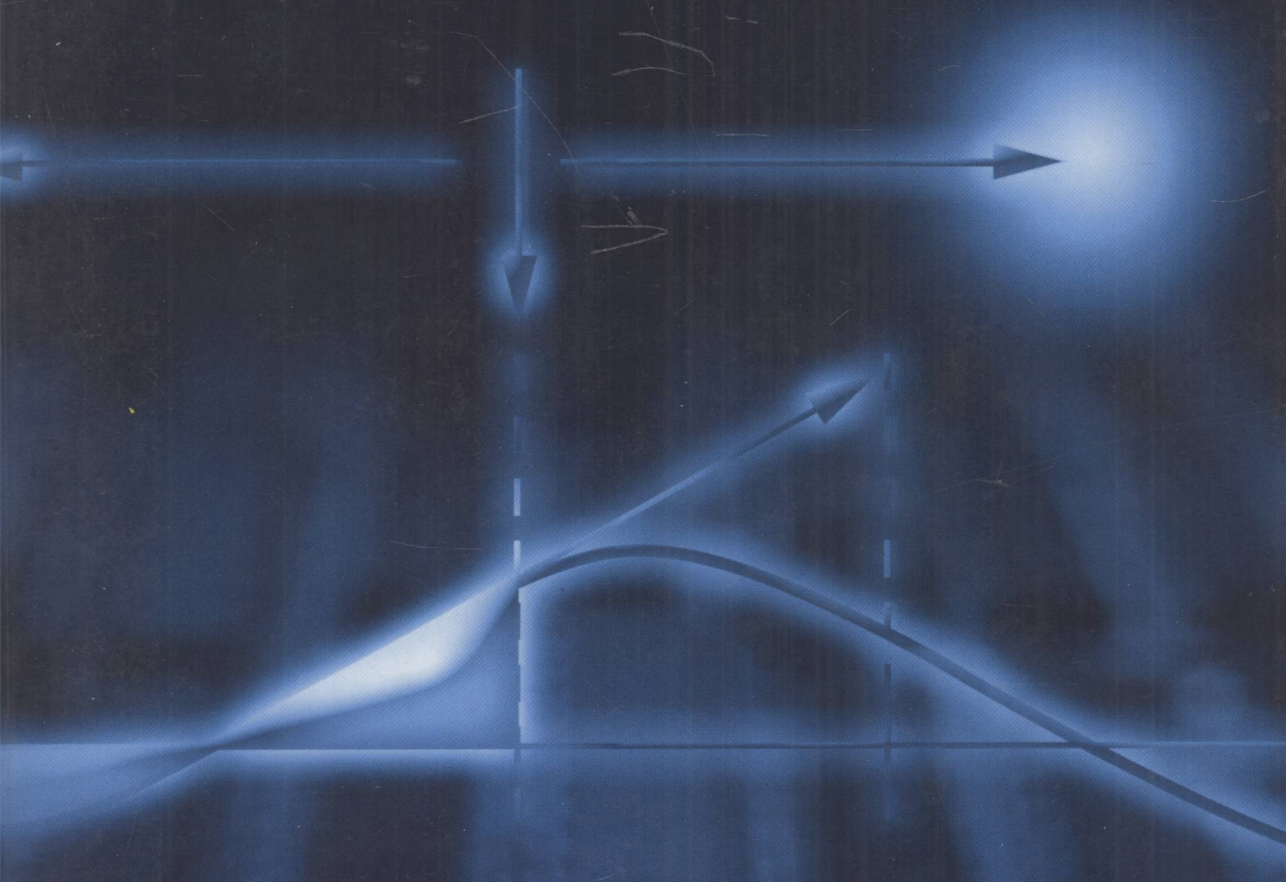


# ADVANCED PID CONTROL

Karl J. Åström and Tore Hägglund



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# Advanced PID Control

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# Advanced PID Control

# Preface

The PID controller is the most common solution to practical control problems. Although controllers with proportional and integral action have been used from the time when windmills and steam engines were the dominant technologies, the current form of the PID controller emerged with the pneumatic controllers in the 1930s. The controllers have been implemented in many different ways using mechanical, pneumatic, electronic, and computer technology. The development accelerated when the microprocessor implementations appeared in the 1980s. One reason was that the computer implementations made it possible to add features like auto-tuning and diagnostics, which are very beneficial for users. From an engineering perspective, it is particularly interesting to analyze what happened at the technology shifts, when some important features were rediscovered and others were added.

