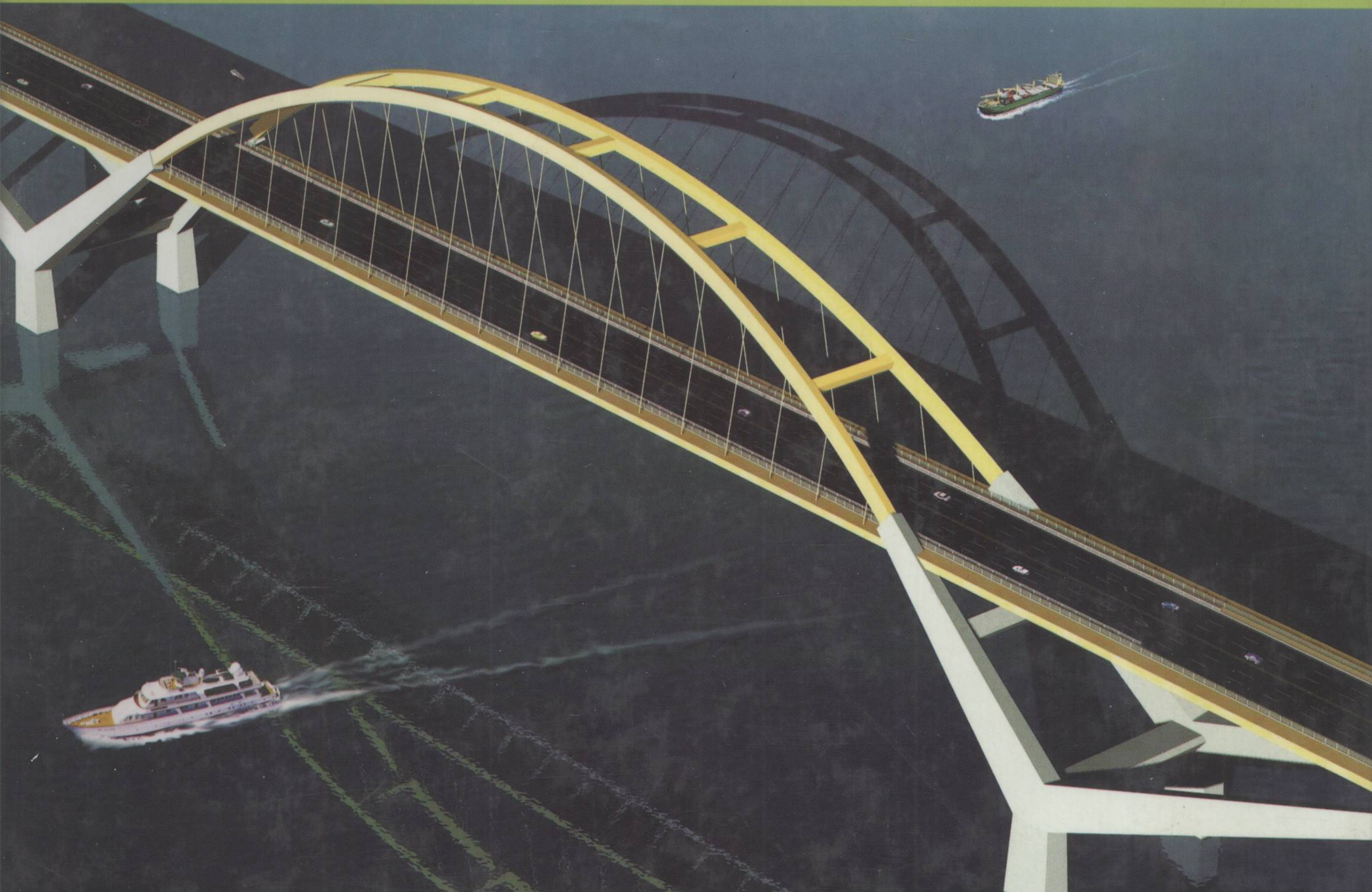
2nd Edition

PROBABILITY CONCEPTS IN ENGINEERING

Emphasis on Applications to Civil and Environmental Engineering



ALFREDO H-S. ANG • WILSON H. TANG

2ND EDITION

Probability Concepts in Engineering*

Emphasis on Applications in Civil & Environmental Engineering

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WILEY
JOHN WILEY & SONS, INC.

