

CONTEMPORARY LINEAR SYSTEMS

— *using* **MATLAB[®]** —

Robert D. Strum

Donald E. Kirk

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BookWare Companion Series™

CONTEMPORARY LINEAR SYSTEMS **USING MATLAB®**

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LIST OF NOTATION

$\delta(t), \delta(n)$	Unit impulse, unit sample	$u(t), u(n)$	Unit step, continuous/discrete
LTI	Linear, time-invariant system	$x(t), x(n)$	System inputs
$y(t), y(n)$	System outputs	$h(t), h(n)$	Unit impulse and unit sample response
DE	System differential/difference equation		

$$\sum_{k=0}^N a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^L b_k \frac{d^k x(t)}{dt^k} \quad \text{and}$$

$$\sum_{k=0}^N a_k y(n - k) = \sum_{k=0}^L b_k x(n - k)$$

$\mathbf{v}(t), \mathbf{v}(n)$ State-variable vector, continuous/discrete

Matrix DE State or matrix differential/difference equation

$$\dot{\mathbf{v}}(t) = \mathbf{A}\mathbf{v}(t) + \mathbf{B}\mathbf{x}(t) \quad \text{and}$$

$$\mathbf{v}(n + 1) = \mathbf{A}\mathbf{v}(n) + \mathbf{B}\mathbf{x}(n)$$

$\mathbf{y}(t), \mathbf{y}(n)$ System output equation, continuous/discrete

$$\mathbf{y}(t) = \mathbf{C}\mathbf{v}(t) + \mathbf{D}\mathbf{x}(t) \quad \text{and}$$

$$\mathbf{y}(n) = \mathbf{C}\mathbf{v}(n) + \mathbf{D}\mathbf{x}(n)$$

$F(s), f(t)$ Bilateral Laplace transform and inverse

$$\mathcal{L}[f(t)] = F(s) = \int_{-\infty}^{\infty} f(t)e^{-st} dt \quad \text{and}$$

$$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)e^{st} ds$$

$F(z), f(n)$ Bilateral z transform and inverse

$$\mathcal{Z}[f(n)] = F(z) = \sum_{n=-\infty}^{\infty} f(n)z^{-n} \quad \text{and}$$

$$\mathcal{Z}^{-1}[F(z)] = f(n) = \frac{1}{2\pi j} \oint_C F(z)z^{n-1} dz$$

About the Series

“The purpose of computing is insight, not numbers.”

—R. W. Hamming, *Numerical Methods for Engineers and Scientists*,
McGraw-Hill, Inc.

It is with this spirit in mind that we present the BookWare Companion Series.TM

Increasingly, the latest technologies and modern methods are crammed into courses already dense with important theory. As a result, many instructors now ask, “Are we simply teaching students the latest technology, or are we teaching them to reason?” We believe that these two alternatives need not be mutually exclusive. In fact, this series was founded on the belief that computer solutions and theory can be mutually reinforcing. Properly applied, computing can illuminate theory and help students to think, analyze, and reason in meaningful ways. It can also help them to understand the relationships and connections between new information and existing knowledge and to cultivate problem-solving skills, intuition, and critical thinking. The BookWare Companion Series was developed in response to this mission.

Specifically, the series is designed for educators who want to integrate computer-based learning tools into their courses, and for students who want to go further than their textbook alone allows. The former will find in the series

the means by which to use powerful software tools to support their course activities without having to customize the applications themselves. The latter will find relevant problems and examples quickly and easily available and will have electronic access to them. Important for both educators and students is the premise on which the series is based: that students learn best when they are actively involved in their own learning. The BookWare Companion Series will engage them, provide a taste of real-life issues, demonstrate clear techniques for solving real problems, and challenge them to understand and apply these techniques on their own.

