MATHEMATICAL ASPECTS of COMPUTER ENGINEERING

Edited by
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K. A. VOLOSOV

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PREFACE

The present collection of articles is the result of many years of research conducted by our team into various aspects of designing and building the component base of promising high-speed computational systems. The articles deal with the following topics:

(a) the optimal design and functioning of parallel computational systems, (b) the optimal recognition of optical and acoustic fields in synthesizing an optimal dynamic analyzer, and (c) the modeling of nonlinear transfer processes in the component base of a computer.

We discuss new mathematical methods that can be applied in solving specific problems arising in the construction of mathematical models for handling the above-mentioned three topics. Although various countries have developed devices and technological processes for creating new generations of computers, there is still no general theoretical approach. In this respect the present collection fills an important gap in the literature on the subject.

All results set forth in this collection are new and obtained only recently. Here we give a brief survey.

The article written by S. M. Avdoshin, V. V. Belov, V. P. Maslov, and A. M. Chebotarev is devoted to constructing a theory of the optimization problems that emerge in the development of the architecture, the organization of parallel computations, and the design of flexible manufacturing systems for homogeneous multiprocessor computational systems. The following concept lies at the base of the suggested approach: all the optimization problems considered here are linear in a space of functions with values in semirings. Depending on the choice of the semimodule, for instance, the Hamilton-Jacobi equation and the Bellman equation prove to be linear in the new sense. For these equations analogs of the Duhamel principle and the Fredholm alternatives prove valid. This concept leads to a new definition of an integral corresponding to the semigroup operation of the "sum" type, the concept of a measure additive in this new sense, and an analog of the scalar product (say, $(\varphi_1, \varphi_2) = \min_x [\varphi_1(x) + \varphi_2(x)]$ or $(\varphi_1, \varphi_2) = \max_x [\min_x \varphi_1(x), \varphi_2(x)]$), which makes it possible to go over to adjoint operators and define functions "generalized" in the new sense. On the basis of the "linearity" concept and the notion of generalized convergence in optimization problems dealing with homogeneous computational systems, we consider the passage to the limit in a natural large parameter proportional to the number of elementary processors in the computational system. For a broad class of problems the solutions to the limiting equations can he set up on the basis of Pontryagin's maximum principle.

The article by V. P. Belavkin and V. P. Maslov has a direct bearing on the problem of mathematical synthesis of an optimal dynamic analyzer, a device intended for automatic sound recognition. As is known, establishing a verbal link between man and computer is one of the key problems in the design of fifth-generation computational systems. The article provides a systematic exposition of the wave theory of representations and measurements, the theory that is based on similarities with quantum mechanics and used to solve problems of detection, separation, identification, and estimation of the parameters of acoustic and visual images within the framework of the noncommutative theory of wave hypothesis testing. The idea of applying quantum mechanics to the problem of recognizing wave images emerged at the beginning of the 1970s, when a seminar devoted to quantum mechanics and image recognition was opened in the Physics Department of Moscow State University under the direction of Yu. P. Pyt'ev and this author.

The article by V. G. Danilov, V. P. Maslov, and K. A. Volosov is directly related to the key issue of creating the component base of computers, namely, the calculation and design of new technological methods for the various stages of designing integrated circuits and other computer elements. Mathematical modeling in this case is a preliminary stage. Most of the modeling problems can be reduced to a quasilinear parabolic equation or a system of such equations.

The article suggests new methods for building asymptotic equations to quasilinear parabolic equations. From the mathematical view the class of problems considered is characterized by two effects: localization of a perturbation and the finiteness of the speed with which the perturbation travels. In other words, the support of the solution is a compact set or a semibounded set, and the boundary of the support propagates with a certain speed. At the support boundary the solution undergoes a weak discontinuity; hence, along with the problem of constructing the asymptotics in a small parameter there emerges the problem of the propagation of the singularity (the weak discontinuity). The theory developed in the article is applied to calculating such processes as diffusion, heat conduction, turbulent filtration, adsorption (desorption), epitaxy, and film flow. Application of the findings to the various stages of the technological processes reduces designing time and production costs.

The abundance of basically new material, that is, new methods, notions, definitions, etc., must have posed certain difficulties in preparing the manuscript for print. For this reason I would like to express my sincere gratitude to Mir Publishers for undertaking to introduce the foreign reader to the achievements in this field of knowledge. In particular, I would like to thank the staff of the mathematics editorial office for preparing the Russian version of the manuscript for translation and the English physics and mathematics editorial office for the expert translation.

