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1. Introduction

Transonic flows are those in which the local flow speed is close to the local sonic speed. That is the local Mach number M, the ratio of flow speed q to sound speed a, is close to one: $M = \frac{q}{a} \doteq 1$. This means that the dynamic pressure $\frac{\rho q^2}{2}$ and the static pressure P are the same order of magnitude since $\frac{1}{2}\frac{\rho q^2}{P} \sim \frac{\gamma M^2}{2} \sim 0(1)$.* Since the local flow speed is approaching a critical value, we can expect some special phenomena to occur, in contrast to other flow regimes, and indeed they do. The qualitative features that dominate the situation are the existence of throats in streamtubes when the local Mach number is one and the possible occurrence of shock waves when the local Mach number is supersonic. In general when a local supersonic zone is formed in the flow around an airfoil a shock wave occurs. This also occurs when the flight speed is supersonic. (See Figure 1.1.1)

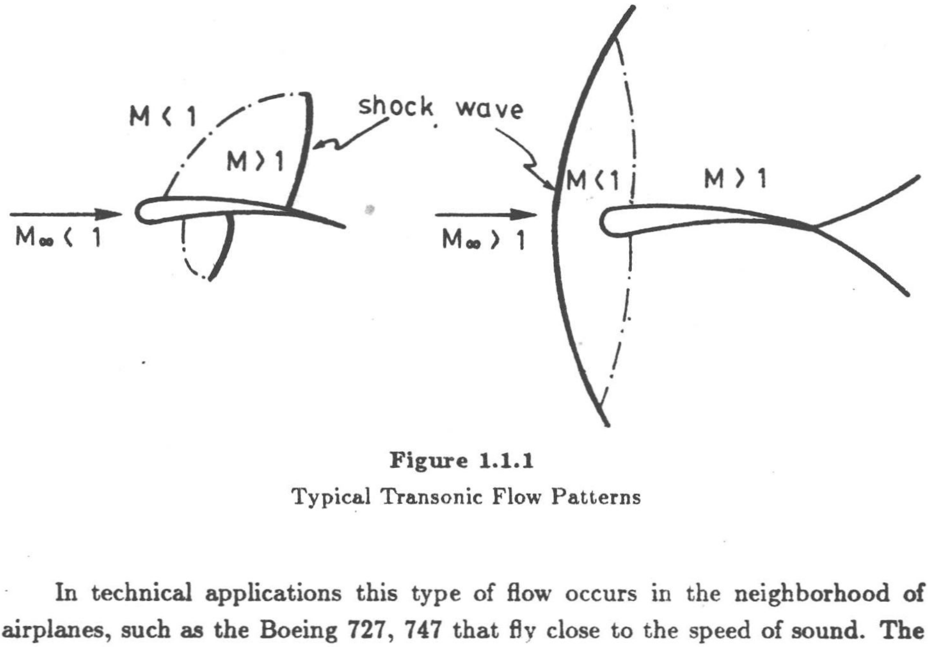

Figure 1.1.1
Typical Transonic Flow Patterns

In technical applications this type of flow occurs in the neighborhood of airplanes, such as the Boeing 727, 747 that fly close to the speed of sound. The next generation of transports might include a "boom-less" airplane which flies at supersonic speed at altitude but is subsonic with respect to sound speed at

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* For an ideal gas $a^2 = \gamma RT = \frac{\gamma P}{\rho}$. At 10 Km altitude, $P/P_0 = 2.6153 \times 10^{-1}$ P_0 = ground pressure = 1.01325×10^5 Newtons/m^3, $\rho = 4.153 \times 10^{-1}$ Kg $a = 299.53$ m/sec, $T = 223.25$°K.

the ground. Transonic flows also occur in compressors and turbines and around helicopter blades, in the throat regions of supersonic wind tunnels, in inlets and in rocket nozzles. Even at highly supersonic speeds a transonic region appears near the nose of a blunt body. "Quasi" transonic flows appear when an important component of the flow velocity is close to sonic, as for example when a wing is swept back close to the Mach angle $\theta_M = \sin^{-1}\frac{1}{M_\infty}$, then the component normal to the edge is sonic.