*This title is the 2nd edition of *Probability Concepts in Engineering Planning and Design*, Vol. I: Basic Principles.

ASSOCIATE PUBLISHER	Daniel Sayre
ACQUISITIONS EDITOR	Jennifer Welter
SENIOR PRODUCTION EDITOR	William A. Murray
MARKETING MANAGER	Frank Lyman
COVER DESIGN	Hope Miller
ILLUSTRATION COORDINATOR	Mary Alma
MEDIA EDITOR	Stefanie Liebman

COVER PHOTO
The modern-style Caiyuanba Bridge is a tie-arch bridge located in Chongqing, China over the Yangtze River. It has a main arch span of 420 meters with two decks. The upper deck carries six lanes of traffic and two pedestrian paths; the lower deck carries two monorail tracks. Both the girder and the box-arch ribs are constructed of steel.

The cover image was provided by T.Y. Lin International (San Francisco, California), designer of the main span of the Caiyuanba Bridge. The authors and publisher wish to express their thanks to T.Y. Lin International for the use of the image.

This book was set in Times Roman by TechBooks and printed and bound by Hamilton Printing Company. The cover was printed by Phoenix Color.

This book is printed on acid free paper. ♾

ISBN-13 978-0-471-72064-5
ISBN-10 0-471-72064-X

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

CONVERSION FACTORS: U.S. CUSTOMARY AND SI METRIC UNITS

U.S. Units	SI Metric Units	To Convert from U.S. Customary to SI, Multiply by
inch (in.)	meter (m)	0.0254
inch (in.)	centimeter (cm)	2.54
inch (in.)	millimeter (mm)	25.4
feet (ft)	meter (m)	0.305
yard (yd)	meter (m)	0.914
mile (mi)	kilometer (km)	1.609
angle, degree (°)	radian (rad)	0.0174
temp., degree (°F)	Celsius (°C)	(°F − 32)/1.80
acre (acre)	hectare (ha)	0.4052
gallon (gal)	liter (l)	3.79
pound (lb)	kilogram (kg)	0.4536
ton (ton, 2000 lb)	kilogram (kg)	907.2
pound force (lbf)	newton (N)	4.448
pound/in^2 (psi)	newton/m^2 (N/m^2)	6895
pound/ft^2 (psf)	newton/m^2 (N/m^2)	47.88
foot-pound (ft-lb)	joule (J)	1.356
horsepower (hp)	watt (W)	746
British thermal unit (BTU)	joule (J)	1055
British thermal unit (BTU)	kilowatt-hour (kwh)	2.93×10^{-4}

Definitions

1 newton—force that will give a 1-kg mass an acceleration of 1 m/sec^2
1 joule—work done by a force of 1 N over a displacement of 1 m
1 pascal = 1 N/m^2
1 kilogram force (kgf) = 9.807 N
1 gravity acceleration (g) = 9.807 m/sec^2
1 acre (a) = 4052 m^2
1 hectare (ha) = 10,000 m^2
1 kip (kip) = 1000 lb

Probability Concepts in Engineering

Emphasis on Applications in Civil & Environmental Engineering

Dedicated to Myrtle Mae and Bernadette

Preface

OBJECTIVES AND APPROACH

The first edition of this book (originally titled *Probability Concepts in Engineering Planning and Design,* Vol. I, Basic Principles, published in 1975) has served to provide the basics of probability and statistical concepts in terms that are more easily understood by engineers and engineering students. The basic principles are presented and illustrated through problems of relevance to engineering and the physical sciences (particularly to civil and environmental engineering), and the exercise problems in each chapter are designed to further enhance the understanding of the basic concepts and reinforce a working knowledge of the concepts and methods. We firmly believe that the easiest and most effective way for engineers to learn a new set of abstract principles, such as those of probability and statistics, to the extent as to be able to apply them in modeling and formulating engineering problems, is through varied illustrations of applications of such principles. Moreover, the first exposure of engineers to probabilistic concepts and methods should be in physically meaningful terms; this is necessary to properly emphasize and motivate the recognition of the significant roles of the relevant mathematical concepts in engineering.