January 1988 *Academician V. P. Maslov*

1 Design of Computational Media: Mathematical Aspects

S. M. Avdoshin, V. V. Belov, V. P. Maslov,
and *A. M. Chebotarev*

1.0 A Brief Survey

In the present article we aim at a solution of certain problems associated with the architecture and analysis of parallel programs and flexible manufacturing systems (FMS) for homogeneous multiprocessor computational systems (CS), which are characteristic of fifth-generation computers.

These problems constitute examples of optimization problems containing a natural large parameter proportional to N, the number of elementary processors in the CS. As a rule, the complexity of the solution algorithms for these problems increases rapidly with N (at least like N^2). This brings us to the problem of the limiting transition as $N \to \infty$, that is, a limit problem whose solution does not depend on N and approximates the solution of the initial problem all the better as N increases. In some important specific cases this limit problem can be associated with the Bellman equation [1.1]. As a rule, however, even for smooth initial data the limit equation has no differentiable solutions (except for a small number of extremely special problems). Hence, the classical statement of the Cauchy problem for this equation usually has no meaning. More than that, the Bellman equation does not even have generalized solutions in the usual sense. Hence only Pontryagin's maximum principle applied to such cases has a clearly defined mathematical meaning. This principle has been used in solving the corresponding optimization problems [1.2].

The general approach suggested in this paper to solving optimization problems related to multiprocessor computers can also be applied to optimization problems of an entirely different nature. This approach is based on the fact that all optimization problems considered here are "linear" in function spaces whose elements have values in certain semi-rings. Here is what this means. Let us consider a function space in which the common operations of addition and multiplication by numbers are replaced with other semi-group operations, $\oplus$ and $\odot$, related through the distributivity law. For instance, instead of the sum of two functions we take their supremum, and instead of the product of a function by a number we take the infimum. The linearity of equations in such spaces means that, if $y_1(x)$ and

$y_2(x)$, $x \in X$, are solutions, then $\sup_{x\in X}(y_1(x),\ y_2(x))$ and $\inf_{x\in X}(y_1(x), \lambda)$ or $\inf_{x\in X}(y_2(x),\ \lambda)$, $\lambda = \text{const}$, are also solutions. Next we introduce the concept of an "integral" corresponding to a semi-group operation of the "sum" type, the concept of measure that is additive in this new sense, and an analog of the scalar product, which makes it possible to introduce conjugate operators and define functions that are "generalized" in the new sense. For example, the scalar product in a space of functions with values in a semi-ring A where the sum is replaced with min and the product with the common sum, $+$, has the form

$$\langle \varphi_1,\ \varphi_2\rangle = \min_{x\in X}(\varphi_1(x) + \varphi_2(x)) \overset{\text{def}}{=} \int_{\oplus} \varphi_1(x) \odot \varphi_2(x)\, dx.$$

In this space, for the Hamilton-Jacobi equation

$$\frac{\partial u}{\partial t} + H\left(x,\ \frac{\partial u}{\partial t},\ t\right) = 0 \qquad (*)$$

we have the superposition principle for solutions, that is, if y_1 and y_2 are solutions, then $\lambda_1 \odot y_1 \oplus \lambda_2 \odot y_2$, with $\lambda_i = \text{const}$ $(i = 1, 2)$, are also solutions. This leads to a formula that represents a solution of the equation in terms of a source, or

$$u(x,\ t) = \int_{\oplus} k(x,\ \zeta,\ t) \odot u_0(\zeta)\, d\zeta$$
$$= \min_{\zeta\in X}(k(x,\ \zeta,\ t) + u_0(\zeta)), \qquad (1.0.1)$$

where $k(x,\ \zeta,\ 0) = \delta(x - \zeta)$, and $\delta(x - \zeta)$ is understood to be the functional $\min_{\xi\in X}(\delta(x - \zeta) + \varphi(\zeta)) = \varphi(x)$, say, $\delta(x - \zeta) = \lim_{\varepsilon\to 0}[(x - \zeta)^2/\varepsilon]$. It is easy to see that $k(x,\ \zeta,\ t) = \min \int \mathcal{L}\, dt$, where $\mathcal{L}$ is the Lagrangian, and formula (1.0.1) proves to be the well-known representation of a solution to the Hamilton-Jacobi equation (*) in the small in terms of a generating function.

