Dynamics of Fractal Surfaces

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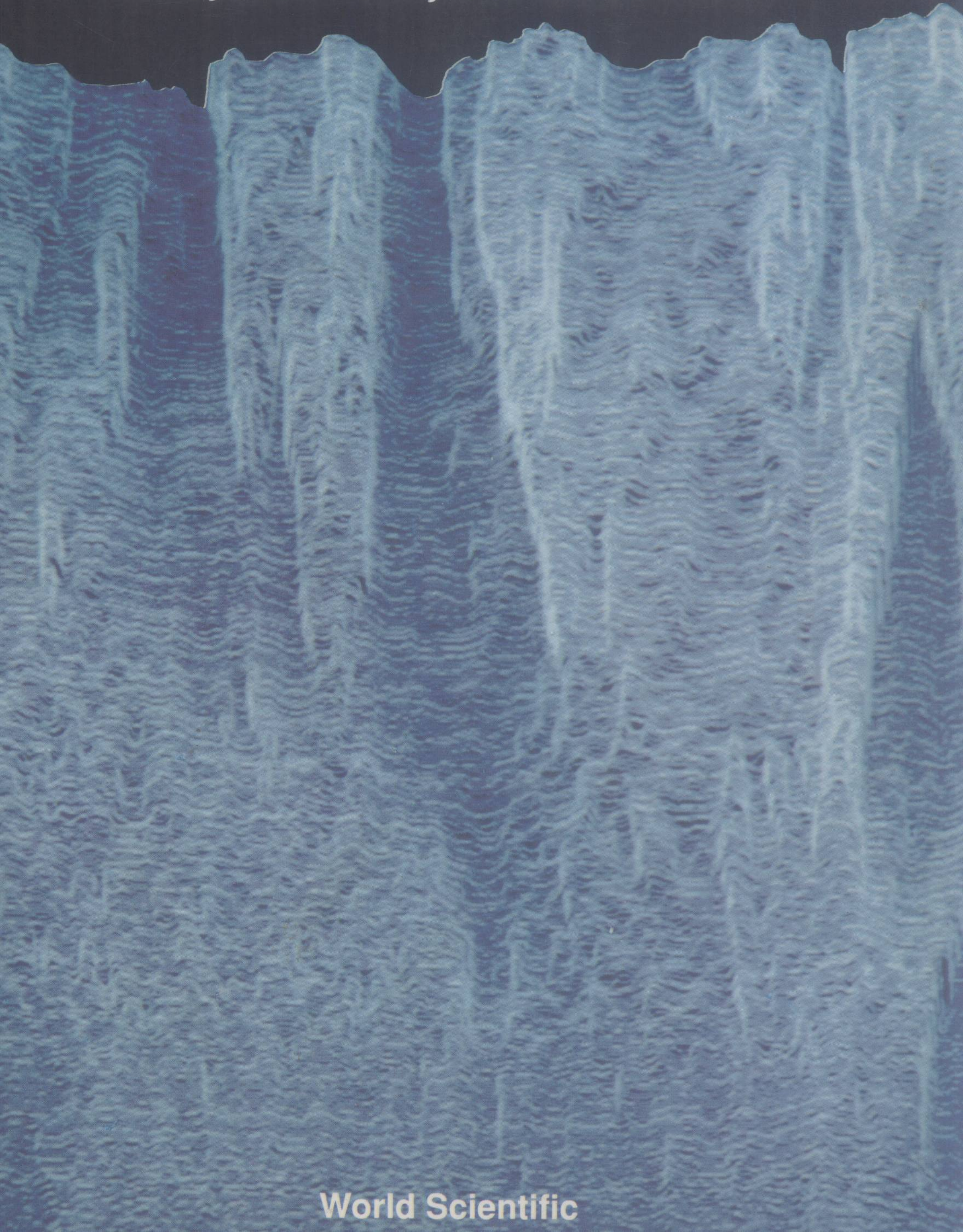
Fereydoon Family and Tamás Vicsek



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PREFACE

In the past few years there has been an explosion of activity in the field of dynamics of fractal surfaces, which, through the convergence of important new results from computer simulations, analytical theories and experiments, has led to significant advances in our understanding of non-equilibrium surface growth phenomena. This is a rare example where these three major approaches of physics can be successfully applied to a far-from-equilibrium phenomenon. There is also considerable interest in surface growth from a practical point of view, because rough surfaces and interfaces are formed in a wide variety of natural and industrial processes. We think it is timely to present these developments in a single volume, with the central theme of dynamic scaling of marginally-stable self-affine fractal surfaces and interfaces. Our goal is to bring together all the seminal papers in this specific area and avoid material from outside this field or on controversial and highly specialized issues. To this end, we have consciously tried to include only works that are considered to be conceptually sound and of fundamental importance.