NEW TO THIS SECOND EDITION

This second edition follows the same general approach expounded in the first edition; however, the text in all the chapters has been improved and has been thoroughly revised, updated, or completely replaced from that of the first edition. In particular, almost all the illustrative examples, as well as the problems, in each chapter of the first edition have been replaced with new ones. Also, where actual data are illustrated, they have been updated; more recent or new data are presented or added in this edition. Several new topics and sections have been added or expanded, including the following:

- **Two Types of Uncertainty** In this edition, we have emphasized (introduced in Chapter 1) the importance of distinguishing the two broad types of uncertainties; namely, the *aleatory* and the *epistemic* uncertainties, and the need to evaluate their respective significances separately in engineering applications. Engineers and engineering students need to be made aware of this approach, especially in practical engineering decision making. Nonetheless, the tools for such evaluations require the same basic principles of probability and statistics as presented here.
- **Extreme Values** Chapter 4 now includes the distributions of *extreme values,* which are of special interest to engineers dealing with natural and extreme hazards.
- **Hypothesis Testing** The topic of *Hypothesis Testing* is added as part of Chapter 6.
- **Anderson–Darling Method** The *Anderson–Darling* method for goodness-of-fit test is now included in Chapter 7 (of relevance when the tails of distributions are important).
- **Confidence Intervals in Regression Analysis** Linear regression in Chapter 8 has been expanded to include the determination of *confidence intervals.*
- **Regression and Correlation Analyses** Chapter 9 on Bayesian probability now includes Bayesian *regression and correlation analyses.*

- **Computer-Based Numerical and Simulation Methods** The new chapter (Chapter 5) on *Computer-Based Numerical and Simulation Methods in Probability* should make this second edition more in tune with modern-day engineering education. The numerical and simulation methods presented in this chapter, particularly with reference to Monte Carlo simulations, extend the practical applicability of probability concepts and methods for formulating and solving engineering problems, beyond those possible with purely analytical tools. These numerical methods are particularly powerful with the present-day availability of personal computers and related commercial software, and should serve to augment the analytical methods, thus extending the general usefulness and utility of probability and statistics in engineering.
- **Quality Assurance** The chapter on *Elements of Quality Assurance and Acceptance Sampling* (Chapter 9 of the first edition) is now designated as Chapter 10 but is available only on the Web. The material in this chapter is beyond the scope of this book, which is devoted to the basic fundamentals of probability and statistics; however, it is useful in the specialized area of quality assurance. For these reasons, the material in this chapter is made available online at the Wiley Web site, www.wiley.com/college/ang.

INTENDED AUDIENCE

The material in the book is intended for a first course on applied probability and statistics for engineering students at the sophomore or junior level, or for self-study, stressing probabilistic modeling and the fundamentals of statistical inferences. The primary aim is to provide an in-depth understanding of the fundamentals for the proper application in engineering. Only knowledge of elementary calculus is required, and thus the material can be taught to undergraduate engineering students at any level. It may be used for a course taught either in the engineering departments or offered for engineers by the departments of mathematics and statistics.

The book is self-contained and thus is also suitable for self-study by practicing engineers who desire a reading and working knowledge of the basic concepts and tools of probability.

SUGGESTED SYLLABUS

One-semester course A suggested outline for a one-semester (or one-quarter) course may be as follows: Chapter 1 (assigned as required reading with guidance from instructors) through Chapter 5 stressing the modeling of probabilistic problems, plus Chapters 6 through 8 stressing the fundamentals of inferential statistics, can be covered in a one-semester course.
One-quarter course For a one-quarter course, the same chapters may be covered with less emphasis on selected sections (e.g., discuss fewer types of useful probability distributions) and limit the number of illustrations in each chapter.
Senior-level course For a course at the senior level, all the chapters, including the first part of Chapter 9, may be covered in one semester.