The "Fourier transform" in a space of functions with the values in a semi-ring A is the eigenfunction expansion of the translation (or shift) operator T_Δ, that is, $T_\Delta\varphi(x) = \varphi(x + \Delta)$; the eigenfunctions $\psi_\mu(x)$ of T_Δ have the form μx: $T_\Delta \mu x = \mu(x + \Delta x) = \mu\Delta + \mu x = \mu\Delta \odot \mu x$, and the corresponding eigenvalues are $\mu\Delta$. The "Fourier transform" of a function $\varphi(x)$ has the form $\int_{\oplus} \psi_\mu(x) \odot \varphi(x)\, dx = \min_{x\in X}(\mu x + \varphi(x))$ and in the case at hand coincides with the Legendre transform, which is "linear" in this space. It has been established that if $H(p,\ x,\ t)$ is a function homogeneous of degree one in p, then the Cauchy problem for equation (*) is also "linear" in the space of functions with the values in the semi-ring A: $\oplus = \min$, $\odot = \max$. The Cauchy problem for the Bellman equation also proves to be "linear" in appropriate function spaces of functions with values in a semi-ring A.

The general differential equation in spaces of functions of a continuous argument that generalizes both the Bellman equation and the Hamilton-Jacobi equation is an equation for which the resolving operator is linear in function spaces with values in the appropriate semi-rings and whose solution is generalized in the above-mentioned new sense (that is, is a "linear" continuous functional with respect to the new "scalar product"). We will call this equation the generalized Hamilton-Jacobi equation and in the discrete case the generalized Bellman equation. For such equations there exists an analog of Fredholm alternative theorems. For a broad class of problems the solutions of these equations can be constructed using Pontryagin's maximum principle as a basis.

The concept described above makes it possible to determine the limiting values when $N \to \infty$ and overcome the difficulty that arises from the fact that usually the solutions of such problems assume only two values, 0 and 1. For the sake of comparison we first turn to the linear case, where the discrete problem converges to a continuous one.

Example 1. Suppose a discrete problem is described by the difference scheme

$$a_n^{k+1} = La_n^k, \quad n \in \mathbb{Z}, \quad k \in \mathbb{Z}_+, \tag{1.0.2}$$

where L is a linear difference operator with constant coefficients on an integer lattice, with the initial value a_n^0 a nonzero constant (say, c) for $n \geqslant 0$ and zero for $n < 0$. This problem has no limit in the ordinary sense of the word as $k \to \infty$. Nevertheless, the concept of a generalized solution introduced by S. L. Sobolev makes it possible to find the weak limit of the solution to this problem. To this end we take a family of functions of continuous independent variables, $v_h(t, x)$, $t \in [0, T]$, $T = \text{const}$, $x \in R^1$, that depends on parameter $h \in (0, 1]$ and is such that

$$v_h(kh, nh) = a_n^k.$$

With the initial problem (1.0.2) we associate the problem for $v_h(t, x)$:

$$v_h(t+h, x) = \widetilde{L}_h v_h(t, x), \quad v_{h\,|t=0} = v^0(x), \quad t = kh. \tag{1.0.2$'$}$$

Here $\widetilde{L}_h$ is the natural continuation of L on functions of a continuous independent variable. For example, if $La_n^k = \sum_i c_i a_{n+i}^k$, then

$\tilde{L}_h v_h(t, x) = \sum_i c_i v_h(t, x + ih)$. The condition $k \to \infty$ is equivalent to $h \to 0$, since $kh \leqslant T$. Let us assume that on smooth functions the operator $(\tilde{L}_h)^l$ converges to the operator e^{tL_0} as $l \to \infty$, $lh \to t$, where L_0 is a linear differential operator that, as $h \to 0$, is approximate on smooth functions by the operator $[\tilde{L}_h - 1]/h$. Then, if the initial value $v^0(x)$ is a smooth function, the family of functions $v_h(lh, x)$ converges (in $C(R_x^1)$) to the solution $v(t, x)$ of the differential equation

$$\frac{\partial v}{\partial t} = L_0 v, \quad v|_{t=0} = v^0(x). \tag{1.0.3}$$

If $v^{(0)}(x)$ is a discontinuous function, the solution to the difference problem converges to a solution of the differential equation in the sense of generalized functions. Indeed, for any smooth finite function φ we have

$$\begin{aligned}(v_h(lh, x), \varphi) &= ((\tilde{L}_h)^l v^0, \varphi) = (v^0, (\tilde{L}_h^*)^l \varphi)\\ &\xrightarrow[h=0]{} (v_0, e^{tL_0^*}\varphi) \overset{\text{def}}{=} (e^{tL_0}v^0, \varphi) = (v(t, x), \varphi)\\ &\overset{\text{def}}{=} \int \varphi(x)\, v(t, x)\, dx,\end{aligned}$$

where $v(t, x)$ is a generalized solution to problem (1.0.3).

