

MATHEMATICS

2 107

FOR LIBERAL ARTS

A PROBLEM SOLVING APPROACH

Billstein • Lott



Mathematics for Liberal Arts

A Problem Solving Approach

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Mathematics for Liberal Arts
A Problem Solving Approach

To Esther, Jane, Molly, and Karly
—R. W. B.

To Carolyn and John
—J. W. L.

Preface

Richard Courant once wrote that “understanding mathematics cannot be transmitted by painless entertainment.” In writing this book for nonmathematicians, our feeling that mathematics is not a spectator sport comes through. However, it was also our feeling when writing this book that students in liberal arts, elementary education, business, and social sciences need not be overpowered and intimidated by mathematics. Thus, our goals in writing were the following:

OUR GOALS

- To survey the appropriate mathematics in a way that is both intelligible and entertaining
- To present and use the heuristics of problem solving as an integral part of mathematics
- To encourage our students to extend their learning beyond the classroom by providing a variety of discussion topics and a diversity of problems (both elementary and challenging).

PROBLEM SOLVING

This book reflects a great commitment to the goal that problem solving should be a focus of mathematics. Because of this, we have emphasized problem solving wherever possible:

- *Two complete chapters* (Chapters 1 and 13) are devoted to the problem solving techniques used throughout the text. Chapter 1 develops a four-step problem solving method, based upon Polya’s work:

Understanding the problem

Devising a plan

Carrying out the plan

Looking back

This chapter presupposes only minimal mathematics skill from students. Chapter 13 reviews problem solving strategies and presents challenging problems based upon topics covered in the preceding chapters.

- A *preliminary problem* begins Chapters 1 through 12. Each problem poses a question that students can answer after mastering the material in the chapter. We encourage our students to attempt to solve the preliminary problem before starting the chapter so that they might develop a sense of what is needed to solve the problem. The final section of each of these chapters gives a solution to the preliminary problem using the four-step method presented in Chapter 1.
- *Example problems* have been introduced throughout the text. These problems are solved in detail using the four-step problem solving format.

FEATURES

We have included a number of study aids and incorporated end-of-section, end-of-chapter, and design features to make the book as useful and interesting as possible.


Study Aids



COMPUTER CORNER



BRAIN TEASER

- *Computer Corners* are included throughout the book. These illustrate content in the corresponding sections and are written in BASIC or Logo.
- *Brain Teasers* supplement many of the problem sets. They are challenging and entertaining problems related to the subject matter of the sections in which they appear. Solutions to the Brain Teasers are in the Instructor's Resource Manual.
- *Cartoons* are included throughout the book to add a lighter touch to the text and to illustrate the content in sections.
- Problems emphasizing *calculator usage* are included in problem sets by a calculator symbol .
- *Historical Notes* are included throughout the book. The study of mathematics is sometimes more palatable when it is realized that mathematics was created by humans.

End-of-Section Features

- *Problem sets* at the end of each section include large numbers of problems generally arranged in order of increasing difficulty. Stars ★ indicate the most *challenging problems*.
- *Review problems* are included in problem sets. The review problems constitute a basic review of material from previous sections in the chapter.
- *Time Outs* are suggested at the end of many sections. These may be used to aid in the learning of the mathematics content.
- *Answers* to odd-numbered problems are included in the back of the book.

End-of-Chapter Features

- *Chapter outlines* are included to help students review the chapter.
- *Chapter Tests* provide an opportunity for students to test themselves on important concepts developed in each chapter. Asterisks indicate questions from optional sections.

Design Features

- *Key terms, definitions, theorems*, and other important concepts are highlighted in boldface type. Key terms are repeated in the margins to help students review the material.
- A functional *use of color* in the text material and illustrations helps to emphasize various concepts.
- *Graphs, charts, geometric drawings, cartoons*, and other kinds of illustrations reinforce the content presented.

CONTENT

Because the mathematics preparations of students who take this course vary widely, we have written the book so that the material can be used by students with diverse backgrounds. We have built in flexibility for instructors: We have included enough topics to allow instructors to adapt the text to a variety of course lengths and organizations, including sections preceded by asterisks (*) that are optional and can be omitted without loss of continuity.

As we mentioned previously, Chapters 1 and 13 provide an explanation and review of problem solving. Further chapters cover the following topics:

Sets and logic. We present these topics (Chapter 2) in a way that allows instructors to cover less than the complete chapter if they wish.

Numeration systems, whole numbers, and integers. Chapter 3 is a review of operations on whole numbers and integers with an introduction to algebra.

Number theory. Number theory concepts (Chapter 4) afford an excellent opportunity to study mathematics. We have developed many of the properties in this chapter in a way that we believe is most meaningful to students at this level.

Rational and real numbers. Chapter 5 deals with fractions, decimals, and exponents.

Applications of mathematics. Chapter 6 focuses upon ratio, proportion, and percentage as used in daily life.

Computers. Chapters 7 and 12 are concerned with the computer languages BASIC and Logo. BASIC is and has for years been the computer language learned by college-bound students and college students. Logo, a language developed at the Massachusetts Institute of Technology, is rapidly being assimilated by students at all levels.

Geometry. Chapter 8 covers geometrical notions of symmetry, measurement, topology, and the Pythagorean Theorem.

Linear mathematics. Chapter 9 presents the fundamentals of coordinate geometry and linear programming with a special section on matrices.

Probability. Topics in probability are presented in Chapter 10 through the use of tree diagrams. Formulas for combinations and permutations are also developed in this chapter.

Statistics. Chapter 11 presents statistics with an emphasis on organizing, presenting, and interpreting data with sections on normal distributions and misuses of statistics.

THE INSTRUCTOR'S GUIDE

This supplement includes:

- Answers to odd- and even-numbered problems
- Complete solutions to the problems in Chapter 13
- Sample chapter tests that may be used as test questions or as make-up tests
- Answers to Brain Teasers

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The reviewers of our work (listed below) offered us valuable guidance, and we are grateful to them for the care they took with their reviews. We also acknowledge the cooperativeness of The Benjamin/Cummings Publishing Company and Shlomo Libeskind in allowing us to use portions of the book, *A Problem Solving Approach to Mathematics for Elementary Teachers*, in preparing this manuscript. Finally, we would like to thank our editor, Craig Bartholomew, and the staff at Benjamin/Cummings, who have worked extremely hard on this text.

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1 Introduction to Problem Solving

PRELIMINARY PROBLEM

Professor Noah Little designed a 24-question true-false test on which he hoped to discourage guessing. On the test grading, a student would receive 5 points for each correct answer and lose 7 points for each incorrect answer. Stu Dent took the test, answered every question, and scored zero. How many problems did he answer correctly?

