Formal Logic

A Philosophical Approach

Paul Hoyningen-Huene

Translated by Alex Levine

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University of Pittsburgh Press

Published by the University of Pittsburgh Press, Pittsburgh, Pa., 15261

Manufactured in the United States of America

Printed on acid-free paper

10 9 8 7 6 5 4 3 2 1

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Hoyningen-Huene, Paul, 1946–

[Formale Logik. English]

Formal logic : a philosophical approach / Paul Hoyningen-Huene ; translated by Alex Levine.

p. cm.

Includes bibliographical references (p.).

ISBN 0-8229-5847-3 (alk. paper)

1. Logic, Symbolic and mathematical—Textbooks. I. Title.

BC135.H6913 2004

160—c22

Translator's Preface

This book differs in several respects from other introductory texts currently on the market. As the most important of these are canvassed in the author's preface, I see no need to comment on them further, except to note that the approach taken here is as unique among English-language treatments of logic as it is in the German-speaking world. But a few remarks on other features of this translation are perhaps in order.

I begin with a note on symbolic notation. Over the course of the late nineteenth and twentieth centuries, three major approaches to the symbolic representation of the formulas of first-order predicate logic were developed: the notations of Gottlob Frege's (1879) *Begriffschrift*, of Polish logician Jan Lukasiewicz, and of Giuseppe Peano, later revised by Bertrand Russell. Despite the undisputed importance of Frege's *Begriffschrift*, its cumbersome notational system never gained wider currency. Lukasiewicz's system, also known as "Polish notation," is in many respects the most economical (it dispenses with the need for parentheses). Its peculiar elegance makes it particularly suited to computational applications, where it has secured its own niche. But the overwhelming majority of logic texts, on both sides of the Atlantic, employ some variant or another of the Peano-Russell notation. None of the numerous variants, however, has attained the status of consensual standard.

This translation preserves the variant of Peano-Russell notation employed in the original German, one more common in Continental Europe than in the English-speaking world. The differences among the common notational variants are essentially trivial; anyone who has successfully mastered one of them should have little difficulty grasping any of the others. Still, the variant presented here has a few advantages. The most important of these is perhaps the resemblance between the symbolic representations for conjunction and disjunction ('$\wedge$' and '$\vee$', respectively), and those for universal and existential quantifiers ('$\bigwedge$' and '$\bigvee$'). This similarity serves to remind us of the respects in which predicate logic is a generalization of statement logic, thus easing the transition from the latter to the former. In any case, whenever a new symbol is introduced over the course of chapters 2 and 3, the common alternatives employed in other notational variants are always listed.

This translation incorporates several changes to the original text, all made in close consultation with the author, most of them too minor to mention. Several exercises have been changed, usually in deference to English colloquial usage, and corresponding changes have been made to appendix 2. Appendix 3, "Suggestions for Further Reading," lists several additional English-language texts. Most significant, a new section has been added to chapter 1 (section 3, "Validity and Soundness"). This section introduces the notion of logical soundness, applied to valid inferences with true premises. While this concept has no conventional name in German, it is a commonplace of English-language discussions of formal logic, and so the addition of this section was deemed necessary.

This translation has benefited greatly from the patience of Cynthia Miller at the University of Pittsburgh Press, and from the incomparable diligence and care of Paul Hoyningen-Huene, whose corrections to successive drafts have improved every page. He should not be blamed for any residual errors. I am immensely grateful for the support my colleagues in the Philosophy Department at Lehigh University have given this project. Finally, I owe an incalculable debt to Adriana Novoa.

Alex Levine, June 2003

Preface

The publication of a book entitled *Formal Logic: A Philosophical Approach* requires some justification. Surely by now there are quite enough introductions to logic, to mathematical logic, to the philosophy of logic, and to the logical tools required by particular disciplines, including philosophy. As is often the case, this book owes its existence to the author's frustration with earlier texts. Most texts in logic proper are written by logicians (usually of a mathematical stripe) or by logically or mathematically inclined philosophers. Such texts are primarily concerned with the development of a formal apparatus and its associated techniques. So-called philosophical problems are treated, at best, in passing. These approaches, in particular the more mathematically oriented of them, leave the connection between the arcane formalisms and the ordinary understanding of what logic is about almost entirely obscure. In more philosophical texts, by contrast, the philosophy of logic is frequently presented as a piece of residual philosophizing about issues for which technical solutions have long-since been devised, solutions that function flawlessly. Doing the philosophy of logic is thus a luxury, an indulgence that many expert logicians view with considerable skepticism.