The extensive variety of problems at the end of each chapter provides wide choices for class assignments and also opportunities for self-measuring a reader's understanding.

INSTRUCTOR RESOURCES

These instructor resources are available on the Instructor section of the Web site at www.wiley.com/college/ang. They are available only to instructors who adopt the text:

- **Solutions Manual:** Solutions to all the exercise problems in the text.

- **Image Gallery:** All figures and tables from the text, appropriate for use in PowerPoint presentations.

These resources are password protected. Visit the Instructor section of the book Web site to register for a password to access these materials.

MATHEMATICAL RIGOR

We have not emphasized mathematical rigor throughout the book; such rigor may be supplemented with treatises on the mathematical theory of probability and statistics. We are concerned mainly with the practical applications and relevance of probability concepts to engineering. The necessary mathematical concepts are developed in the context of engineering problems and through illustrations of probabilistic modeling of physical situations and phenomena. In this regard, only the essential principles of mathematical theory are discussed, and these principles are explained in non-abstract terms in order to stress their relevance to engineering. This is necessary and essential to enhance the appreciation and recognition of the practical significance of probability concepts.

MOTIVATION

Uncertainties are unavoidable in the design and planning of engineering systems. Properly, therefore, the tools of engineering analysis should include methods and concepts for evaluating the significance of uncertainty on system performance and design. In this regard, the principles of probability (and its allied fields of statistics and decision theory) offer the mathematical basis for modeling uncertainty and the analysis of its effects on engineering design.

Probability and statistical decision theory have especially significant roles in all aspects of engineering planning and design, including: (1) the modeling of engineering problems and evaluation of systems performance under conditions of uncertainty; (2) systematic development of design criteria, explicitly taking into account the significance of uncertainty; and (3) the logical framework for quantitative risk assessment and risk-benefit tradeoff analysis relative to decision making. Our principal aim is to emphasize these wider roles of probability and statistical decision theory in engineering, with special attention on problems related to construction and industrial management; geotechnical, structural, and mechanical design; hydrologic and water resources planning; energy and environmental problems; ocean engineering; transportation planning; and problems of photogrammetric and geodetic engineering.

The principal motivation for developing this revised edition of the book is our firm belief that the principles of probability and statistics are of fundamental importance to all branches of engineering, although the examples and exercise problems included in this text are mostly from civil and environmental engineering. These principles are essential for the quantitative analysis and modeling of uncertainties in the assessment of risk, which is central in the modern approach to decision making under uncertainty.

The concepts and methods expounded in this book constitute only the basics necessary for the proper treatment of uncertainties. These basic principles may need to be supplemented with more advanced tools for specialized applications. See Volume II of Ang and Tang (1984) for some of these advanced topics.

Over the years, we have received numerous compliments from former students and professional colleagues regarding the way we elucidated the concepts and methods in the first edition, particularly for those wishing to learn and apply the principles of probability and statistics. In this regard, we are encouraged that the first edition of this book has contributed to the education of several generations of engineering students, and of professional colleagues through self-studies. The work for this second edition of the book is also inspired by the hope that this work will continue to contribute to the education of future generations of engineering students in the practical roles and significance of probability and statistics in engineering, enhanced further nowadays by the general availability of personal computers and associated commercial software.

VOLUME II

The first edition of this text was published in two volumes. For the second edition, only Volume I (this text) is being revised. If you would like to obtain a copy of the original Volume II, you may contact Professor Ang directly at ahang2@aol.com.

