Understanding Digital Signal Processing

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DEDICATION

I dedicate this book to my two daughters Julie and Meredith, I wish I could go with you; to my mother Ruth for making me finish my homework; to my father Grady who didn't know what he started when he built that workbench in the basement; to my brother Ray for improving us all; to my brother Ken who succeeded where I failed; to my sister Nancy for running interference for us; to John Lennon for not giving up; to Dr. Laura Schlessinger for keeping us honest; to my advisor Glenn Caldwell and to the Iron Riders Motorcycle Club (Niles, CA) who keep me alive.

PREFACE

Learning Digital Signal Processing

Learning the fundamentals, and how to speak the language, of digital signal processing does not require profound analytical skills or an extensive background in mathematics. All you need is a little experience with elementary algebra, knowledge of what a sinewave is, this book, and enthusiasm. This may sound hard to believe, particularly if you've just flipped through the pages of this book and seen figures and equations that appear rather complicated. The content here, you say, looks suspiciously like the material in technical journals and textbooks, material that is difficult to understand. Well, this is not just another book on digital signal processing.

This book's goal is to gently provide explanation followed by illustration, not so that you may understand the material, but that you must understand the material.† Remember the first time you saw two people playing chess? The game probably appeared to be mysterious and confusing. As you now know, no individual chess move is complicated. Given a little patience, the various chess moves are easy to learn. The game's complexity comes from deciding what combinations of moves to make and when to make them. So it is with understanding digital signal processing. First we learn the fundamental rules and processes and, then, practice using them in combination.

If learning digital signal processing is so easy, then why does the subject have the reputation of being difficult to understand? The answer lies partially in how the material is typically presented in the literature. It's difficult to convey technical information, with its mathematical subtleties, in written form. It's one thing to write equations, but it's another matter altogether to explain what those equations really mean from a practical standpoint, and that's the goal of this book.

Too often, written explanation of digital signal processing theory appears in one of two forms: either mathematical miracles occur and you are simply given a short and sweet equation without further explanation, or you are engulfed in a flood of complex variable equations and phrases

† "Here we have the opportunity of expounding more clearly what has already been said" (Rene Descartes).

such as "it is obvious that," "such that $W(f) \geq 0 \; \forall \; f$," and "with judicious application of the homogeneity property." Authors usually do provide the needed information, but, too often, the reader must grab a pick and shovel, put on a miner's helmet, and try to dig the information out of a mountain of mathematical expressions. (This book presents the results of several fruitful mining expeditions.) How many times have you followed the derivation of an equation, after which the author states that he or she is going to illustrate that equation with a physical example—and this turns out to be another equation? Although mathematics is necessary to describe digital signal processing, I've tried to avoid overwhelming the reader because a recipe for technical writing that's too rich in equations is hard for the beginner to digest.†

The intent of this book is expressed in a popular quote from E. B. White in the introduction of his *Elements of Style* (New York: Macmillan Publishing, 1959):

> Will (Strunk) felt that the reader was in serious trouble most of the time, a man floundering in a swamp, and that it was the duty of anyone attempting to write English to drain the swamp quickly and get his man up on dry ground, or at least throw him a rope.

I've attempted to avoid the traditional instructor-student relationship, but, rather, to make reading this book like talking to a friend while walking in the park. I've used just enough mathematics to develop a fundamental understanding of the theory, and, then, illustrate that theory with examples.

The Journey

Learning digital signal processing is not something you accomplish; it's a journey you take. When you gain an understanding of some topic, questions arise that cause you to investigate some other facet of digital signal processing. Armed with more knowledge, you're likely to begin exploring further aspects of digital signal processing much like those shown in the following diagram. This book is your tour guide during the first steps of your journey.

You don't need a computer to learn the material in this book, but it would help. Digital signal processing software allows the beginner to ver-

† "We need elucidation of the obvious more than investigation of the obscure" (Oliver Wendell Holmes).