This book has grown out of more than 25 years of development of auto-tuners for PID controllers in close collaboration with industry. Through this work, we have been exposed to a large number of real industrial control problems. We have benefited much from participating in development, commissioning, and troubleshooting of industrial controllers. The practical work has also inspired research.

This book is the last part of a trilogy. The first book, *Automatic Tuning of PID Controllers, 1988*, which had 6 chapters, gave a short description of our early experiences with development of relay auto-tuners. The second book, *PID Controllers: Theory, Design, and Tuning, 1995*, which has 7 chapters, grew out of the need for a broader coverage of many aspects of PID control. In particular, it reviews many design methods for PID controllers that we investigated in connection with our work on auto-tuners.

The knowledge about PID control in 1995 still was not satisfactory for design of auto-tuners. One drawback was that the user had to provide the controller with design choices. It is particularly difficult for a user to assess if dynamics is dead-time or lag dominated. This question stimulated further research. Because of the drastic increase in computing power, it was also possible to use design algorithms that require more computations.

Tuning and design of PID controllers have traditionally been based on special techniques. Robust control was a major development of control theory that matured in the late 1990s, resulting in powerful design methods based on robust loop shaping. This stimulated us to initiate a research program to adapt

## Preface

these methods to PID control. At the same time, it seemed natural to bring PID control closer to the mainstream ideas in control. When working with industrial auto-tuners, we also saw a great need to include diagnostics in the controller, because it is no use to tune a controller if the process has severe malfunctions. The present book, *Advanced PID Control*, is the result of this effort.

With a total of 13 chapters, this new book substantially expands on some of the topics covered in the previous versions and provides several new chapters that deal with controller design, feedforward design, replacement of the Ziegler-Nichols tuning rules, predictive control, loop and performance assessment, and interaction. At this point in our book trilogy, we assume that the reader is highly familiar with control theory.

Our research has given a deeper understanding of the trade-offs between load disturbance attenuation, injection of measurement noise, and set-point response. We have also been able to answer questions like: Should a controller be tuned for response to load disturbances or set points? What information is required to design a PID controller? When can derivative action give significant improvements? When are more complicated controllers justified? When is it justified to develop more accurate process models? With the knowledge developed, it is now possible to design auto-tuners that can make these assessments autonomously. In addition, we have developed new simple methods for designing PID controllers.

As an example of the insight gained we can mention that control theory tells that it is not necessary to make a compromise between tuning for load disturbance response and set-point response. Both requirements can be satisfied by using a controller with two degrees of freedom, which combines feedback and feedforward. The feedback gains should be chosen to satisfy requirements on disturbance attenuation and robustness. The desired response to set-point changes can then be obtained by proper use of feedforward. Set-point weighting is a simple form of feedforward for PID control. In some cases, it is justified to use more elaborate feedforward. For this reason, we have included a chapter on controller design and another chapter on feedforward in the new book.

The robustness analysis also shows the advantage of having low controller gain at high frequency, high frequency roll-off. This can be accomplished by filtering the process output by a second order filter. Based on the insight obtained, we recommend extended use of set-point weighting or more advanced feedforward. We also recommend that the process output is filtered using a second order filter.

We would like to thank many people who have given knowledge, insight, and inspiration. Our interest in PID control was inspired by Axel Westrenius and Mike Somerville of Eurotherm in the early 1980s. We have learned much from working with students; particular thanks are due to Lars Göran Elfgrén (Eurotherm), Göran Grönhammar (LTH), Ari Ingimundarson (UPC), Oskar Nordin (Volvo), Helene Panagopoulos (Volvo), Per Persson (Volvo), Mikael Petersson (ABB), Ola Slättke (ABB), and Anders Wallén (Ericsson Mobile Platforms), who continue to give us valuable insight even if they are now pursuing careers in industry.