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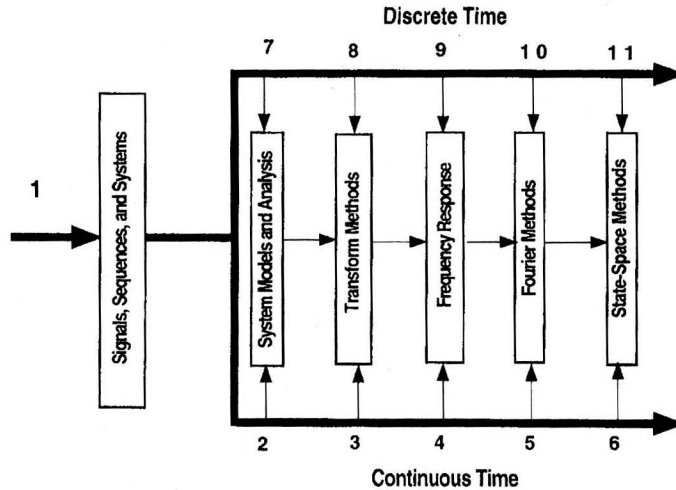
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You can recommend ways to make the series even better, share your ideas about using technology in the classroom with your colleagues, suggest a specific problem or example for the next edition, or just let us know what's on your mind. We look forward to hearing from you, and we thank you for your continuing support.

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Preface

Computers have been an indispensable engineering tool for more nearly four decades. They are used extensively for analysis and design and often as system components. Recently, inexpensive and easy-to-use software packages have become widely available, making it possible to focus on the problem being solved rather than on the programming necessary to obtain a solution. This book is built around one such software package, MATLAB[®], which is in widespread use in both academia and industry.

Our goal is to provide an effective and efficient environment for students to learn the theory and problem-solving skills for linear systems. The material is designed to provide appropriate background to proceed into areas such as communications, control systems, digital filter design and signal processing, and analog filter design.

To accomplish this we have used a “computer-biased” approach in which computer solutions and theory are viewed as mutually reinforcing rather than as an either-or proposition. We adhere to the axiom that one learns by doing rather than by listening, and, consequently, more than 200 problems are provided for the reader to solve. We also believe that students need feedback on their work, preferably sooner rather than later. To that end, answers for all problems are either in the book or are available as MATLAB scripts at the BookWare Companion Resource Center, online at <http://www.brookscole.com/engineering/ee/bookware.html>. We also have provided the briefest of introductions to the Symbolic Math Toolbox. For a complete presentation of the use of this toolbox, we recommend appropriate publications of The MathWorks, Inc.

The prerequisites are a basic background in calculus, experience with the formulation of differential equations, ability to manipulate complex numbers, and some familiarity with matrix operations. The background that comes with the completion of a first course in electrical engineering is also helpful. It is also assumed that the reader has used a computer. While expertise in programming is not required, familiarity with a programming language such as Basic, Pascal, Fortran, or C is helpful. In addition, the capability to perform some fundamental

computer operations, such as creating, editing, printing, and managing files, is needed.

BOOK ORGANIZATION

We begin with an introductory chapter (Chapter 1) that presents the basics of how signals and sequences are described, both analytically and graphically. Some fundamental concepts, nomenclature, and notation for linear continuous-time and discrete-time systems are also discussed. Chapter 2 introduces several important aspects of continuous systems, including differential equations, unit impulse response, convolution, and state-space models. Laplace transforms are presented in Chapter 3 and are used to solve differential equations, perform convolution, and draw signal flowgraphs or block diagrams. Chapter 4 describes frequency response methods for continuous-time systems. Fourier series and Fourier transform methods for continuous systems are presented in Chapter 5, and these techniques are applied to determine the output of linear systems when periodic or nonperiodic signals are applied as inputs. State-variable methods for continuous systems are discussed in Chapter 6.

Chapter 7 begins the coverage of discrete-time systems with a discussion parallel to that of Chapter 2 for continuous systems. Included are difference equations, unit sample response, and state-space models, as well as the convolution operation. In Chapter 8 we enter the z transform domain and learn how to use z transforms to solve difference equations, carry out convolution, and create a signal flowgraph or block diagram model. Frequency response of discrete systems is the topic of Chapter 9 where we also see a simple approach for designing digital filters. Chapter 10 presents Discrete Fourier Transforms (DFTs) and introduces the Fast Fourier Transform (FFT) algorithm for finding DFTs. State-space techniques for discrete systems are introduced in Chapter 11 where we see that solutions can be obtained in either the time or z transform domains.

Although the principal coverage of state-space methods is given in Chapters 6 and 11, we have presented enough of the basics in Chapters 2 and 7 to enable the reader to use state-equation-oriented MATLAB functions.