The book is divided into chapters consisting of reprints related to a single topic. We have written an introductory section for each chapter in order to help unify the different approaches to each topic. In addition, to make the contents of the reprints more accessible and to standardize the notation, every reprint is preceded by a short descriptive note.

We would like to acknowledge and thank many of the authors that are represented here for the collaborations and the interactions that we have had with them over the years. We wish to thank all the authors for granting us permission to reproduce their work in this book. We would also like to thank Scott Anderson for a careful reading of the manuscript and for his helpful suggestions.

Atlanta, Budapest, 1991

Fereydoon Family and Tamás Vicsek

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Chapter 4: MODELS

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Chapter 1

INTRODUCTION

A rich variety of natural and technological processes lead to the formation of complex interfaces (Thomas 1982). If the conditions of the growth process are such that the development of the interface is neither stable or unstable and the fluctuations are relevant, the resulting structure is a rough surface and can be well-described in terms of self-affine fractals (Mandelbrot 1982). The investigation of fractal growth phenomena (Stanley and Ostrowsky 1986, Vicsek 1989) has attracted great interest during the past decade and, in particular, much of the most exciting advances have been in understanding the dynamics of self-affine fractal interfaces. In many cases rough surfaces are generated by a growing interface which advances as new parts are added according to some dynamical process. Examples include crystal growth, vapor deposition, electroplating, spray painting and coating, and biological growth. Fractally rough surfaces may also be formed during the removal of material, as in chemical dissolution, corrosion, grinding, erosion, blasting, wear and many types of polishing. There also exists a third class of processes in which rough interfaces are spontaneously formed without addition or removal of material. Fracture and interfaces between different states of matter are examples of such surfaces. Therefore, one of the most challenging problems in surface science is the understanding of the dynamics of rough interfaces.

The general interest in the study of interface growth also stems from the fact that, in addition to their practical significance in surface science, the dynamics of interfaces is intimately related to a variety of other processes, including the propagation of flame fronts, the long-time behavior in randomly stirred fluids, impurity roughening and pinning of interfaces, and the problem of directed polymers in random media. These relations provide powerful connections among seemingly different phenomena, which can be exploited in the development of various approaches for understanding the evolution of rough interfaces.

Dynamic roughening of interfaces is an example of a far-from-equilibrium phenomenon. As in many nonequilibrium (and irreversible) processes, one is dealing with an open system without a Hamiltonian formulation. At the present time there does not exist a systematic formalism for treating such processes. This implies that the standard approaches of statistical mechanics are not suitable for describing the interface growth problem. However, the discovery that stochastically growing surfaces exhibit non-trivial scaling behavior and naturally evolve to a steady-state without a characteristic time or spatial scale, has led to the development of a general scaling approach for describing growing interfaces (Family and Vicsek 1985). This formalism, which is based on the general concepts of scale-invariance (Ma 1974) and fractals (Mandelbrot 1982), has become a standard tool in the study of growing surfaces. In particular, the dynamic scaling approach has been applied to the study of a variety of theoretical models of growing interfaces, and some recent experiments.

In the past decade, fractal and scaling concepts have provided effective methods for analyzing systems having no characteristic spatial or temporal scale. In the absence of a natural scale, rough surfaces should evolve into fractal patterns. However, the existence of a specific growth direction and smoothing effects, such as surface tension and surface diffusion, introduce a preferred direction in the formation of growing surfaces. This implies that growing interfaces are locally rough, but are anisotropic and globally flat. This type of growth is in contrast to the kind of large scale instabilities that are usually associated with diffusion-limited growth processes (Witten and Sander 1981), such as solidification, viscous fingering, electrodeposition, and chemical dissolution. Since marginally stable interfaces have an anisotropic pattern, they are not self-similar on all length scales. They are examples of self-affine fractal patterns which occur quite widely in many dynamical processes.

There have been significant advances in our understanding of the dynamics of fractal surfaces in recent years, through the convergence of important new results from computer simulations, analytical theories and experiments. This is a rare example where these three major approaches of physics can be successfully applied to a nonequilibrium phenomenon. In this book we present these developments in a single volume by bringing together some of the most important papers in this field.