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# 1

## Introduction

### 1.1 Introduction

The idea of feedback is deceptively simple and, yet, extremely powerful. Feedback can reduce the effects of disturbances, it can make a system insensitive to process variations and it can make a system follow commands faithfully. Feedback has also had a profound influence on technology. Application of the feedback principle has resulted in major breakthroughs in control, communication, and instrumentation. Many patents have been granted on the idea.

The PID controller is a simple implementation of feedback. It has the ability to eliminate steady-state offsets through integral action, and it can anticipate the future through derivative action. PID controllers, or even PI controllers, are sufficient for many control problems, particularly when process dynamics are benign and the performance requirements are modest. PID controllers are found in large numbers in all industries. The controllers come in many different forms. There are stand-alone systems in boxes for one or a few loops. The PID controller is a key part of systems for motor control. The PID controller is an important ingredient of distributed systems for process control. The controllers are also embedded in many special-purpose control systems. They are found in systems as diverse as CD and DVD players, cruise control for cars, and atomic force microscopes. In process control, more than 95 percent of the control loops are of PID type; most loops are actually PI control. Many useful features of PID control have not been widely disseminated because they have been considered trade secrets. Typical examples are techniques for mode switches and anti-windup.

PID control is often combined with logic, sequential functions, selectors, and simple function blocks to build the complicated automation systems used for energy production, transportation, and manufacturing. Many sophisticated control strategies, such as model predictive control, are also organized hierarchically. PID control is used at the lowest level; the multivariable controller gives the set points to the controllers at the lower level. The PID controller can thus be said to be the “bread and butter” of control engineering. It is an important component in every control engineer’s toolbox.

PID controllers have survived many changes in technology, ranging from

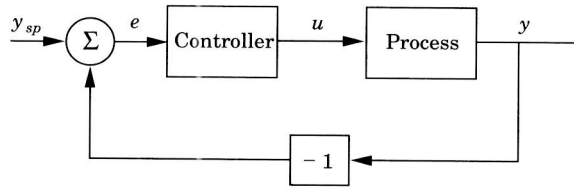
pneumatics to microprocessors via electronic tubes, transistors, and integrated circuits. The microprocessor has had a dramatic influence on the PID controller. Practically all PID controllers made today are based on microprocessors. This has created opportunities to provide additional features like automatic tuning, gain scheduling, continuous adaptation, and diagnostics. Most new PID controllers that are produced today have some capability for automatic tuning. Tuning and adaptation can be done in many different ways. The simple controller has in fact become a test bench for many new ideas in control. There has also been a renaissance of analog implementation in micro-mechanical systems because analog implementation requires less silicon surface than digital implementations. The PID controller is also implemented using field programmable gate arrays for applications where very fast control is required.

A large number of instrument and process engineers are familiar with PID control. There is a well-established practice of installing, tuning, and using the controllers. In spite of this there are substantial potentials for improving PID control. Evidence for this can be found in the control rooms of any industry. Many controllers are put in manual mode, and among those controllers that are in automatic mode, derivative action is frequently switched off for the simple reason that it is difficult to tune properly. The key reasons for poor performance are equipment problems in valves and sensors, process constraints and bad tuning practice. The valve problems include wrong sizing, hysteresis, and stiction. The measurement problems include poor or no anti-aliasing filters; excessive filtering in “smart” sensors, excessive noise, and improper calibration. Substantial improvements can be made. The incentive for improvement is emphasized by demands for improved quality, which is manifested by standards such as ISO 9000. Knowledge and understanding are the key elements for improving performance of the control loop. Specific process knowledge is required as well as knowledge about PID control.

Based on our experience, we believe that a new era of PID control is emerging. This book will take stock of the development, assess its potential, and try to speed up the development by sharing our experiences in this exciting and useful field of automatic control. The goal of the book is to provide the technical background for understanding PID control.

## 1.2 Feedback

A simple feedback system is illustrated by the block diagram in Figure 1.1. The system has two major components, the process and the controller, represented as boxes with arrows denoting the causal relation between inputs and outputs. The process has one input, the manipulated variable (MV), also called the control variable. It is denoted by  $u$ . The control variable influences the process via an actuator, which typically is a valve or a motor. The process output is called process variable (PV) and is denoted by  $y$ . This variable is measured by a sensor. In Figure 1.1 the actuator and the sensor are considered part of the block labeled “Process”. The desired value of the process variable is called the set point (SP) or the reference value. It is denoted by  $y_{sp}$ . The control error  $e$  is the difference between the set point and the process variable, i.e.,  $e = y_{sp} - y$ .