The aim of this book is to present relatively a self-contained treatment, based on an elementary knowledge of fluid mechanics.

1.1 Framework

The book covers mainly ideal inviscid flow theory (gasdynamics). The resu for external flows, are then applicable to flows at high Reynolds numbers pa streamlined bodies. The viscous effects are assumed to be confined to the interio of very thin regions. These shapes are desirable for technical applications and experience shows that wide classes of engineering problems are amenable to this theory. For example, lift, drag, and moment for three-dimensional wings can be calculated.

Viscous effects and interactions may be important but in any case one must know how to calculate the inviscid flows.

1.2 Mathematics

Perturbation methods will be used to give a systematic discussion of transonic small disturbance theory. The use of this theory is justified by the fact the simplified equations exhibit all the essential features and provide in many cases a good numerical approximation to experimental results. Various important similarity rules appear which are not available for the exact equation. Further, since a systematic procedure is employed corrections to the first-order theory can be studied.

Some special problems for more exact equations will also be studied.

Significant mathematical areas which enter the discussion are:

— Partial Differential Equations of Mixed Type

— Weak Solutions (Shock Waves)

— Hodograph Transformations

— Similarity Solutions

— New Numerical Methods for Equations of Mixed Type

1.3 Historical Note

Transonic flows have been studied theoretically since the beginning of the century: (e.g.): S.A. Chaplygin "On Gas Jets", Moscow University Press (1902) (O Газobix Cmpyax).

Shock waves as isolated phenomena have been known for a long time. Early U.S. experiments were done in the 1930's (NACA Briggs, Dryden, Stack). These were motivated by sonic effects near propeller tips.

Pioneering work in the field was done by Guderley (early 1950's) and Frankl, as evidenced by many references throughout this book. In more recent years a vast literature has accumulated on the subject of transonic flow. In particular many papers dealing with computations have appeared for both approximate and more exact equations.

It is not possible in this work to review all of these developments. We try here to give a detailed theoretical picture of the basis of transonic flow and some discussion of the ideas behind recent numerical approaches.

References

Useful books dealing especially with transonic flow are:

[1.1] Guderley, K. G., *Theorie Schallnahe Strömungen*, Springer-Verlag Berlin 1957. English translation: Addison-Wesley 1962.

[1.2] Bers, L., *Mathematical Aspects of Subsonic and Transonic Gas Dynamics*. John Wiley, N.Y. 1958.

[1.3] Ferrari, C. and Tricomi, F., *Transonic Aerodynamics*, Academic Press 1968.

REMARK: Equations of mixed type appear also in other contexts such as oceanography, elastic shell theory, viscoelastic fluids.

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2. Linearized Theory – Transonic Breakdown

Airplanes and slender objects cause only a small disturbance to the ambient state on passage through the air. The theory of "Acoustics" describes the propagation of such small disturbances usually in a uniform medium at rest. Thus all of linearized aerodynamics (subsonic, supersonic, unsteady) is equivalent to acoustics. The solutions of acoustics are solutions to the classical wave equation. However, in aerodynamics new and typical boundary value problems appear.

For technical applications it would be very useful if linear theory gave a good approximation. Linear solutions are easy to compute and further very general problems can be formulated and solved. For example, R. T. Jones has shown, in linearized supersonic theory, how to distribute the lift on a wing of given span so as to obtain the minimum wave drag. Unfortunately, linearized theory cannot give the correct answer in the transonic range.

In order to understand the breakdown of linearized theory, we can consider the development of the acoustic field around a body flying at sonic speed. For this we need the equations of acoustics. The assumption of isentropy is adequate for the weak disturbances of acoustics. This point will be discussed in some detail later.