We have therefore found that the weak limit of the solution to a difference problem is a generalized solution to the respective limiting equation, to which the initial difference equation converges only on smooth functions.

Let us now study the analogy between the example just considered and the solution to the respective discrete optimization problem.

Example 2. Consider the process $\{a^k, k = 0, 1, \ldots\}$ with a discrete space of states $\mathbb{Z} \times \mathbb{Z}$, $a^k: \mathbb{Z} \times \mathbb{Z} \to R^1$, and satisfying the Bellman equation

$$\begin{aligned}&a^{k+1}(m, n) = \min\{a^k(m-1, n), a^k(m, n-1)\},\\ &n, m \in \mathbb{Z}, \quad k = 0, 1, \ldots,\end{aligned} \tag{1.0.4}$$

and the initial data of the form

$$a^0(m, n) = \begin{cases} 0 \text{ if } n = 0, m \geqslant 0 \text{ or } m = 0, n \geqslant 0, \\ +\infty \text{ otherwise.}\end{cases} \tag{1.0.5}$$

Note that this equation is linear in the space of functions with discrete arguments with values in the semi-ring $A = (R^1 \cup \{\pm\infty\}, \oplus = \min, \odot = +)$, where $\mathbb{1} = 0$ and $\mathbb{0} = +\infty$. We may rewrite it in a form quite similar to the one discussed in Example 1:

$$a^{k+1}(m, n) = L_V a^k(m, n), \tag{1.0.6}$$

where the "linear" operator L_V is given by the formula

$$L_V a^k(m, n) = \bigoplus_{(i,j)\in V} c_{ij} \odot a^k(m-i, n-j),$$

$$c_{ij} = \mathbb{1} = 0, \quad V = \{(0, +1), (+1, 0)\},$$

and the initial value is

$$a^0(m, n) = \begin{cases} \mathbb{1} & \text{if } n=0,\ m \geqslant 0 \text{ or } m=0,\ n \geqslant 0, \\ \mathbb{O} & \text{otherwise.} \end{cases} \qquad (1.0.7)$$

Just as in the linear case, this problem has no limit if we send k to ∞. Nevertheless, the concept of "generalized" solutions makes it possible, as in the linear case, to obtain the weak limit of problem (1.0.6), (1.0.7). With problem (1.0.6), (1.0.7) we associate the following problem for functions of continuous arguments $(x, y) \in R^2 \cup \{\pm\infty\}$, $t \in [0, T]$, $T = \text{const} > 0$:

$$u_h(t+h, x, y) = \tilde{L}_{h,V} u_h(t, x, y), \quad t = kh, \quad k \in \mathbb{Z}, \qquad (1.0.6')$$

$$u_h(0, x, y) = u^0(x, y) = \begin{cases} \mathbb{1} & \text{if } x=0,\ y \geqslant 0 \text{ or } y=0,\ x \geqslant 0, \\ \mathbb{O} & \text{if } x \neq 0 \text{ or } y \neq 0, \end{cases} \qquad (1.0.7')$$

in such a manner that $u_h(kh, mh, nh) = a^k(m, n)$.

The operator $\tilde{L}_{h,V}$ in the given case acts according to the formula

$$\tilde{L}_{h,V} u_h(t, x, y) = \min(u_h(t, x-h, y),\ u_h(t, x, y-h)). \qquad (1.0.8)$$

At $t = kh$ the solution to problem (1.0.6′), (1.0.7′) assumes the form

$$u_h(t, x, y) = (\tilde{L}_{h,V})^k u^0(x, y).$$

Allowing for (1.0.8), we can calculate the right-hand side of this equation explicitly:

$$u_h(t, x, y) = \min_{\substack{\zeta_1+\zeta_2=k \\ \zeta_i \in \mathbb{Z}_+ \\ i=1,2}} \{u^0(x-h\zeta_1, y-h\zeta_2)\}.$$

Let us introduce the "scalar product" for A-valued functions assuming that

$$\langle \varphi, \psi \rangle_{\oplus} = \int_{\oplus} \varphi(x, y) \odot \psi(x, y)\, dx\, dy \overset{\text{def}}{=} \inf_{x,y} \{\varphi(x, y) + \psi(x, y). \qquad (1.0.9)$$