ACKNOWLEDGEMENTS

Finally, it is our pleasure to acknowledge the many constructive comments and suggestions offered by the prepublication reviewers of our original manuscript, including:

C. H. Aikens, University of Tennessee

B. Bhattacharya, University of Delaware

V. Cariapa, Marquette University

A. Der Kiureghian, University of California, Berkeley

S. Ekwaro-Osire, Texas Tech University

B. Ellingwood, Georgia Institute of Technology

T. S. Hale, Ohio University

P. A. Johnson, Pennsylvania State University

J. Lee, University of Louisiana

M. Maes, University of Calgary

S. Mattingly, University of Texas, Arlington

P. O'Shaughnessy, The University of Iowa

C. Polito, Valparaiso University

J. R. Rowland, University of Kansas

Y. K. Wen, University of Illinois, Urbana-Champaign

as well as a number of other anonymous reviewers. Many of their suggestions have served to improve the final manuscript. We also greatly appreciate the many compliments from several of the reviewers, including the phrase "the authors seem to understand what Socrates knew a long time ago . . . 'Analytical tools that are understood have a higher probability of being used' "; relating our work to Socrates certainly represents the height of compliments. Last but not least, our thanks to T. Hu, H. Lam, J. Zhang and L. Zhang for their assistance in the solutions to some of the examples, and in the preparation of the Solutions Manual for the problems in the book.

A. H-S. Ang and W. H. Tang

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CHAPTER 1

Roles of Probability and Statistics in Engineering

1.1 INTRODUCTION

In dealing with real world problems, uncertainties are unavoidable. As engineers, it is important that we recognize the presence of all major sources of uncertainty in engineering. The sources of uncertainty may be classified into two broad types: (1) those that are associated with natural randomness; and (2) those that are associated with inaccuracies in our prediction and estimation of reality. The former may be called the *aleatory* type, whereas the latter the *epistemic* type. Irrespective of the type of uncertainty, probability and statistics provide the proper tool for its modeling and analysis. In the ensuing chapters we will present the fundamental principles of probability and statistics, and illustrate their applications in engineering-type problems. The main aim of this work is to present the concepts and methods of probability and statistics for the modeling and formulation of engineering problems under uncertainty; this is in contrast to books that are devoted to statistical data analysis, although the fundamentals of statistics are also presented here.

The effects of uncertainties on the design and planning of an engineering system are important, to be sure; however, the quantification of such uncertainty and the evaluation of its effects on the performance and design of the system, should properly include the concepts and methods of probability and statistics. Furthermore, under conditions of uncertainty, the design and planning of engineering systems involve risks, which involves probability and associated consequences, and the formulation of related decisions may be based on quantitative risk-benefit trade-offs which are properly also within the province of applied probability and statistics. In this light, and with reference to problems containing randomness and uncertainty, the significance of the concepts of probability and statistics in engineering parallels those of the principles of physics, chemistry, and mechanics in the formulation and solution of engineering problems.

In light of the above, we see that the role of probability and statistics is quite pervasive in engineering; it ranges from the description of basic information to the development and formulation of bases for design and decision making. Specific examples of such imperfect information, and of applications in engineering design and decision-making problems, are described in the following sections.

1.2 UNCERTAINTY IN ENGINEERING

The presence of uncertainty in engineering, therefore, is clearly unavoidable; the available data are often incomplete or insufficient and invariably contain variability. Moreover, engineering planning and design must rely on predictions or estimations based on idealized models with unknown degrees of imperfections relative to reality, and thus involve additional uncertainty. In practice, we might identify two broad types of uncertainty: namely, (i) uncertainty associated with the randomness of the underlying phenomenon that is exhibited as variability in the observed information, and (ii) uncertainty associated with imperfect models of the real world because of insufficient or imperfect knowledge of reality. As we said earlier, these two types of uncertainty may be called, respectively, the *aleatory uncertainty* and the *epistemic uncertainty*. See Ang (1970, 2004) for a basic framework for defining and treating these two types of uncertainties. The two types of uncertainty may be combined and analyzed as a total uncertainty, or treated separately. In either case, the principles of probability and statistics apply equally.

