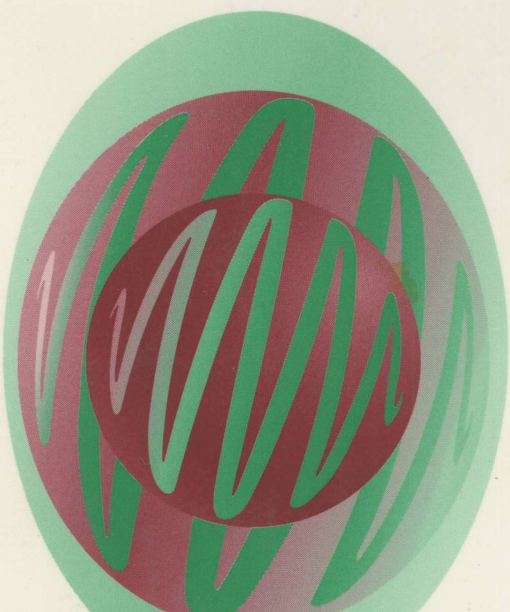
Quantum Classical Correspondence

The 4th Drexel Symposium on Quantum Nonintegrability

Drexel University

editors

D.H. Feng B.L. Hu



International Press

Quantum Classical Correspondence

Proceedings of the 4th Drexel Symposium on Quantum Nonintegrability

Drexel University, Philadelphia, USA
September 8-11, 1994

Edited by

Da Hsuan Feng
Drexel University, Philadelphia, Pennsylvania

Bei Lok Hu
University of Maryland, College Park, Maryland

International Press

International Press
96 Mt Auburn Str
P.O. Box 2872
Cambridge, MA 02238-2872

Quantum Classical Correspondence
Proceedings of the 4th Drexel Symposium
on Quantum Nonintegrability.
Edited by Da Hsuan Feng and Bei Lok Hu.

ISBN 1-57146-099-3

Printed on acid free paper, in the United States of America

To our Parents

PREFACE

A serendipitous development of theoretical physics in the past decade was the apparent confluence of some major issues in several areas of physics: quantum measurement, quantum cosmology and semiclassical gravity, quantum chaos and mesoscopic physics. Although these areas address vastly different aspects of physics, covering atomic, molecular and quantum optics, condensed matter, nuclear as well as particle physics and general relativity, they all share the common concern of how the many quantum and classical features of matter and spacetime and their dynamics are related to each other. This fundamental issue, which lies at the base of all aspects of physics, is the theme of this conference.

The series of three conferences held at Drexel University since 1988 on Quantum Nonintegrability was designed to be the forum on "quantum chaos" and related topics. In view of the rapid recent development in the physics of the quantum / classical interface, the organizers of this conference feel that it is timely to broaden the scope of the symposium to encompass the above mentioned areas on one unifying theme: that of quantum-classical correspondence.

Through this conference and the sequential ones we hope to initiate and establish a meaningful dialogue (and the unavoidable debates) between researchers in these apparently disparate yet intrinsically connected areas of physics. From the talks and reports presented in this conference we think at least we have succeeded in calling into attention and bringing into focus the basic issues these areas share in common. We hope that in time more interdisciplinary effort will be devoted to understanding this profound question with practical implications.

The organizers wish to thank Drexel University's Betsy and Louis Stein Family Foundation who provided funds to support the attendence of this conference by many outstanding Isreali scientists, the National Science Foundation for its grant to support a number of graduate students to actively participate, the Physics Departments of Drexel University, Chung-Yuan Christian University of Chung-Li, Taiwan, the University of Maryland, as well as World Scientific Publication Corporation of Singapore and the International Press of Boston/Hong Kong.

Many colleagues have given us valuable suggestions on the scientific content of this meeting. We are grateful specially to Rolf Landauer, our co-chairman, who offered many timely and useful advices. (Though unable to attend, he did write up his talk which is included here.) The dedication and skills of the local organizing committee members were essential to making this conference a success. We express our appreciation to the unfailing assistance of Jackie Sampson, Janice Murray, Michel Vallieres, Jian-Min Yuan, Lorenzo Narducci and Vincent Daniels.