In my view, what is missing is an attempt to unite the motives underlying both sorts of text. On the one hand, a philosophical approach to logic that fails to do justice to the formal niceties is clearly inadequate. On the other hand, even the advanced student frequently finds the more formally rigorous treatments full of puzzling, indeed seemingly arbitrary, steps—steps demanding far greater reflection and explanation than is usually provided. Clearly the present (short) book can only go a short distance in either direction toward uniting philosophical with formal motives. But I would like to show how an introduction to formal logic that gives proper consideration to the philosophical problems of logic might look. In the process, I hope to address some of the most common problems I have found to arise in logic classes designed for nonmathematicians. It strikes me that there are pedagogical advantages to be gained from linking philosophical with formal issues. I also aim to contribute both critically and constructively to the substantive explication of some of the more notorious quandaries of introductory logic, including the notions of logical form, material implication, and valid inference, as well as the so-called paradoxes of material implication, the circularity of attempts to justify logic itself, and the (in)adequacy of classical logic.

This book grew out of lectures I have given since 1976 at the Universities of Zurich, Berne, and Constance. The various ways in which those present at these lectures have understood, or occasionally misunderstood them, have served as a constant source of inspiration. Many of their suggestions have been incorporated. Dr. Gertrude Hirsch (formerly of the University of Zurich, now of the ETH Zurich, Switzerland, and the University of Constance, Germany) produced a creative transcription from recordings of lectures given during the winter term, 1983–84, thus paving the way for a book whose previous existence was confined to fragmentary handwritten notes. Christopher von Bülow of the University of Constance read several drafts with exacting attention, correcting the manuscript in countless matters of stylistic and substantive detail. He also contributed to the English version of appendix 2. Most of his suggestions were accepted. Over the years I have had numerous stimulating conversations on diverse logical problems, most notably with Prof. André Fuhrmann, formerly of the University of Constance and now at the Universidade São Judas Tadeu in São Paulo, Brazil; Prof. Gottfried Gabriel, formerly of the University of Constance and now at the University of Jena, Germany; Prof. Rolf George at University of Waterloo, Canada; Dr.

Stephen Read at the University of St. Andrews, United Kingdom; and Prof. Hans Rott, formerly of the University of Constance and now at the University of Regensburg, Germany. André Fuhrmann and Prof. Hubert Schleichert (University of Constance) generously provided me with the manuscripts of their own logic lectures, from which I took relatively little. Some of the exercises are indebted to the textbooks of Quine and von Savigny. Solutions to these exercises were prepared by Christopher von Bülow, who at one time served as my teaching assistant at the University of Constance, and were adapted for the English reader by Alex Levine. Cynthia Miller of Pittsburgh University Press oversaw the process of transforming a German book into an English one, which is an operation in many respects more complicated than initially expected. Finally, Alex Levine again did a splendid job of translating this book into English, which due to its particular style was not always easy. In addition, he made extremely useful suggestions on where to amend the text, even with respect to one of the proofs that turned out—much to my surprise—to be incomplete, and contributed the new section in chapter 1 on soundness. To all of the above, I extend my heartfelt thanks. Without their efforts, this book might have been completed much sooner—but it would have been much worse.

Paul Hoyningen-Huene, Hannover, July 2003

Contents

1 * Introduction

1. An Example

This chapter introduces the subject of formal logic. Toward this end it seems natural to begin with a definition, one that explains what formal logic is. This is the normal practice in teaching a whole range of specialized fields, but in philosophy a definition is usually a false start, at least it is if we take the definition seriously and plan to stick to it. A definition determines how the thing being defined is to be understood. In philosophy, however, we must first come to some preliminary understanding of the thing (in some broad sense of the word 'thing'), an understanding that is then critically explored and perhaps altered. For this purpose, the device of the initial definition, held constant through all subsequent discussion, is ill suited.

In philosophy, we generally start out with some prior knowledge of the subject under study, albeit not the sort of knowledge we really want. Frequently, we find our subject somehow implicit in some more-or-less concrete fact or example. The analysis of such examples is thus an extremely useful way of approaching the subject itself. In this way we shall approach one of the central notions of formal logic, the notion of valid inference. As to why this notion is central to formal logic, and why this logic is called "formal," these are questions we cannot yet answer. But our analysis of examples will get us started in the right direction.