Now we wish to calculate the weak limit, as $h \to 0$, of the solution to problem (1.0.6′), (1.0.7′) on smooth functions with respect to the "scalar product" introduced above (this, as noted earlier, is equivalent to finding the limit as k tends to ∞). Suppose $k \to \infty$, $kh \to t_0$, $t_0 \in [0, T]$. Then for every smooth function $\varphi(x, y)$ we have

$$\langle u_h(t_0, x, y), \varphi(x, y)\rangle_\oplus = \langle(\widetilde{L}_{h,V})^k u^0(x, y), \varphi(x, y)\rangle_\oplus$$
$$= \langle u^0(x, y), (\widetilde{L}^*_{h,V})^k \varphi(x, y)\rangle_\oplus,$$

where operator $\widetilde{L}^*_{h,V}$ is the conjugate of $\widetilde{L}_{h,V}$ with respect to the scalar product $\langle\ ,\ \rangle_\oplus$ introduced above.

It can easily be verified that

$$(\widetilde{L}^*_{h,V})^k \varphi(x, y) = \min_{\substack{\zeta_1+\zeta_2=k \\ \zeta_i \in \mathbb{Z}_+}} \{\varphi(x+h\zeta_1, y+h\zeta_2)\}.$$

Hence

$$\langle u_h(t_0, x, y), \varphi(x, y)\rangle_\oplus$$
$$= \langle u^0(x, y), \min_{\substack{\zeta_1+\zeta_2=k \\ \zeta_i \in \mathbb{Z}_+}} \{\varphi(x+h\zeta_1, y=h\zeta_2)\}\rangle_\oplus$$

We denote by $u_0(t_0, x, y)$ the weak limit of the solution to problem (1.0.6′), (1.0.7′): $u_0(t_0, x, y) = s - \lim_{h\to 0} u_h(t_0, x, y)$, where s is a fixed vector function whose meaning will be defined later on Sending h to 0 and kh to t_0 in the last equation, we get

$$\langle u_0(t_0, x, y), \varphi(x, y)\rangle_\oplus$$
$$= \lim_{h\to 0} \langle u_h(t_0, x, y), \varphi(x, y)\rangle_\oplus$$
$$= \langle u^0(x, y), \min_{\substack{\eta_1+\eta_2=t_0 \\ \eta_i \in R^1_+.}} \{\varphi(x+\eta_1, y+\eta_2)\}\rangle. \tag{1.0.10}$$

We will call the generalized function $u_0(t_0, x, y)$ defined by (1.0.10) a generalized solution to the limiting generalized Hamilton-Jacobi equation to which Eq. (1.0.6′) converges on smooth functions. This limiting equation has the form

$$\frac{\partial u}{\partial t} = \min\left\{-\frac{\partial u}{\partial x}, -\frac{\partial u}{\partial y}\right\}.$$

The solution to this equation can be obtained by employing Pontryagin's maximum principle [1.2, 1.3].

In contrast to the linear case, the statement of smooth initial data for discrete optimization problems has no meaning, as a rule. For this reason a study of the limiting equation of an optimization problem is justified only in constructing generalized solutions to this equation in the above sense.

The example of the discrete optimization problem (1.0.4), (1.0.5) is closely related to an analysis of the activity of homogeneous multiprocessor computational systems (see Sec. 1.2). When the number of processors, N, in such a system grows, that is, $N \to \infty$ ($h \sim 1/N \to 0$), the support of the generalized solution (1.0.10) to the limiting Hamilton-Jacobi equation determines, at each moment $t > 0$, the set of processors carrying out calculations at time t.

The suggested approach enables considering generalized solutions for general optimization problems, too. But here we will give a brief description of the properties of discrete optimization problems that arise when the operation of homogeneous computational systems is analyzed.

It appears that all such problems can be studied using solutions (generalized solutions in the limiting case of $N \to \infty$) to the generalized Hamilton-Jacobi equation in the space of functions with values in semi-rings. It has also been found that in problems related to the architecture of multiprocessor homogeneous CS, the range of the sought functions has the structure of a crystal lattice with certain symmetry properties. For instance, in the simplest case of a matrix processor, a draft of which was proposed in 1982 by a group of US scientists [1.4-1.7], the common Bravais lattice [1.8] serves as such a range.

For optimization problems connected with the estimation of the effectiveness of parallel programs, a discrete lattice with nontrivial symmetry groups (a nonempty set of nonelementary translations) serves as a natural range of independent variables of the functions involved in the problems. The symmetry of such lattices is uniquely determined by the text of the program, while the execution time of the program operators is determined by the values of the coefficients of the appropriate system of generalized Hamilton-Jacobi equations (systems of discrete generalized Bellman equations).