We might point out that there are good reasons to view the significance of the two types of uncertainty, and their respective effects on engineering, differently. First of all, the aleatory (databased) uncertainty is associated with the inherent variability of basic information, which is part of the real world (within our ability to observe and describe). Much of the aleatory uncertainty that civil engineers must deal with are inherent in nature and, therefore, may not be reduced or modified. On the other hand, epistemic (or knowledge-based) uncertainty is associated with imperfect knowledge of the real world, and may be reduced through application of better prediction models and/or improved experiments. Second, the respective consequences of these two types of uncertainty may also be different—the effect of the aleatory randomness leads to a calculated probability or risk, whereas the effect of the epistemic type expresses an uncertainty in the estimated probability or risk. In many application areas of engineering and the physical sciences, the uncertainty (or error bounds) of a calculated risk or probability is as important as the risk itself; e.g., the National Research Council (1994) has emphasized the importance of quantifying the uncertainty in the calculated risk, and a number of U.S. government agencies, such as the U.S. Department of Energy (1996), the Environmental Protection Agency (1997), NASA (2002), NIH (1994), as well as in the UK (2000), have applied this approach in the quantitative assessment of risk. In some practical applications, however, the two types of uncertainty are combined and their aggregate effects estimated accordingly. Again, irrespective of whether the two types of uncertainties are combined or treated separately, the concepts and methods covered in the ensuing chapters are equally applicable.

Finally, there should be no problem in delineating between the two types of uncertainty—the aleatory type is essentially databased, whereas the epistemic type is knowledge based. For practical purposes, the epistemic uncertainty may be limited to the estimation of the mean or median values, even though in theory it includes inaccuracies in the prescribed form of probability distributions and in all the parameters.

1.2.1 Uncertainty Associated with Randomness—the Aleatory Uncertainty

Many phenomena or processes of concern to engineers, or that engineers must contend with, contain randomness; that is, the expected outcomes are unpredictable (to some degree). Such phenomena are characterized by field or experimental data that contain significant *variability* that represents the natural randomness of an underlying phenomenon; i.e., the observed measurements are different from one experiment (or one observation) to another, even if conducted or measured under apparently identical conditions. In other words, there is a range of measured or observed values of the experimental results; moreover, within

TABLE 1.1 Rainfall Intensity Data Recorded over a Period of 29 Years

Year No.	Rainfall Intensity, in.	Year No.	Rainfall Intensity, in.	Year No.	Rainfall Intensity, in.
1	43.30	11	54.49	21	58.71
2	53.02	12	47.38	22	42.96
3	63.52	13	40.78	23	55.77
4	45.93	14	45.05	24	41.31
5	48.26	15	50.37	25	58.83
6	50.51	16	54.91	26	48.21
7	49.57	17	51.28	27	44.67
8	43.93	18	39.91	28	67.72
9	46.77	19	53.29	29	43.11
10	59.12	20	67.59		

this range certain values may occur more frequently than others. The variability inherent in such data or information is statistical in nature, and the realization of a specific value (or range of values) involves probability. The inherent variability in the observed or measured data can be portrayed graphically in the form of a *histogram or frequency diagram*, such as those shown in Figs. 1.1 through 1.23, all of which demonstrate information on physical phenomena of relevance particularly to civil and environmental engineering. Furthermore, if two variables are involved, the joint variability may similarly be portrayed in a *scattergram.*

A histogram simply shows the relative frequencies of the different observed values of a single variable. For example, for a specific set of experimental data, the corresponding histogram may be constructed as follows.

From the range of the observed data set, we may select a range on one axis (for a two-dimensional graph) that is sufficient to cover the largest and smallest values among the set of data, and divide this range in convenient intervals. The other axis can then represent the number of observations within each interval among the total number of observations, or the fraction of the total number. For example, consider the 29 years of annual cumulative rainfall intensity in a watershed area recorded over a period of 29 years as presented in Table 1.1.