I begin with an example of the familiar, trivial variety so often employed in logic:

Example 1.1	All logicians are human
	All humans need sleep
	*
Therefore,	All logicians need sleep

To simplify our discussion of this and future examples, we will call the sentences above the asterisk "premises" and the sentence below it the "conclusion." Being a premise or a conclusion is clearly not a property of a sentence in and of itself—it is merely a function of whether a given sentence is found above or below the asterisk. Now, in this case the move from premises to conclusion is somehow compelling, and so we call the whole a (logically) valid inference. It is of the utmost importance to realize that the expression "valid inference" refers to the entire system of three sentences and not merely to the conclusion. In other words, validity is a feature of the relationship between premises and conclusion and is not a property of the conclusion itself. Of course there are things that *can* be said about the conclusion, such as that it is true, but then we are not talking about the inference, which again is the particular relationship between premises and conclusion. I will return to this distinction in greater detail below (see II.2.4.b), but for now suffice it to insist that the following are two separate questions:

1. Is the inference valid?
2. Is the conclusion true?

(The ease with which we confuse these questions is due to the fact that there is, after all, a connection between the validity of an inference and the truth of its conclusion; I will return to this later.) As we shall see, the distinction between these two questions is central to logic.

It is a truly remarkable fact that one rarely encounters anyone who fails to agree that Example 1.1 constitutes a logically valid inference, in the sense that the transition from premises to conclusion is somehow compelling. Without having to come to any agreement, or demand any sort of explanation, anyone with sufficient mastery of the English language to understand the sentences in question recognizes the inference as valid. We would be hard pressed to find someone who, having accepted the premises, would refuse to accept the conclusion. It is worth asking why this is so. At this stage, however, I cannot pursue this question. Let us extract five features of logically valid inferences from our example. Anyone who accepts Example 1.1 as logically valid will also be prepared to ascribe all of these features to it and to related inferences.

Feature 1. When the premises are true, then the conclusion is also true.

Here we must pay close attention to what is being said. We are not claiming *that the premises are true,* but only that *if* they are true, *then* the conclusion is also true. This feature holds with all logically valid inferences and thus deserves a name. I shall call it the "truth-transferring" property of logically valid inferences. So, when the premises of a logically valid inference happen to be true, the conclusion is also true—the truth of the premises is *transferred* to the conclusion.[1]

This truth-transferring quality of our example is, in the end, something one must see for oneself; anyone incapable of seeing it will never understand what logic is all about. Thus, as far as this fundamental insight into truth transfer is concerned, logic cannot be taught. To be sure, I can hint and gesture toward the insight, toward the experience of a certain thought process, with some degree of clarity. But no such descriptions of the relevant thought process will serve as a set of instructions such that, if one only follows the recipe, one will come to understand what truth transfer is. Experience of the thought process in question cannot be replaced by descriptions of it, and this experience is fundamental to logic. In this

1. Here the limitations of the truth transfer metaphor become apparent. When funds are transferred from account *A* to account *B*, they are obviously no longer in account *A*. By contrast, successful truth transfer from premises to conclusion does not mean that the premises are no longer true (provided they were true in the first place). Furthermore, in our present example, the conclusion "All logicians need sleep" is true anyway. No "transfer" of truth from the premises is required in order to make it true.

regard, our situation is far from unique. There are other experiences for which no description can be substituted, such as the experience of being in love, of the effects of alcohol, or of swimming in a strong current. Here, too, only someone who has had the right sort of experience will understand what the discussion is about.

Feature 2. The truth of the premises plays no role in our assessment of the validity of the inference.

Here it is claimed that the validity of an inference cannot be judged on the basis of the truth of its premises. Before defending this assertion, which may strike some as implausible, I hasten to point out that this second feature is not inconsistent with the first. Feature 1 is a claim about what happens *when* the premises are true; it does not assert *that* the premises are true. Now for the defense of Feature 2: Let us substitute the expression 'have characteristic *S*' for the expression 'need sleep' wherever the latter occurs in Example 1.1. This yields:

Example 1.2. All logicians are human.
All humans have characteristic *S*.
*
Therefore, All logicians have characteristic *S*.

As is plain to see, we have another logically valid inference before us. But it must also be noted that the sentences 'All humans have characteristic *S*' and 'All logicians have characteristic *S*' are not straightforwardly true or false statements. For the meaning of the expression 'characteristic *S*' is indeterminate, thus leaving the two sentences in which it occurs equally indeterminate. Nonetheless, the logical validity of Example 1.2 may be seen without the slightest difficulty, even more easily than the validity of Example 1.1. Example 1.2 thus demonstrates that our assessment of the validity of an inference cannot depend on the truth of its premises, for in this example the second premise is so indeterminate as to be neither true nor false. Accordingly, ascertaining the validity of Example 1.1 does not require that we consult the Institute for Sleep Study on the distribution of sleep requirements among various segments of the population. The truth of the premises of this earlier example is equally irrelevant to its validity.