USING THE BOOK

We are advocates of studying discrete systems first and have previously stated the case for doing this.¹ In fact, in an earlier draft Chapters 7–11 on discrete

¹R. D. Strum and D. E. Kirk, “Linear Systems: Be Discrete—Then Continuous,” *IEEE Transactions on Education*, vol. 32, no. 3, pp. 335–342, August 1989.

systems were Chapters 2–6. So why the change? We decided that in spite of our preference, the typical electrical engineering curriculum is strongly biased in its early phases toward continuous-time systems with differential and integral calculus, differential equations, and analog circuit theory. Even fearless authors like us shrink from the task of trying to re-orient the entire curriculum, though there are indications that it is beginning to happen. So, persuaded by comments from several reviewers, we decided to cover continuous-time systems first. We did, however, attempt to minimize cross references (leaving these to the instructor) so that it is quite natural to study the material in the order: Chapters 1–11; or Chapter 1, Chapters 7–11, Chapters 2–6. Another arrangement, which is based upon parallel coverage of continuous and discrete system concepts, would start with Chapter 1 (introductory material), followed by the chapter pairs (in either order) of 2 and 7 (time-domain treatment of continuous and discrete systems), 3 and 8 (Laplace and z transform applications), 4 and 9 (frequency response), 5 and 10 (Continuous and Discrete Fourier Transforms), and 6 and 11 (state-space topics).

CHAPTER ORGANIZATION

Each chapter is organized in the following format.

Preview: An overview, motivation, and an historical perspective.

Basic Concepts: A development of the important concepts and relationships, including *Illustrative Examples* to clarify the concepts and computational details.

Solved Examples and MATLAB Applications: Comprehensive illustrations of problem-solving techniques and additional concepts.

Reinforcement and Exploration Problems: Practice in applying the basic concepts (reinforcement) or guidance for the reader to probe new issues or make extensions (exploration).

Definitions, Techniques, and Connections: A summary of the important relationships of the chapter.

MATLAB Functions Used: A list of the computer functions that were used in the chapter.

Annotated Bibliography: Sources of additional information together with a brief statement of where the relevant topics are covered.

Answers: Results for all problems.

In addition, after Chapters 4, 6, 9, and 11, *Retrospectives* are provided which attempt to integrate the material by pausing and reconsidering the interrelationships of the material that has gone before.

Note: It should be emphasized that the *Solved Examples and MATLAB Applications* sections are important reading. Frequently, a point covered lightly, or not at all, in the *Basic Concepts* section of a chapter is amplified or introduced in a solved example. It is recommended that the reader attempt to work these solved examples as if solutions were not available, using the solutions to check or to get back on track when necessary.

BOOKWARE COMPANION RESOURCE CENTER WEBSITE

The website online at <http://www.brookscole.com/engineering/ee/bookware.html> is an important adjunct to the text and provides MATLAB scripts in both Macintosh and Windows formats. The Macintosh version contains a folder of Some More Functions and 11 other folders (one for each chapter) that have the scripts for answers to problems, solved examples, chapter figures plotted with MATLAB, and plots used as problem statements. The Windows version has a directory containing the Some More Functions and a second directory with the script files for all 11 chapters. On both versions, the Some More Functions augment the capabilities of MATLAB. For both the Macintosh and Windows versions, the script files for the chapters begin with the letters A (Answers), E (Examples), F (Figures), and P (Problems) followed by a one- or two-digit number 1 through 11 corresponding to the chapter. The final characters indicate the number of the appropriate answer (A5_2), example (E7_4), figure (F4_2), or problem (P8_20). The underbar “_” character is used where a period “.” appears in a text caption because MATLAB doesn’t like periods in file names. All run with the *Student Edition of MATLAB*.

To execute the scripts discussed above requires that either the *Student Edition of MATLAB* or the Professional version is installed on your system. Experience has shown that it is relatively easy to install MATLAB if the installation directions provided by the documentation are followed. A discussion of how to get started with MATLAB is found in Appendix A.

OPEN MANUSCRIPT

It is our goal that this book be an “open” manuscript. In that spirit, the user sees everything, and we hope that faculty members and students pass on their suggestions for new and interesting problems and examples. These recommendations will receive careful consideration for inclusion in future versions of the book. And, of course, anything we use will be specifically acknowledged. The open-manuscript approach will also be fostered by publishing updated versions more frequently than traditional revised editions. The original *Contemporary Linear Systems Using MATLAB*, for example, was published in 1994.

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Tom Robbins, the initiator and former editor of the BookWare Companion Series, deserves a great deal of credit (and no blame) for our finished product.

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Carmel, California

R. D. S.

D. E. K.

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