In optimization problems related to the operation of CS, these equations are usually nonhomogeneous steady-state equations (the right-hand side of the equations describes the interaction of the set of processors in a CS with the external memory of the system). The solution to these equations is found as the limit, as $N \to \infty$, of the solutions of appropriate evolutionary nonhomogeneous equations. Solving the latter can be reduced to solving homogeneous equations, using an analog of Duhamel's principle.

The difference between this case and the common linear case lies in the following: although for a steady-state problem there is no limiting generalized Hamilton-Jacobi equation, in the limit the corresponding nonstationary problem can be reduced to the evolutionary generalized Hamilton-Jacobi equation. This makes it possible, by means of Duhamel's principle, to write the limiting problem in terms of generalized solutions of a certain Cauchy problem for the generalized Hamilton-Jacobi equation. We call such a problem a stabilization one. Thus, a generalized solution of a stabilization Cauchy problem is the limit (in the new sense of the word), as $t \to \infty$, of the solution to the Cauchy problem for the generalized Hamilton-Jacobi equation.

Let us illustrate the aforesaid with two examples. In the first example we will consider a nonhomogeneous steady-state scalar equation on a simple one-dimensional lattice, so as to demonstrate how Duhamel's principle can be employed. In the second example we will study an optimization problem on a two-dimensional discrete lattice with a nontrivial symmetry group.

Example 3. Let us consider the simplest one-dimensional "tracing" problem, the problem of finding the shortest route [1.9-1.11] on a discrete lattice $\Omega_\varepsilon = \{x = x_n = n\varepsilon, n = 0, \pm 1, \ldots\}$ with spacing ε, where ε is a positive parameter. The following relation exists for the length $s_\varepsilon(n)$ of the shortest route at point n:

$$s_\varepsilon(n) = \min\{s_\varepsilon(n-1) + \varepsilon c_1,\ s_\varepsilon(n+2) + \varepsilon c_2,\ \mathcal{F}_\varepsilon(n)\}, \quad n \in \mathbb{Z}, \tag{1.0.11}$$

where c_1 and c_2 are constants, and $\mathcal{F}_\varepsilon(n) = g(n\varepsilon)$, with $g(x)$, $x \in R^1$, a continuous function bounded below.

Problem (1.0.11) is a steady-state problem with a right-hand side equal to $\mathcal{F}_\varepsilon(n)$ and is linear in the space of functions with values in the semi-ring

$$A = \{R^1 \cup \{\pm\infty\},\quad \oplus = \min,\quad \odot = +\},$$

where $\mathbb{0} = +\infty$, and $\mathbb{1} = 0$. We rewrite it in the form

$$s_\varepsilon(n) = L_\varepsilon s_\varepsilon(n) \oplus \mathcal{F}_\varepsilon(n)$$

where operator L_ε acts according to the rule

$$L_\varepsilon s_\varepsilon(n) = \bigoplus_{v \in V} c_\varepsilon(v) \odot s_\varepsilon(n - v),$$

with $V = \{v_1 = 1, v_2 = -2\}$, $c_\varepsilon(v_1) = \varepsilon c_1$, and $c_\varepsilon(v_2) = \varepsilon c_2$.

With this problem we associate the following problem for the family of functions of a continuous variable $u_\varepsilon(x)$, $x \in R^1$, $\varepsilon \in (0, 1]$:

$$u_\varepsilon(x) = \widetilde{L}_\varepsilon u_\varepsilon(x) \oplus g(x), \tag{1.0.11'}$$

where operator $\widetilde{L}_\varepsilon$ is defined as follows:

$$L_\varepsilon u_\varepsilon(x) = \min\{u_\varepsilon(x - \varepsilon) + \varepsilon c_1,\ u_\varepsilon(x + 2\varepsilon) + \varepsilon c_2\}.$$

Obviously, $u_\varepsilon(n\varepsilon)$ is the solution to the initial discrete problem (1.0.11). The solution to problem (1.0.11′) is the limit, as $t \to \infty$, of the solution to the evolutionary nonhomogeneous equation

$$f_\varepsilon(t + \varepsilon, x) = \widetilde{L}_\varepsilon f(t, x) \oplus g(x),\ t = k\varepsilon, \quad k = 0, 1, \ldots, \tag{1.0.12}$$