**Figure 1.1** Block diagram of a process with a feedback controller.

Assume for simplicity that the process is such that the process variable increases when the manipulated variable is increased. The principle of feedback can then be expressed as follows:

Increase the manipulated variable when the error is positive, and decrease it when the error is negative.

This type of feedback is called *negative feedback* because the manipulated variable moves in opposite direction to the process variable since  $e = y_{sp} - y$ .

The PID controller is by far the most common form of feedback. This type of controller has been developed over a long period of time, and it has survived many changes in technology, from mechanical and pneumatic to electronic and computer based. Some insight into this is useful in order to understand its basic properties as is discussed in Section 1.4.

Some properties of feedback can be understood intuitively from Figure 1.1. If the feedback works well the error will be small, and ideally it will be zero. When the error is small the process variable is also close to the set point irrespective of the properties of the process. To realize feedback it is necessary to have appropriate sensors and actuators and a mechanism that performs the control actions.

Feedback has some interesting and useful properties.

- Feedback can reduce effects of disturbances
- Feedback can make a system insensitive to process variations
- Feedback can create well-defined relations between variables in a system

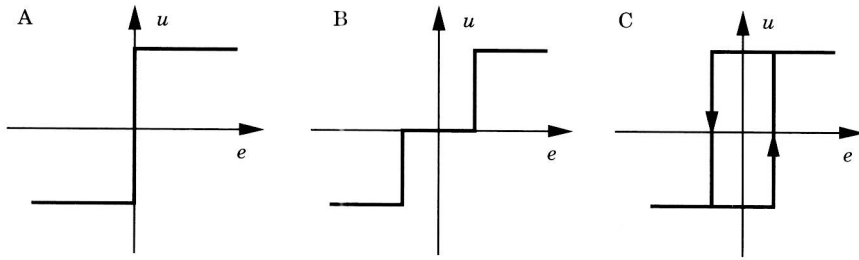
## 1.3 Simple Forms of Feedback

Many of the nice properties of feedback can be accomplished with simple controllers. In this section we will discuss some simple forms of feedback, namely, on-off control, proportional control, integral control, and PID control.

### On-Off Control

The feedback can be arranged in many different ways. A simple feedback mechanism can be described as

$$u = \begin{cases} u_{\max}, & \text{if } e > 0 \\ u_{\min}, & \text{if } e < 0, \end{cases} \quad (1.1)$$



**Figure 1.2** Controller characteristics for ideal on-off control (A), and modifications with dead zone (B) and hysteresis (C).

where  $e = y_{sp} - y$  is the control error. This control law implies that maximum corrective action is always used. This type of feedback is called *on-off control*. It is simple and there are no parameters to choose. On-off control often succeeds in keeping the process variable close to the set point, but it will typically result in a system where the variables oscillate. Notice that in Equation 1.1 the control variable is not defined when the error is zero. It is common to have some modifications either by introducing hysteresis or a dead zone (see Figure 1.2).

### Proportional Control

The reason why on-off control often gives rise to oscillations is that the system overreacts since a small change in the error will make the manipulated variable change over the full range. This effect is avoided in proportional control where the characteristic of the controller is proportional to the control error for small errors. This can be achieved by making the control signal proportional to the error

$$u = K(y_{sp} - y) = Ke, \quad (1.2)$$

where  $K$  is the controller gain.

### Integral Control

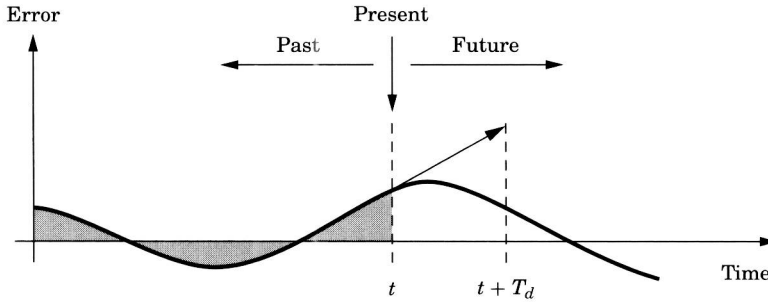
Proportional control has the drawback that the process variable often deviates from the set point. This can be avoided by making the control action proportional to the integral of the error