Da Hsuan Feng and Bei Lok Hu
November 1995

TABLE OF CONTENTS

I. QUANTUM DECOHERENCE AND MEASUREMENT

II. QUANTUM AND SEMICLASSICAL GRAVITY

III. QUANTUM AND CLASSICAL DYNAMICS

IV. QUANTUM CHAOS

V. MESOSCOPIC PHYSICS

Part I

QUANTUM DECOHERENCE AND MEASUREMENT

QUANTUM CLASSICAL CORRESPONDENCE:
PROCEEDINGS OF THE 4TH DREXEL SYMPOSIUM
ON QUANTUM NON-INTEGRABILITY
DREXEL UNIVERSITY, PHILADELPHIA, USA
SEPTEMBER 8-11, 1994 PP. 3-35,

Eds. D.H. Feng and B.L. Hu

Strong Decoherence

MURRAY GELL-MANN AND JAMES B. HARTLE

We introduce a condition for the strong decoherence of a set of alternative histories of a closed quantum-mechanical system such as the universe. The condition applies, for a pure initial state, to sets of homogeneous histories that are chains of projections, generally branch-dependent. Strong decoherence implies the consistency of probability sum rules but not every set of consistent or even medium decoherent histories is strongly decoherent. Two conditions characterize a strongly decoherent set of histories: (1) At any time the operators that effectively commute with generalized records of history up to that moment provide the pool from which — with suitable adjustment for elapsed time — the chains of projections extending history to the future may be drawn. (2) Under the adjustment process, generalized record operators acting on the initial state of the universe are approximately unchanged. This expresses the permanence of generalized records. The strong decoherence conditions (1) and (2) guarantee what we call "permanence of the past" — in particular the continued decoherence of past alternatives as the chains of projections are extended into the future. Strong decoherence is an idealization capturing in a general way this and other aspects of realistic physical mechanisms that destroy interference, as we illustrate in a simple model. We discuss the connection between the reduced density matrices that have often been used to characterize mechanisms of decoherence and the more general notion of strong decoherence. The relation between strong decoherence and a measure of classicality is briefly described.

1. Introduction.

In this article we continue our efforts to explore the quantum mechanics of closed systems, most generally and realistically the universe as a whole, and within that framework to understand the significance of the quasiclassical realm[1] that includes familiar experience.

We introduce a strong realistic principle of decoherence for sets of alternative coarse-grained histories of a closed system and discuss the relationships between this and other principles that have been put forward. We also examine the concept of classicality, including the role played by the realistic principle of decoherence in characterizing it.

The most general predictions of quantum mechanics are the probabilities of individual members of a set of alternative coarse-grained histories of the universe.[2] A set of coarse-grained histories is a partition of one of the sets of fine-grained histories (which are the most refined possible descriptions of the closed system) into mutually exclusive classes. The classes are the individual coarse-grained histories, which are thus "bundles" of fine-grained histories.

The absence of quantum-mechanical interference between the individual histories in a set is necessary, at the very least, for quantum theory to assign consistent probabilities to the alternative possibilities. Such sets of histories for which interference is absent are said to *decohere*. Except for pathological cases, coarse-graining is necessary for decoherence.

Various conditions for the decoherence of sets of histories have been proposed. Some authors have tried to weaken the condition as much as possible to get the minimum condition necessary for probabilities to be defined. Our point of view has always been to try and describe a realistic principle of decoherence that characterizes in a general way the physical processes by which the dissipation of interference occurs. We have therefore been led to investigate conditions of decoherence that are as strong as possible compatible with the physical mechanisms that destroy interference. We shall investigate such a strong condition for decoherence in this paper.

Implementing a strong form of decoherence is part of a program to understand how the quasiclassical realm that includes everyday experience arises in quantum mechanics from the Hamiltonian of the elementary particles and the initial condition of the universe [1]. By a quasiclassical realm we mean an exhaustive set of mutually exclusive coarse-grained alternative histories

[1] In previous work we called a decoherent set of alternative coarse-grained histories a "domain". However, that term can be confusing because of its other uses in physics. We do not want to call such a set a "world" because that word connotes a single history and not a set of alternative ones. Hence, we now call a decoherent set of alternative coarse-grained histories a "realm".