This book is organized as follows. The material is divided into chapters consisting of reprints related to a particular aspect of the physics of growing fractal surfaces. Each chapter has a general introduction to provide scientific background for the papers reproduced in the main part of these chapters. They are written in a pedagogical style and contain only the most necessary information. To make the content of the reprints more accessible to the reader, each of the papers is also preceded by a short description of what we find to be their most important results. In addition, in these brief descriptions the notation used in the paper is related to the standard notation introduced at the beginning of each chapter.

The following chapter (Chapter 2) is devoted to papers about the mathematical aspects of rough surfaces. Self-affine fractals have a number of distinct features from self-similar ones which become clear from the selected reprints. The papers reproduced in this chapter also demonstrate the richness of self-affine geometry and describe many properties which are useful in the analysis of related numerical and experimental results.

Chapter 3 contains the most important papers on the dynamic scaling of growing interfaces. These papers illustrate how the dynamic scaling picture used today emerged from the studies of various models of surface growth. The papers provided in this chapter formulate the most interesting questions one can ask about fractal surfaces and introduce the theoretical framework and formalism which has been used widely in the works reproduced in the following chapters.

The application of the concept of dynamic scaling to a large class of growth models is demonstrated by the reprints collected in Chapter 4. The selected papers concentrate on determining the most important quantities related to dynamic scaling, such as the exponents and the scaling function. Among the further important issues discussed are the behavior of the exponents for high spatial dimensions and their dependence on the various quantities which may affect the growth process.

The last chapter is devoted to experimental studies of self-affine growth. There are only a limited number of such reprints; however, they are expected to play an increasingly important role in the future. This is especially true since the available experimental results are apparently inconsistent with the predictions based on the simplest models of fractal growth.

Chapter 2

SELF-AFFINE GEOMETRY

Self-affine fractals are objects which are invariant under affine transformations (Mandelbrot 1982, 1985, Voss 1985a, 1985b, 1988). This means that if a small piece of the fractal is blown up in an *anisotropic* way, the enlarged version can be made to match the whole object. The stress here is on the anisotropic rescaling of the lengths. Self-affine fractals should be distinguished from self-similar fractals, which can be magnified isotropically to observe the similarity at different scales. Anisotropic rescaling means that the factor which is used to calculate the coordinates of the magnified object depends on the direction.

A rough interface can be well described in terms of nowhere differentiable, single-valued self-affine functions. A self-affine function $h(x)$ has the property

$$h(x_1, \ldots, x_n) \simeq \lambda_1^{-H_1} \ldots \lambda_n^{-H_n} h(\lambda_1 x_1, \ldots, \lambda_n x_n) \tag{2.1}$$

where H_i is called the roughness or Hurst exponent (Mandelbrot 1982). Typically there is only one characteristic roughness exponent H and the x_i are equivalent from the point of view of scaling and (2.1) has a simpler form $h(x) \simeq \lambda^{-H} h(\lambda x)$. For example, in the case of a single variable x, (2.1) expresses the fact that the function is invariant under the following rescaling: shrinking along the x axis by a factor of $1/\lambda$, followed by rescaling of the values of the function (measured in the perpendicular direction) by a different factor λ^{-H}. For some deterministic self-affine functions this can be done exactly, while for random functions the above considerations are valid only in a stochastic sense (expressed by using the sign $\simeq$). There are self-affine fractals which are different from single-valued functions; however, the growth of marginally stable interfaces typically leads to surfaces which can be well approximated by single-valued functions.

It can be shown that (2.1) is equivalent to the statement that the average width $w(x)$ of the function h scales with the exponent H if parts of the function are considered over different linear regions of length x. In the one-dimensional case this means that

$$w(x) \sim x^H \tag{2.2}$$

where the left-hand side is the average of the widths $w(x) = \langle h^2(x)\rangle - \langle h(x)\rangle^2$, taken over intervals of length x. Alternatively, the average height difference between points separated by distance x scales as

$$\langle |h(x' + x) - h(x')|\rangle \sim x^H \,. \tag{2.3}$$

Thus, expressions (2.1)–(2.3) represent equivalent definitions of self-affine surfaces and the various methods based on these definitions can be looked at as analogous approaches.

Let us review a few examples of self-affine functions in order to get more insight into the many possible cases.

i) As a deterministic self-affine fractal defined on a finite interval we consider the series

$$C(x) = \sum_{n=-\infty}^{\infty} \frac{1 - \cos(b^n x)}{b^{(2-D)n}} \,, \tag{2.4}$$

which is the real part of the more general Weierstrass-Mandelbrot function (see Fig. 2.1). It is easy to see that the first derivative of the above function diverges everywhere for $1 < D < 2$ and $1 < b$, although the function itself remains continuous. Formally replacing n with $n + 1$ in (2.4) leads to the scaling relation $C(x) = b^{-(2-D)}C(bx)$ which is equivalent to the definition (2.1).