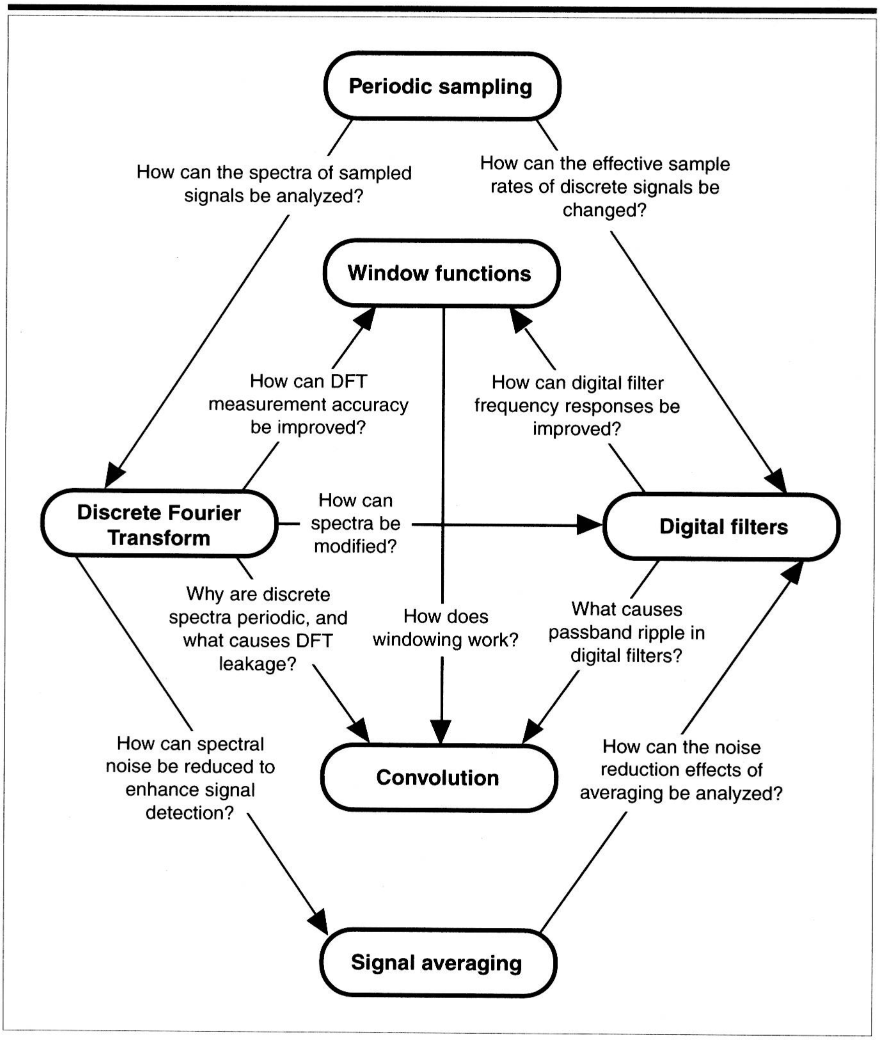

Figure P-1

ify signal processing theory through trial and error.† In particular, software routines that plot signal data, perform the fast Fourier transform, and analyze digital filters would be very useful.

As you go through the material in this book, don't be discouraged if your understanding comes slowly. As the Greek mathematician Menaechmus

† "One must learn by doing the thing; for though you think you know it, you have no certainty until you try it" (Sophocles).

curtly remarked to Alexander the Great, when asked for a quick explanation of mathematics, "There is no royal road to mathematics." Menaechmus was confident in telling Alexander that the only way to learn mathematics is through careful study. The same applies to digital signal processing. Also, don't worry if you have to read some of the material twice. While the concepts in this book are not as complicated as quantum physics, as mysterious as the lyrics of the song "Louie Louie," or as puzzling as the assembly instructions of a metal shed, they do get a little involved. They deserve your attention and thought. So go slow and read the material twice if you have to; you'll be glad you did. If you show persistence, to quote a phrase from Susan B. Anthony, "Failure is impossible."

Coming Attractions

Chapter 1 of this book begins by establishing the notation used throughout the remainder of our study. In that chapter, we introduce the concept of discrete signal sequences, show how they relate to continuous signals, and illustrate how those sequences can be depicted in both the time and frequency domains. In addition, Chapter 1 defines the operational symbols we'll use to build our signal processing system block diagrams. We conclude that chapter with a brief introduction to the idea of linear systems and see why linearity enables us to use a number of powerful mathematical tools in our analysis.

