

Professor Likins prepared and presented Lectures 1 - 9; Dr. Wittenburg, Lectures 11 and 14, and I the remaining lectures.

Udine, September 1971

Peter W. Likins

Robert E. Roberson

Jens Wittenburg

Introductory remarks

As I have remarked previously, spacecraft problems have been responsible for a resurgence and growth of the dynamical theory of rotating systems. For many purposes during the last two decades it has been possible to model the spacecraft as a rigid bodies or gyrostats, or perhaps simple two-body systems. However, even from the earliest days of real satellites, cases have been known where non-rigid characteristics have dominated the dynamical behavior.

Within the last few years, elastic deformations have become of increasingly great importance in both spin-stabilized and passively stabilized systems, because the elastic behavior can be central to the stability of the desired state of motion. Furthermore, even in actively controlled systems elastic behavior has become increasingly important as the size of the proposed structures increases and the accuracy of pointing control becomes greater for certain applications.

We no longer can afford to focus solely on the rigid body aspects of spacecraft rotation, but must begin to consider its elastic behavior as well. The resulting class of problems lie at an interesting triple point between classical rigid body dynamics, the theory of structures, and control theory, presenting new facets to the disciplines now familiar in each of these fields.

This lecture series is intended to convey some of the flavor of this relatively new aspect of spacecraft dynamics. It is not a definitive treatment, for such is not yet possible. The field is growing and changing, and currently represents a forefront research. It is felt, nevertheless, that the material presented here is a sound foundation on the basis of which the listener can further develop his own interests.

1. (*) Mathematical modeling of spacecraft

A lecture series devoted to the Dynamics of Flexible Spacecraft is concerned not with a single problem but with a family of related problems. Accordingly, there is not a single correct approach to solution, but a spectrum of methods to be applied to a family of spacecraft idealizations. Although analysts will differ in their preference for various ways of formulating equations, one man choosing Lagrange's equations, a second preferring Hamilton's principle, and a third relying upon a Newton-Euler formulation, these differences are much less fundamental

(*) Lectures 1 through 9 by Likins are based largely on work sponsored by NASA, through either the Jet Propulsion Laboratory or Marshall Space Flight Center. Nomenclature for these lectures is listed following section 9, commencing on page 93.

than the initial choice of a mathematical model of the vehicle.

The spacecraft mathematical model consists firstly of an idealization and mathematical description of the physical system (this we might call the mechanical model), and secondly of a mathematical statement of the motions which can be experienced by the idealized spacecraft (this then becomes the kinematical model). A representative list of options to be considered in each of these modeling decisions might be drawn up as follows:

A. Mechanical models

(a) Rigid body
(b) Elastic or viscoelastic continuum
(c) Collection of elastic elements ("finite elements") interconnected at nodal rigid bodies or particles
(d) Collection of interconnected substructures, each of which is modeled as in a), b), or c).

B. Kinematical models

(a) Unrestricted coordinates of the mechanical model
(b) Coordinates restricted to allow only "small" deformations
(c) Coordinates partially prescribed by interpolation functions

(d) Combinations of a), b), and c).

Although mechanical model Aa), the rigid body, has served to represent most early spacecraft after launch, and mechanical model Ab), the elastic continuum, has proven useful in the representation of launch vehicles modeled as elastic beams, still these basic models must be said to have quite limited utility in application to modern space vehicles. Model Ac) is appropriate for any spacecraft which may be idealized as linearly elastic and subject to small deformations, but this is still in a modern context quite a restricted class of vehicles. In most current applications one must resort to model Ad), which involves the subdivision of a spacecraft into a collection of substructures, and independent idealization of the individual substructures. In this fashion one might accomodate an actively controlled scanning antenna of great flexibility mounted on a spacecraft frame which is itself essentially rigid, or one might connect two or more rigid bodies or two or more elastic bodies. The combinations are many and varied, as are the vehicles to be analyzed.

In order to develop a rationale for adopting a particular idealized mechanical model, one must give some thought to the anticipated kinematical model. A rigid body (model Aa)) is, of course, fully characterized in its motions by six scalar coordinates, and six second-order (or twelve first order) ordinary differential equations will always suffice to predict its motions. The number of independent coordinates increases for a collection of rigid bodies in a manner established by the constraints among the bodies, being always no greater than 6n. In working with such systems, it is customary to deal with some collection of scalar coordinates each of which describes a kinematical property of a particular rigid body of the system. Such coordinates may be characterized as discrete, in contrast to the distributed coordinates to be described next.