The framework of acoustics is that of an inviscid ideal gas. Viscous effects are supposed to be confined to thin layers, such as boundary layers adjacent to solid surfaces, vortex sheets, and the interior of "discontinuous" jumps in pressure (shock waves). The main interest here is the calculation of forces normal to solid surfaces and this can be done if flow separation does not occur. The fact that viscous effects may modify the downstream flow considerably does not affect the calculation of forces on the solid surfaces producing this flow. The ideal gas assumption is not necessary since only small disturbances appear and an arbitrary equation of state could be treated. However it is convenient and sufficiently accurate for most technical applications.

This same framework will also cover most of our considerations on transonic flows.

2.1 Equations of Acoustics

Let $\mathbf{q}\ (x, y, z, t)$ be the flow velocity in the rest frame. (cf Figure 2.1.1). For acoustics this is assumed to be small in some sense, e.g.,

$$\frac{|\mathbf{q}|}{a_\infty} \ll 1; \quad a_\infty = \text{sound speed at infinity} = \left(\frac{\gamma P_\infty}{\rho_\infty}\right)^{\frac{1}{2}},$$

$$\gamma = \frac{c_p}{c_v} = \frac{7}{5} \quad \text{for a diatomic gas,}$$

$$= \frac{5}{3} \quad \text{for a monotomic gas,}$$

$$= \text{ratio of specific heats.}$$

BASIC EQUATIONS:

continuity $$\frac{\partial \rho}{\partial t} + \nabla{\scriptstyle\bullet}\rho\mathbf{q} = 0, \quad \nabla{\scriptstyle\bullet} \equiv \mathbf{div};$$

momentum $$\rho\left\{\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q}{\scriptstyle\bullet}\nabla\mathbf{q}\right\} = -\nabla P, \quad \nabla \equiv \mathbf{grad}; \tag{2.1.1}$$

isentropy $$\frac{P}{\rho^\gamma} = \frac{P_\infty}{\rho_\infty^\gamma}.$$

The acoustic equations are derived by assuming small disturbances

$$\rho/\rho_\infty = 1 + s,$$

$$p, s \ll 1. \tag{2.1.2}$$

$$P/P_\infty = 1 + p,$$

These forms are substituted in the basic equations and squares and higher powers of small quantities are neglected. Isentropy gives

$$1 + p = (1 + s)^\gamma = 1 + \gamma s + \cdots$$

or

$$p = \gamma s. \tag{2.1.3}$$

Continuity and momentum are:

$$\frac{\partial s}{\partial t} + \nabla{\scriptstyle\bullet}\mathbf{q} = 0, \tag{2.1.4}$$

$$\rho_\infty \frac{\partial \mathbf{q}}{\partial t} = -P_\infty \nabla p. \tag{2.1.5}$$

Nonlinear convective effects are thus neglected. The kinematic consequence follows from (2.1.5)

$$\frac{\partial \omega}{\partial t} = 0, \quad \omega = \nabla \times \mathbf{q} \equiv \mathbf{curl}\,\mathbf{q} = \text{vorticity}. \tag{2.1.6}$$