An examination of these data will reveal that the observed rainfall intensities range from 39.91 to 67.72 in. Therefore, choosing a uniform interval of 4 in. between 38 and 70 in. the number of observations within each interval and the corresponding fraction of the total observations are calculated as summarized in Table 1.2.

The uniform intervals indicated in Table 1.2 may then be scaled on the abscissa, and the corresponding number of observations (column 2 in Table 1.2) can be shown as a bar

TABLE 1.2 Number and Fraction of Total Observations in Each Interval

Interval	No. of Observations	Fraction of Total Observations
38–42	3	0.1034
42–46	7	0.2415
46–50	5	0.1724
50–54	5	0.1724
54–58	3	0.1034
58–62	3	0.1034
62–66	1	0.0345
66–70	2	0.0690

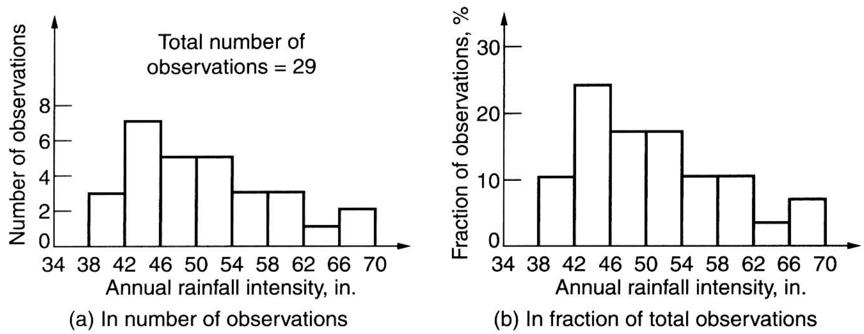

Figure 1.1 Histograms of Annual Rainfall Intensity.

on the vertical axis, as illustrated in the histogram of Fig. 1.1a for the rainfall intensity of the watershed area. Alternatively, the vertical bar may be in terms of the fraction of the total observations (column 3 in Table 1.2) and would appear as shown in Fig. 1.1b. Oftentimes, there may be reasons to compare an empirical frequency diagram, such as a histogram, with a theoretical frequency distribution (such as a *probability density function*, PDF, discussed later in Chapter 3).

For this purpose, the area under the empirical frequency diagram must be equal to unity; we obtain this by dividing each of the ordinates in a histogram by its total area; e.g., we obtain the empirical frequency function of Fig. 1.1a by dividing each of the ordinates by $29 \times 4 = 116$; whereas the corresponding empirical frequency function may also be obtained from Fig. 1.1b by dividing each of the ordinates by $4 \times 1 = 4$. In either case, we would obtain the empirical frequency function of Fig. 1.1c for the rainfall intensity in

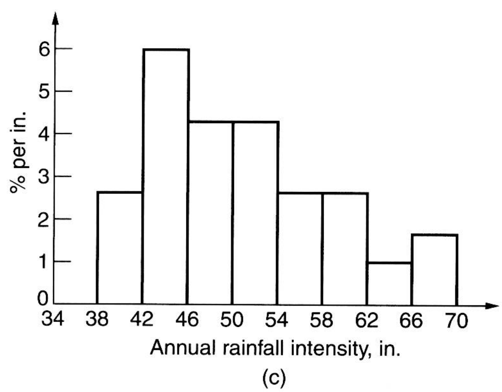

Figure 1.1c Empirical Frequency Function.

the watershed area. We may then observe that the total area under the empirical frequency function is equal to 1.0, and thus the area over a given range may be used to estimate the probability of rainfall intensity within the given range.

A large number of physical phenomena are represented in Figs. 1.1 through 1.23; these are purposely collected here to demonstrate and emphasize the fact that the state of most engineering information contains significant variability. For examples, the properties of most materials of construction vary widely; in Figs. 1.2 and 1.3 we present the histograms demonstrating the variabilities in the bulk density of soils and the water–cement (w/c) ratio of concrete specimens, respectively, whereas in Figs. 1.4 and 1.5 are shown the yield strength of reinforcing bars and the ultimate shear strength of steel fillet welds.