A continuum (model Ab) must be characterized kinematically not by scalar coordinates depending only on time, but by scalar functions of space and time; the equations of motion must be partial differential equations. Even for the simplest continuous model of a spacecraft (e.g., a uniform elastic beam), one normally finds it advantageous to replace the partial differential equation by a large but finite number of ordinary differential equations, expressed in terms of coordinates each of which describes a motion or deformation in which the entire vehicle (or substructure) participates; these are called distributed coordinates. The substitution of a finite number of ordinary differential equations for a partial equation evidently involves an approximation, since the continuous system originally postulated could be described kinematically only by an infinite number of scalar coordinates varying only with time. In the simplest case (e.g., a uniform elastic beam vibrating freely about a state of rest in inertial space), the transition from partial to ordinary differential equations is accomplished formally by representing the solution to the unknown function in the partial differential equation as a product of two functions, one of which depends only on spatial coordinates and the other only on time. The latter then provides the unknowns in the ordinary differential equations, while the former provides a shape function which describes the spatial distribution of motion (or deformation) for a unit value of the latter. The expeditious decision to work with a finite number of distributed coordinates is then implemented by simply truncating the infinity of coordinates formally obtained and electing to proceed with a smaller number judged to be representative of the salient features of the system dynamics. This is not a rigorous step mathematically, but it need not be inconsistent with the level of validity of the mathematical model originally adopted for the spacecraft (no real space vehicle is a uniform, homogeneous, isotropic, elastic beam).

In most realistic situations, it is impractical to begin with a partial differentail equation of motion for a material continuum model of a space vehicle, and alternatives must be found.

It is a common practice among those attempting to use a continuous model of a spacecraft or complex spacecraft component to avoid the partial differential equation from the outset, relying upon an initial formulation in terms of a finite number of distributed coordinates which the analyst judges to be adequate to represent all dynamically significant structural deformations. Note that when one adopts this practice he begins with a continuous mechanical model but immediately imposes a kinematical model which restricts the number of degrees of freedom of the system. It then becomes equivalent to the adoption in the first place of a mechanical model having a limited number of degrees of freedom, as in model Ac).

Probably the mechanical model most commonly adopted for the representation of a complex flexible substructure of a spacecraft is model Ac), which idealizes an elastic body as a collection of nodal bodies (either particles or rigid bodies) interconnected by elastic members, often called <u>finite elements</u>. The deformable finite elements may be massless, or mass may be distributed throughout each deformable element; in the latter case the nodal bodies might be massless.

Since small deformation theory is generally employed in analyzing the deformations of the elastic elements in mechanical model Ac), one must generally sacrifice the generality of the unrestricted coordinates in kinematical model Ba). If the finite elements have been idealized as massless, then kinematical model Bb) is appropriate, so that nodal bodies of a given substructure are permitted to experience only small relative motions (perhaps in conjunction with large common motions in inertial space). If the continuous finite elements are idealized as having distributed mass, then the mechanical model again has an infinite number of degrees of freedom. The number of coordinates is reduced by employing kinematical model Bc), for which interpolation functions are introduced to provide the deformations within a finite element explicitly in terms of the relative motions of the nodal bodies.

It is not possible to consider in intelligible detail within the limits of this lecture series the dynamic analysis of each of the mathematical models listed, although the engineer responsible for spacecraft analysis really should have some knowledge of the advantages of each. Rather than provide a superficial survey of all potentially valuable procedures, we have elected to examine in depth a limited number of approaches. Specifically, we shall explore initially the formulation of equations of motion of a flexible substructure modeled as interconnected nodal bodies as in Ac), when attached to spacecraft components modeled as rigid bodies as in Aa); this combination will involve a combination of discrete and distributed coordinates, so it is called a hybrid coordinate formulation.(*) After deriving the appropriate equations for this special case (and indicating briefly how one might approach the more general case involving the coupling of flexible substructures), we shall explore some of the results recently obtained by means of hybrid coordinate analysis. The second half of our lecture series will deal

(*) The material immediately following is drawn from Ref. 18 of the Bibliography.

with the formulation and use of equations of motion of mathematical models consisting of collections of interconnected rigid bodies.

2. Appendage idealization

Any portion of a vehicle which can reasonably be idealized as linearly elastic and for which "small" oscillatory deformations may be anticipated (perhaps in combination with large steady-state deformations) is called a flexible appendage.

A flexible appendage is idealized as a finite collection of ε numbered structural elements, with element number s having n_s points of contact in common with neighboring elements or a supporting rigid body, $s = 1, \ldots, \varepsilon$. Each contact point is called a node, and at each of the n nodes there may be located the mass center of a rigid body (called a nodal body), but the elastic structural elements may also have distributed mass.