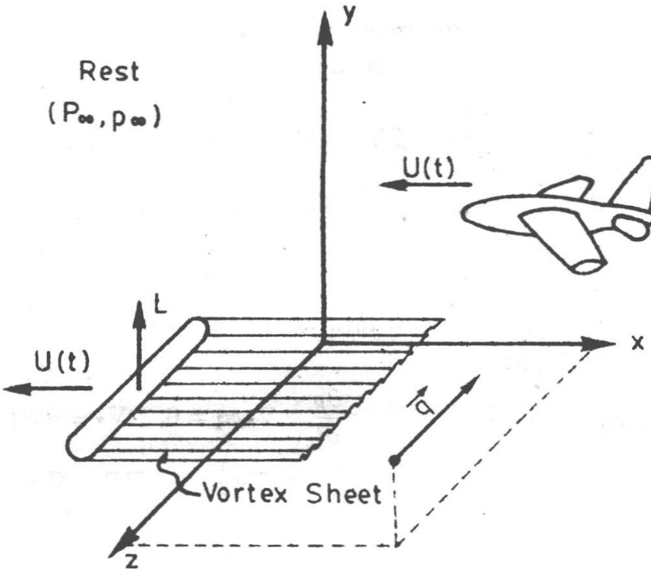

Figure 2.1.1

Coordinates for acoustics

The vorticity at any point in the flow cannot vary with time. Since we assume no distributed vorticity initially there can never be any distributed vorticity. In aerodynamics, however, a concentrated vortex sheet must appear behind a lifting wing. The vorticity so introduced obeys (2.1.6). In general, viscosity is the mechanism by which vorticity is introduced into an aerodynamic flow. For a subsonic trailing edge a Kutta condition is applied that the flow leave the trailing edge smoothly. This is an indirect expression of viscous effects as Reynolds numbers $Re \rightarrow \infty$ and makes possible the unique specification of the flow with a vortex sheet. At a supersonic trailing edge this condition of smooth exit is taken care of by a trailing edge wave system. In linearized theory, an edge can be classified as supersonic or subsonic according to the undisturbed component of flow normal to the edge.

Thus a perturbation velocity potential exists such that

$$\mathbf{q} = \nabla\phi, \quad \phi = \phi(x, y, z, t). \tag{2.1.7}$$

The momentum equation reads $\nabla(\rho_\infty \frac{\partial\phi}{\partial t} + P_\infty p) = 0$ so that integration yields

$\rho_\infty \frac{\partial \phi}{\partial t} + P_\infty p = f(t) = 0$, since disturbances vanish at infinity. Thus we have a linearized Bernoulli equation

$$p = \gamma s = -\frac{\rho_\infty}{P_\infty}\frac{\partial \phi}{\partial t} \quad \text{or} \quad \left\{ \begin{aligned} P - P_\infty &= -\rho \frac{\partial \phi}{\partial t}, \\ s &= -\frac{1}{a_\infty^2}\frac{\partial \phi}{\partial t}, \end{aligned} \right\} \tag{2.1.8}$$

relating the pressure (and force) field to the potential. The equation for the potential comes from the continuity equation

$$-\frac{1}{a_\infty^2}\frac{\partial^2 \phi}{\partial t^2} + \nabla \cdot \nabla \phi = 0, \quad a_\infty^2 = \frac{\gamma P_\infty}{\rho_\infty}.$$

Thus we obtain the classical wave equation

$$\nabla^2 \phi - \frac{1}{a_\infty^2}\frac{\partial^2 \phi}{\partial t^2} = \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) - \frac{1}{a_\infty^2}\frac{\partial^2 \phi}{\partial t^2} = 0. \tag{2.1.9}$$

The most typical property of solutions of the wave equation is that the effect of a concentrated disturbance spreads isotropically at a finite speed a_∞. This speed a_∞ is a property of the medium and is independent of the nature of the disturbance. A signal at the point P_0 at time $t = 0$ spreads to a distance $a_\infty t$ at time t (Figure 2.1-2). A basic solution illustrating this property is obtained from the spherically symmetric solution of (2.1.9) representing outgoing waves. In spherical coordinates $(R = \sqrt{x^2 + y^2 + x^2}, t)$ the wave equation is

$$\frac{\partial^2 \phi}{\partial R^2} + \frac{2}{R}\frac{\partial \phi}{\partial R} - \frac{1}{a_\infty^2}\frac{\partial^2 \phi}{\partial t^2} = 0. \tag{2.1.10}$$

This can, because of good luck, be written

$$\frac{\partial^2 (R\phi)}{\partial R^2} - \frac{1}{a_\infty^2}\frac{\partial^2 (R\phi)}{\partial t^2} = 0. \tag{2.1.11}$$