[2] These are *a priori* probabilities. They can also be thought of as the statistical probabilities for an ensemble of universes, but in that case we have access only to one member of this ensemble.

that obey a realistic principle of decoherence, that consist largely of similar but branch-dependent alternatives at a succession of times, with individual histories exhibiting patterns of correlations implied by effective classical equations of motion subject to frequent small fluctuations and occasional major ones, the whole set being maximally refined given these properties. The theory may exhibit essentially inequivalent quasiclassical realms, but there is certainly at least one that includes familiar experience. This is the *usual* quasiclassical realm, described, at least in part, by alternative values of hydrodynamic operators that are integrals, over suitable volumes, of densities of conserved or nearly conserved quantities. Examples are densities of energy, momentum, baryon number, and, in late epochs of the universe, of nuclei and even chemical species. The sizes of the volumes and the spacing of the alternatives in time are limited above by maximality. The size and spacing are limited below by decoherence and the requirement that the volumes have sufficient "inertia" to enable them to resist deviations from predictability caused by quantum spreading and by the noise that typical mechanisms of decoherence produce [1, 2].

A key property of the usual quasiclassical realm is the persistence of the past. Histories of quasiclassical alternatives up to a given time can be extended into the future to give further such histories without endangering the decoherence of the past alternatives. This persistence of the past is not guaranteed by quantum mechanics alone. Extending a set of histories into the future is a kind of fine graining and this carries the risk of losing decoherence.[3] However, the persistence of the past is critical to the utility of the quasiclassical realm. It is the reason that we do not need to do an elaborate calculation verifying the preservation of past decoherence on every occasion when we want to predict the probability of a quasiclassical alternative in the future conditioned on our experience of the past. We proceed, secure in the understanding that in a quasiclassical realm the past (including the decoherence of past alternatives) will continue to persist.

In this article we discuss a strong form of decoherence that guarantees the persistence of the past. The idea is closely related to the notion of "generalized " records that we treated in our earlier work [4]. The physical picture is that, at every branching of the coarse-grained histories of the universe, each of the exhaustive and mutually exclusive possibilities is correlated with a different state of something like a photon or neutrino going off to infinity and unaffected by subsequent alternatives. The orthogonality of

[3]Indeed, Dowker and Kent [3] have given examples with special final conditions where a quasiclassical realm cannot be extended at all.

those states is the realistic mechanism underlying decoherence. For each of the alternative coarse-grained histories up to some time, a projection operator R describes the information of that kind that has been stored up. The projections constitute the "generalized records" associated with the different histories. They are all orthogonal to one another and that orthogonality gives rise to the decoherence of histories.

The present work assumes a pure state for the universe, that is, a density matrix ρ of the form $|\Psi\rangle\langle\Psi|$. In some earlier articles [4] we allowed ρ to be more general. We then defined another kind of "strong decoherence" which was equivalent, for a pure state, to medium decoherence. We now suggest restricting the term "strong decoherence" to what we are discussing here and abandoning it as a name for the earlier concept, which may be too restrictive when the state is not pure and is redundant otherwise [2]. For the rest of the article we assume that $\rho = |\Psi\rangle\langle\Psi|$.

The present strong decoherence condition is stronger than our earlier "medium decoherence", which in turn is stronger than our "weak decoherence" condition. Other authors have discussed still weaker conditions, for example, the "consistent histories" condition of Griffiths [5] and Omnès [6] and the linearly positive histories of Goldstein and Page [7]. A simple and instructive case of the present strong decoherence was discussed in an insightful paper by Finkelstein [8], who called it "PT-decoherence" and showed how it is related to the "decoherence of density matrices" that has been discussed by many (*e.g.* [9, 10, 11, 12]). We shall consider this relationship in a more general context and show how a variety of reduced density matrices can be constructed for individual histories up to a given time that are diagonal in appropriate alternatives at the next time as a consequence of strong decoherence.

In Section II we shall review the various decoherence conditions after introducing some necessary notation. Section III introduces strong decoherence and describes the connection with reduced density matrices. Section IV explores these ideas in simplified models in which the coarse grainings are restricted to those that follow one set of fundamental coördinates while ignoring all others. In Section V we review our program to provide a measure of classicality and discuss the role that strong decoherence might play in such a program.