Figure 1 is a schematic representation of an appendage (enclosed by dashed lines) attached to a rigid body b of a spacecraft, which may consist of several interconnected rigid bodies and flexible appendages. A typical four-node element of the appendage is shown in three configurations of interest: i) prior to structural deformation, ii) subsequent to steady-state deformation, induced perhaps by spin, and iii) in an excited state,

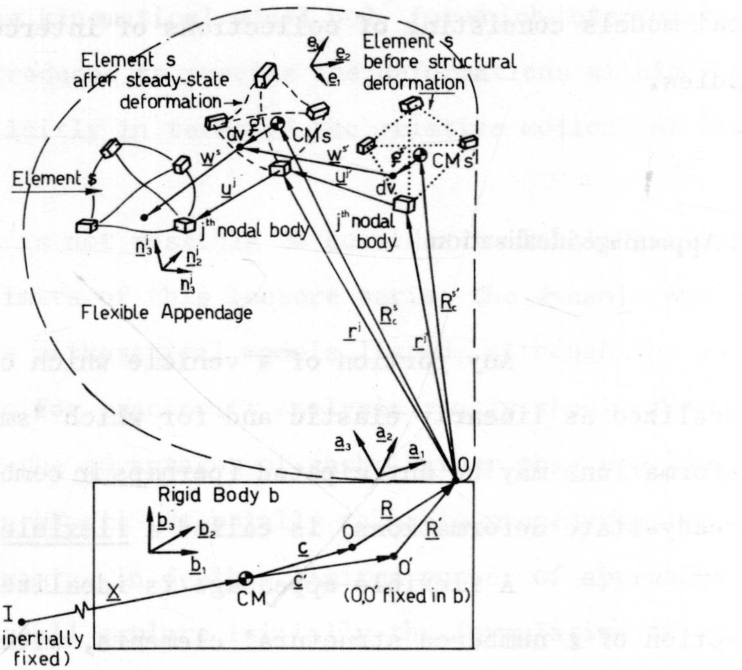

Fig. 1

experiencing both oscillatory deformations and steady state deformations.

The point Q of body b is selected as an appendage attachment point. The dextral, orthogonal unit vectors $\underline{b}_1$, $\underline{b}_2$, $\underline{b}_3$ are fixed relative to b , and the dextral, orthogonal unit vectors $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ are so defined that the flexible appendage undergoes structural deformations relative to a reference frame α established by point Q and vectors $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$. Gross changes in the relative orientation of a and b are permitted, in order to accomodate scanning antennas and such devices; this is accomplish ed by introducing the time-varying direction cosine matrix C relating $\underline{a}_\alpha$ to $\underline{b}_\alpha (\alpha = 1,2,3)$ by

$$\begin{Bmatrix} \underline{a}_1 \\ \underline{a}_2 \\ \underline{a}_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \underline{b}_1 \\ \underline{b}_2 \\ \underline{b}_3 \end{Bmatrix} \tag{1}$$

or, in more compact notation, by

$$\{\underline{a}\} = C\{\underline{b}\} \ . \tag{2}$$

The equations of motion to follow permit arbitrary motion of **b** and arbitrary time variation in C , although practical application of the results requires that the inertial angular velocities of **a** and **b** remain in the neighborhood of constant values over some time interval. These angular velocities will not emerge as solutions of equations to be derived here; the complete dynamic simulation must involve equations of motion of the total vehicle and each of its subsystems, as well as differential equations characterizing necessary control laws for automatic control systems, and only the differential equations of appendage deformations are to be developed here.

As shown in Fig. 1, appendage deformations are described in terms of two increments, one steady-state and the other oscillatory. This separation is necessary because in formulating the equations of motion for the small oscillatory deformations of primary interest here one must characterize the elastic properties of the appendage with a stiffness matrix, and the elements of this matrix are influenced by the structural preload associated with steady-state deformations, as induced for example by spin.

The j^{th} nodal body experiences due to steady-state structural deformation the translation $\underline{u}^{j'} = u^{j'}_{\alpha}\underline{a}_{\alpha}$ (summation convention) of its mass center, and a rotation characterized by $\beta^{j'}_1$, $\beta^{j'}_2$, $\beta^{j'}_3$, for sequential rotations about axes parallel to $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$. The steady-state deformations of a typical element are represented by the function $\underline{w}'$, which is related to the corresponding nodal deformation by the procedures of finite element analysis. The task of solving for the steady-state deformations of appendages on a vehicle with constant angular velocity is mathematically identical to a static deflection problem. Because, at least formally, large deflections and resulting nonlinearities are to be accomodated, this task is not trivial, but it is in this paper assumed accomplished, so that steady-state deformations and structural loads associated with nominal vehicle rotation are assumed known.

Attention is to focus here on the small, time-varying deformations of appendages induced by transient loads or deviations from nominal vehicle motion. The j^{th} nodal body experiences the translation $\underline{u}^{j} = u^{j}_{\alpha}\underline{a}_{\alpha}$ and the rotation $\underline{\beta}^{j} = \beta^{j}_{\alpha}\underline{a}_{\alpha}$ (small angle approximation) in addition to the previously described steady-state deformations. The oscillatory part of the deforma