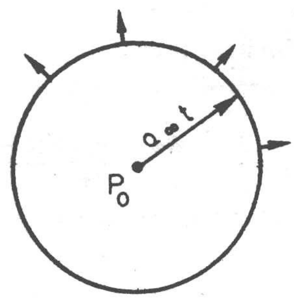

Figure 2.1.2
Spherically spreading disturbance

For only outgoing waves

$$\phi(R,t) = \frac{f(t - R/a_\infty)}{R}, \tag{2.1.12}$$

then the radial velocity $q_R = \frac{\partial\phi}{\partial R}$ has a "near" field and a "far" field

$$\frac{\partial\phi}{\partial R} = -\underbrace{\frac{f(t - R/a_\infty)}{R^2}}_{\text{"near field"}} - \underbrace{\frac{f'(t - R/a_\infty)}{a_\infty R}}_{\text{"far field"}}.$$

The solution can be considered to be produced by a source of fluid at the origin and the "near" field shows an essentially incompressible flow (why?). As $R \to 0$ the outward mass flux (units of ρ_∞) is

$$\lim_{R\to 0} 4\pi R^2 \frac{\partial\phi}{\partial R}(R,t) = -4\pi f(t) = Q(t) = \text{Source Strength} \tag{2.1.13}$$

For the special case of an impulsive source $Q(t) = \delta(t)$ we obtain the fundamental solution S_3, in 3-dimensional space, of the wave equation,

$$\phi(R,t) = S_3 = -\frac{1}{4\pi}\frac{\delta(t - R/a_\infty)}{R}. \tag{2.1.14}$$

This gives the potential at (R,t) due to unit source at (0,0) and is the solution of

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} - \frac{1}{a_\infty^2}\frac{\partial^2\phi}{\partial t^2} = \delta(x)\,\delta(y)\,\delta(z)\,\delta(t) \tag{2.1.15}$$

with $\phi = \phi_t = 0$ at $t = 0-$.

That is, (2.1.9) is a version of the continuity equation and the right hand side can be taken to represent the source strength.

(2.1.14) shows that indeed the propagation is sharp and that all of the disturbance potential is concentrated on $R = a_\infty t$. The corresponding pressure field

$$P - P_\infty = -\rho_\infty\frac{\partial\phi}{\partial t} = \frac{\rho_\infty}{4\pi}\frac{\delta'(t - R/a_\infty)}{R} \tag{2.1.16}$$

shows the arrival of a (singular) compression followed by an expansion (Figure 2.1.3). The superposition (valid because of linearity) of spherical fields can by envelope construction process produce, for example, cylindrical and plane waves (Figure 2.1-4). It is clear then that when a body travels steadily supersonically it out runs its signals. A moving point produces an envelope at the Mach angle θ_M (Figure 2.1.5).

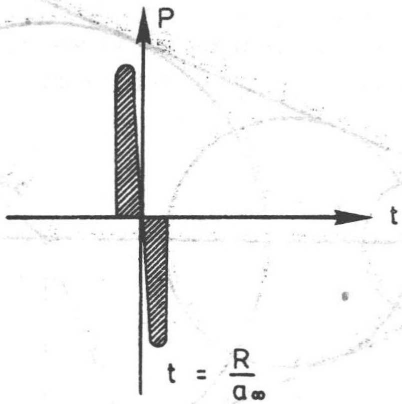

Figure 2.1.3
Pressure signal of an impulsive source

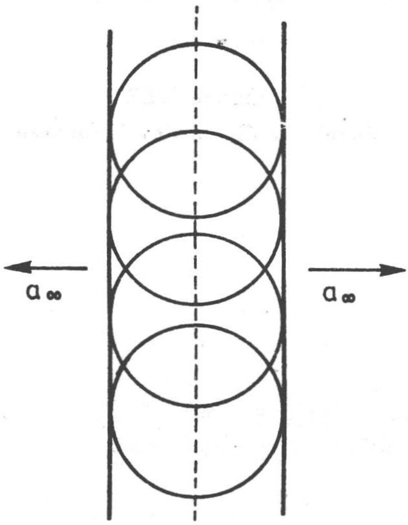

Figure 2.1.4
Envelope construction

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