Indeed, the premises might be manifestly false without undermining the validity of the inference. This can easily be demonstrated by substituting the expression 'have characteristic *S*' as it occurs in Example 1.2 with some other expression that makes the second premise false (for example, 'are reptiles'). Now, one might be tempted to suppose that it is impossible to validly infer anything from false premises, but this temptation must be resisted. Arguments of the following form are, after all, commonplace: "Let us assume that all humans are reptiles. Under this assumption, what would logicians be?" In mathematics, inferences drawn from false premises are systematically employed in a procedure known as indirect proof, in which we assume the exact opposite of what we are trying to prove and infer a contradiction from it. This contradictory consequence shows that our assumption is false and thus that its opposite, the claim we were out to prove in the first place, must be true (for a more detailed exposition of indirect proof, see section II.2.4.f).

Feature 3. From any valid inference, many additional valid inferences may be generated (mechanically).

The procedure by which these additional inferences are generated has actually already put in an appearance, in our explication of Feature 2. Simply replace such expressions as 'logician', 'human,' and 'need sleep' as they occur in Example 1.1 with other expressions, such as 'animal', 'life form', and 'God's creation', respectively. Our replacements must be methodical, in the sense that for *every* occurrence of an expression being replaced, we replace it with *one and the same* new expression; otherwise, our procedure will fail to generate further valid inferences. The substitution proposed above thus yields:

Example 1.3. All animals are life forms.
All life forms are God's creations.
*
Therefore, All animals are God's creations.

This inference, too, is logically valid, as should be immediately clear. In this instance, the procedure of replacing each and every occurrence of a given expression with an occurrence of one and the same new expression has thus yielded a new valid inference. But of course this success hardly justifies the claim made in Feature 3, since it remains conceivable that the same procedure might fail for other examples. However, consideration of the fourth feature should persuade us that, in fact, our procedure will *always* yield new valid inferences.

Feature 4. The meanings of the expressions occurring in an inference are irrelevant to its validity.

The above formulation of Feature 4 is provisional and somewhat imprecise. By way of clarification, we return to Example 1.1, this time replacing every occurrence of 'logician', 'human', and 'need sleep' by *A, B,* and *C,* respectively, yielding:

Example 1.4. All *A*s are *B*s.
All *B*s are *C*s.
*
Therefore, All *A*s are *C*s.

Once again, as with Example 1.2, we observe the logical validity of this inference, this despite the fact that *A, B,* and *C* are completely indeterminate. It follows that logically valid inference has nothing to do with word meaning, for the validity of an inference can be detected even after certain expressions ('logician', 'human', etc.) have been replaced with single letters. The "mechanism" of a valid inference thus depends not on the particular expressions it contains, but on something else. We will return to this something else later.

Having performed the abstraction procedure involved in converting Example 1.1 into Example 1.4, we begin to understand why Features 1–3 really *are* features of valid inferences. Feature 1 asserts that any valid inference is such that if its premises are true, then its conclusion is also true. In establishing the validity of Example 1.4, we consider this: if all *A*s are *B*s, and if all *B*s are also *C*s, is it then the case that all *A*s are *C*s? This question in essence asks whether the inference is truth-transferring—whether the truth of the premises is transferred to the conclusion. We can answer it in the affirmative without having the slightest idea what it would mean for all *A*s to be *B*s or all *B*s to be *C*s, let alone whether both claims—both premises—are actually true. But this is precisely what Feature 2 asserts of valid inferences. This result further underscores the possibility of using any valid inference to generate further valid inferences (Feature 3), for since the meanings of constituent expressions are irrelevant to the validity of the inference, they may be replaced with new expressions, provided we always replace each original expression with one and the same substitute. This reasoning establishes that Features 1–3 are implicit in Feature 4. Anyone who follows this reasoning is well on his or her way to grasping what logic is all about.

It should be noted that in our various manipulations of Example 1.1 we have played around with several of its constituent expressions, but not with all of them. For example, we have always left the word 'all' unchanged. Now let us see what happens when we replace the word 'all', as it occurs in Example 1.4, with the word 'some'. This substitution yields:

Example 1.5. Some *A*s are *B*s
Some *B*s are *C*s
*
Therefore, Some *A*s are *C*s

Is this inference valid, in the sense that the truth of the premises guarantees the truth of the conclusion? To simplify, let us replace '*A*', '*B*', and '*C*' by particular meaningful expressions. If Example 1.5 is valid, then by