Since $C(x)$ can be looked at as a Fourier series, (2.4) suggests that the reason for the absence of a characteristic length-scale for $C(x)$ is a result of the fact that the frequencies b^n form a spectrum spanning the range from zero to infinity. Similarly, the coefficients $A(t)$ of the Fourier spectrum of a stochastic self-affine function with a roughness exponent H are independent Gaussian random variables and their absolute value scales with the frequency f according to a power law

$$|A(f)| \sim f^{-H-1/2} \,. \tag{2.5}$$

ii) The next construction is a function which can be viewed as a deterministic version of the plot of the position of a randomly diffusing particle in one dimension as a function of time. The self-affine fractal is generated on the unit interval by a recursive procedure, replacing the intervals of the previous configuration with the generator having the form of an asymmetric letter z made of four intervals. During each step every interval is regarded as a diagonal of a rectangle becoming increasingly elongated as we go to the next step. The base of the rectangle is divided into four equal parts and the z-shaped generator replaces the diagonal in such a way that its turnovers are always at analogous positions (at the first quarter and the middle of the base). Figure 2.2 shows the first three steps k of the construction. The function becomes self-affine in the limit $k \to \infty$.

It is easy to see that a small part of the $k = 3$ stage has to be dilated horizontally by a factor of 4 and vertically by a factor of 2 to match the $k = 2$ curve. For a true

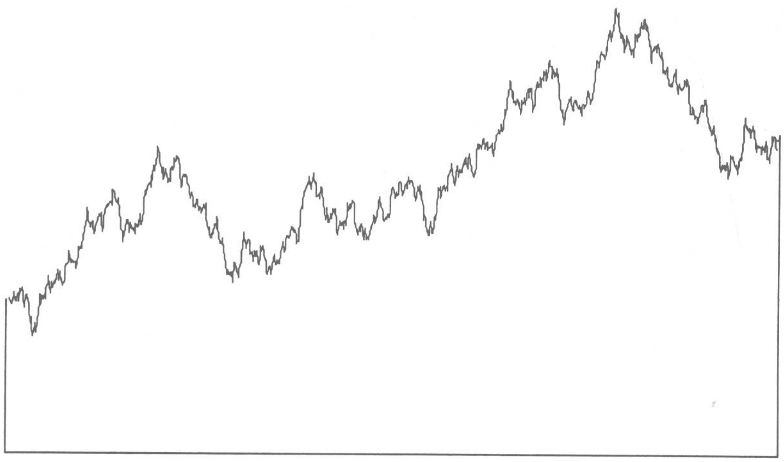

Fig. 2.1. Real part of the Weierstrass-Mandelbrot function on the interval [0.5, 1.0]. In spite of its deterministic origin, this figure strikingly resembles some of the randomly growing self-affine surfaces obtained in computer simulations.

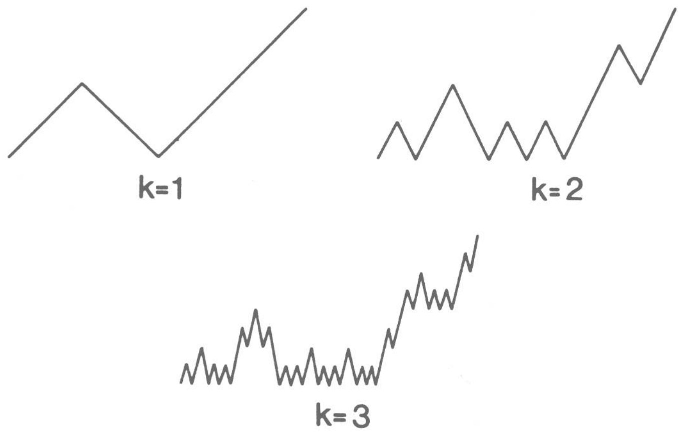

Fig. 2.2. Construction of a Brownian plot-like self-affine function with an iteration procedure.

self-affine fractal ($k \to \infty$) such rescaling leads to a perfect matching on all length scales. Of course, physical objects always have a lower cutoff length below which there is no nontrivial geometrical behavior. For simple shapes, self-affinity is not fulfilled, or it is satisfied in a trivial way. One can, for example, scale parabolas of various sizes onto each other. This, however, cannot be done if one tries to use only a small part of a parabola taken far from its tip.