2. Varieties of Decoherence.

The ideas of the quantum mechanics of closed systems, including the (medium) decoherence of sets of alternative coarse-grained histories, can be formulated in perfect generality for quantum field theory. One can include the effects of a quantized spacetime metric, as in a field theory (Lagrangian) version of superstring theory, by using the principles of generalized quantum theory [13, 14, 15]. However, it is convenient, as well as an excellent approximation for many accessible coarse grainings, to consider a fixed spacetime geometry with well defined timelike directions. In the following brief review of the quantum mechanism of closed systems we shall adopt this approximation, using a time variable t and the associated Hamiltonian H.

One way of specifying a set of alternative histories is to give sets of alternative projection operators as a sequence of times $t_1 < t_2 \cdots < t_n$. At each time t_k, we have a set of Heisenberg picture projection operators $\{P^k_{\alpha_k\alpha_{k-1}\cdots\alpha_1}(t_k; t_{k-1}, \cdots, t_1)\}$ where $\alpha_k = 1, 2, 3\cdots$ denotes the particular alternative in the set. The notation is designed to indicate the branch dependence that is characteristic of useful sets of alternative coarse-grained histories of the universe [2]. The different alternatives in the set at time t_k correspond to different α_k. However, in a branch-dependent set of histories, the set of alternatives at a given time will depend on previous history. Useful sets of alternative histories of the universe (such as those constituting a quasiclassical realm) will be branch-dependent because the efficacy of physical mechanisms of decoherence depends on particular present circumstances and past history. Branch dependence is indicated explicitly by the extended subscript $\alpha_k\alpha_{k-1}\cdots\alpha_1$ and the dependence on previous times $t_{k-1}\cdots t_1$. The projection operators are mutually exclusive and exhaustive as expressed by the relations:

(2.1)
$$P^k_{\alpha_k\alpha_{k-1}\cdots\alpha_1} P^k_{\alpha'_k\alpha_{k-1}\cdots\alpha_1} = \delta_{\alpha_k\alpha'_k} P^k_{\alpha_k\alpha_{k-1}\cdots\alpha_1} \ , \quad \Sigma_{\alpha_k} P^k_{\alpha_k\alpha_{k-1}\cdots\alpha_1} = I \ ,$$

where, as will often be convenient, we have suppressed the time labels for the sake of compactness. The same physical set of alternatives at later times $t > t_k$ can be expressed by way of the Heisenberg equations of motion

(2.2)
$$P^k_{\alpha_k\alpha_{k-1}\cdots\alpha_1}(t; t_{k-1}\cdots t_1) = e^{iH(t-t_k)} P^k_{\alpha_k\alpha_{k-1}\cdots\alpha_1}(t_k; t_{k-1}\cdots t_1) e^{-iH(t-t_k)} \ .$$

(Here, and throughout, we use units such that $\hbar = 1$.)

Each history is then a particular sequence of alternatives $\alpha = (\alpha_1, \cdots, \alpha_k)$ and is represented by the corresponding chain of projections:[4]

(2.3)
$$H_{\alpha_k \cdots \alpha_1} = P^k_{\alpha_k \cdots \alpha_1}(t_k; t_{k-1}, \cdots, t_1) P^{k-1}_{\alpha_{k-1} \cdots \alpha_1}(t_{k-1}; t_{k-2}, \cdots, t_1) \cdots P^1_{\alpha_1}(t_1) \ .$$

As mentioned above, we shall not always indicate the various times. Indeed, since any time label may be altered (preserving the order) by reexpressing the corresponding projections in terms of field operators at another time using the equations of motion, we shall generally suppress these labels.