$$u(t) = k_i \int_0^t e(\tau) d\tau, \quad (1.3)$$

where  $k_i$  is the integral gain. This strategy is called integral control. Integral control has an amazing property. Assume that there is a steady state with constant error  $e_0$  and constant control signal  $u_0$ . It follows from the above equation that

$$u_0 = k_i e_0 t.$$

Since  $u_0$  is a constant it follows that  $e_0$  must be zero. We thus find that if there is a steady state and a controller has integral action the steady-state error is



**Figure 1.3** A PID controller takes control action based on past, present, and future control errors.

always zero. It follows that this is also true for the PI controller

$$u(t) = Ke(t) + k_i \int_0^t e(\tau) d\tau. \tag{1.4}$$

This is one of the reasons why PI controllers are so common.

**PID Control**

An additional refinement is to provide the controller with an anticipative ability by using a prediction of the output based on linear extrapolation. See Figure 1.3. This can be expressed mathematically as

$$u(t) = K \left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right). \tag{1.5}$$

The control action is thus a sum of three terms representing the past by the integral of the error (the I-term), the present (the P-term) and the future by a linear extrapolation of the error (the D-term). The term  $e + T_d \frac{de}{dt}$  is a linear prediction of the error  $T_d$  time units in the future. The parameters of the controller are called: proportional gain  $K$ , integral time  $T_i$ , and derivative time  $T_d$ .

It has been found empirically that the PID controller is capable of solving a wide range of control problems. There are more complicated controllers that differ from the PID controller by using more sophisticated methods for prediction.

**1.4 How the PID Controller Developed**

The PID controller has developed over a period of time that stretches over at least 250 years. It is useful to have some perspective of this development in order to understand many of the issues. The technology used to implement



the controllers has naturally changed significantly over the years. The first controllers were mechanical devices (centrifugal governors) used to control windmills and steam engines. Sensing of angular velocity was combined with actuation of valves. A great deal of cleverness was involved in devising integral action.

A significant change occurred in connection with the development of industrial process control. The functions of sensing, control, and actuation were then separated and special devices that performed the control actions represented by Equation 1.5 were built. An interesting feature was that signal transmission and computing were done pneumatically. A major advance occurred when the tubes used to transmit the pressure and the pressure levels were standardized to 3–15 PSI. This made it possible to combine sensors, controllers, and actuators from different suppliers. It also made it possible to concentrate controllers in separate control rooms that were located far away from sensors and actuators. Much cleverness was again used to obtain the controllers. The use of feedback in the controllers themselves was a major improvement. In this way it was possible to obtain linear action out of components that had strongly nonlinear characteristics.

Starting in 1950s, electronic versions of the PID controller became available. The control actions represented by Equation 1.5 were then obtained by a simple analog computer based on operational amplifiers. The signal transmission was also standardized as current signals in the range 4–20 mA. To represent zero by a nonzero current was useful for diagnostics.

Yet another advance occurred when digital computers were used to implement controllers. Strongly centralized systems were first used when computer control emerged, because digital computing was only cost effective in large systems. With the advent of microprocessors in the 1970s even simple controllers were implemented using computers. When a digital computer is used it is also possible to add many functions such as automatic tuning, adaptation, and diagnostics. This is an area of very active development.

Today we are experiencing other shifts in technology. Analog implementations are reappearing in micro-mechanical electrical systems (MEMS), and digital controllers are also implemented using field programmable gate arrays (FPGA), which admit very short sampling periods. The FPGAs differ significantly from digital computers because they are highly parallel.

Today we find PID controllers in many forms. There are dedicated controllers that can control one or a few loops. PID functions are found in Programmable Logic Controllers that were originally designed to replace relays. There are systems that contain many PID controllers implemented in computers ranging from small systems for a few dozen loops to large distributed systems for process control. PID controllers are commonly used in dedicated systems for motion control. There are also a whole range of special controllers such as autopilots and control systems for CD and DVD players and optical memories that are based on PID control.