Sets of histories consisting of chains of projections like (2.3) are not the only sets potentially assigned probabilities by quantum mechanics. As we have mentioned, the general notion of coarse-graining is a partition of a fine-grained set into classes $c_\beta, \beta = 1, 2, \cdots$. Such histories may consist of *sums* of chains

$$C_\beta = \sum_{(\alpha_1, \cdots, \alpha_k) \in \beta} H_{\alpha_k \cdots \alpha_1} \ . \tag{2.4}$$

Evidently,

$$\sum_\beta C_\beta = I \ . \tag{2.5}$$

Various authors have discussed different conditions for when a set of histories $\{c_\beta\}$ decoheres and can be assigned probabilities $p(\beta)$ in quantum theory. Below we list them in increasing order of strength — first for an initial condition described by a density matrix ρ and then for the special case that ρ is pure, $\rho = |\Psi\rangle\langle\Psi|$.

- The "linearly positive" condition of Goldstein and Page [7]:

$$p(\alpha) = Re\ Tr(C_\alpha \rho) \geq 0 \ , \tag{2.6a}$$
$$p(\alpha) = Re\langle\Psi|C_\alpha|\Psi\rangle \geq 0 \ . \tag{2.6b}$$

- The "consistent histories" condition of Griffiths [5] and Omnès [6] for sets of histories that are chains of the form (2.3) (homogeneous sets):

$$Re\ Tr(C_{\alpha'} \rho C^\dagger_\alpha) = \delta_{\alpha'\alpha}\, p(\alpha) \ , \tag{2.7a}$$
$$Re\ \langle\Psi|C^\dagger_\alpha C_{\alpha'}|\Psi\rangle = \delta_{\alpha'\alpha}\, p(\alpha) \ , \tag{2.7b}$$

provided $C_\alpha + C'_\alpha$ is a chain as well.

[4]These are called *homogeneous* histories by Isham [15]. We used a slightly different notation in [2] with $P^k_{\alpha_k}(t_k; \alpha_{k-1}, t_{k-1}, \cdots \alpha_1)$ instead of $P^k_{\alpha_k \cdots \alpha_1}(t_k; t_{k-1} \cdots t_1)$.

- Weak decoherence:

(2.8a) $$Re\ Tr(C_{\alpha'}\rho C_\alpha^\dagger) = \delta_{\alpha'\alpha}\, p(\alpha)\ ,$$

(2.8b) $$Re\ \langle\Psi|C_\alpha^\dagger C_{\alpha'}|\Psi\rangle = \delta_{\alpha'\alpha}\, p(\alpha)\ ,$$

with no restriction to chains or on the sums of C's.

- Medium decoherence:

(2.9a) $$Tr(C_{\alpha'}\rho C_\alpha^\dagger) = \delta_{\alpha'\alpha}\, p(\alpha)\ ,$$

(2.9b) $$\langle\Psi|C_\alpha^\dagger C_{\alpha'}|\Psi\rangle = \delta_{\alpha'\alpha}\, p(\alpha)\ ,$$

again with no restrictions on the C's.

In this paper we shall discuss a yet stronger condition of decoherence, which for a pure state has the form

(2.10) $$\langle\Psi|C_\alpha^\dagger M^\dagger M' C_{\alpha'}|\Psi\rangle = 0, \quad \alpha \neq \alpha'\ ,$$

for any operators M included in a set $\{M\}_\alpha$ and M' included in a set $\{M\}_{\alpha'}$, both sets including the identity, I. We shall write this as

(2.11) $$\langle\Psi|C_\alpha^\dagger \{M^\dagger\}_\alpha \{M'\}_{\alpha'} C_{\alpha'}|\Psi\rangle = 0 \quad \alpha' \neq \alpha\ .$$

where the occurrence of $\{M\}$ in an equation means that it holds for each $M \in \{M\}$. The probabilities for a set satisfying this condition are

(2.12) $$p(\alpha) = \langle\Psi|C_\alpha^\dagger C_\alpha|\Psi\rangle\ .$$

The properties of the sets $\{M\}_\alpha$ that make this a condition of strong decoherence are discussed in the next section.

3. Strong Decoherence.

We shall now introduce a notion of strong decoherence applying to a set of histories that are chains of projections. This strong decoherence is a special form of medium decoherence and thus permits the assignment of probabilities to any set of histories that is a coarse graining of the set of chains, whether or not the sums of chains involved are themselves chains. Such coarse grainings can also be considered to be strongly decoherent.

The definition of strong decoherence is connected with the properties of generalized records of histories that we have described in earlier work. When