Chapter 2 introduces the most frequently misunderstood process in digital signal processing, periodic sampling. Although it's straightforward to grasp the concept of sampling a continuous signal, there are mathematical subtleties in the process that require thoughtful attention. Beginning gradually with simple examples of low-pass sampling and progressing to the interesting subject of bandpass sampling, Chapter 2 explains and quantifies the frequency-domain ambiguity (aliasing) associated with these important topics. The discussion there highlights the power and pitfalls of periodic sampling.

Chapter 3 is devoted to one of the foremost topics in digital signal processing, the discrete Fourier transform (DFT). Coverage begins with detailed examples illustrating the important properties of the DFT and how to interpret DFT spectral results, progresses to the topic of windows used to reduce DFT leakage, and discusses the processing gain afforded by the DFT. The chapter concludes with a detailed discussion of the various forms of the transform of rectangular functions that the beginner is likely to encounter in the literature. That last topic is included there to clarify and illustrate the DFT of both real and complex sinusoids.

Chapter 4 covers the innovation that made the most profound impact on the field of digital signal processing, the fast Fourier transform (FFT). There we show the relationship of the popular radix-2 FFT to the DFT, quantify the powerful processing advantages gained by using the FFT, demonstrate why the FFT functions as it does, and present various FFT implementation structures. Chapter 4 also includes a list of recommendations to help us when we use the FFT in practice.

Chapter 5 ushers in the subject of digital filtering. Beginning with a simple low-pass finite impulse response (FIR) filter example, we carefully progress through the analysis of that filter's frequency-domain magnitude and phase response. Next we learn how window functions affect and can be used to design FIR filters. The methods for converting low-pass FIR filter designs to bandpass and highpass digital filters are presented, and the popular Remez Exchange (Parks McClellan) FIR filter design technique is introduced and illustrated by example. In that chapter we acquaint the reader with, and take the mystery out of, the process called convolution. Proceeding through several simple convolution examples, we conclude Chapter 5 with a discussion of the powerful convolution theorem and show why it's so useful as a qualitative tool in understanding digital signal processing.

Chapter 6 introduces a second class of digital filters, infinite impulse response (IIR) filters. In discussing several methods for the design of IIR filters, the reader is introduced to the powerful digital signal processing analysis tool called the z-transform. Because the z-transform is so closely related to the continuous Laplace transform, Chapter 6 starts by gently guiding the reader from the origin, through the properties, and on to the utility of the Laplace transform in preparation for learning the z-transform. We'll see how IIR filters are designed and implemented, and why their performance is so different from FIR filters. To indicate under what conditions these filters should be used, the chapter concludes with a qualitative comparison of the key properties of FIR and IIR filters.

Chapter 7 discusses two important advanced sampling techniques prominent in digital signal processing, quadrature sampling and digital resampling. In the chapter we discover why quadrature sampling is so useful when signal phase must be analyzed and preserved, and how this special sampling process can circumvent some of the limitations of traditional periodic sampling techniques. Our introduction to digital resampling shows how we can, and when we should, change the effective sample rate of discrete data after the data has already been digitized. We've delayed the discussion of digital resampling to this chapter because some knowledge of low-pass digital filters is necessary to understand how resampling schemes operate.

Chapter 8 covers the important topic of signal averaging. There we learn how averaging increases the accuracy of signal measurement schemes by reducing measurement background noise. This accuracy enhancement is called processing gain, and the chapter shows how to predict the processing gain associated with averaging signals in both the time and frequency domains. In addition, the key differences between coherent and incoherent averaging techniques are explained and demonstrated with examples. To complete the chapter, the popular scheme known as exponential averaging is covered in some detail.

Chapter 9 presents an introduction to the various binary number formats that the reader is likely to encounter in modern digital signal processing. We establish the precision and dynamic range afforded by these formats along with the inherent pitfalls associated with their use. Our exploration of the critical subject of binary data word width (in bits) naturally leads us to a discussion of the numerical resolution limitations of analog to digital (A/D) converters and how to determine the optimum A/D converter word size for a given application. The problems of data value overflow roundoff errors are covered along with a statistical introduction to the two most popular remedies for overflow, truncation and rounding. We end the chapter by covering the interesting subject of floating-point binary formats that allow us to overcome most of the limitations induced by fixed-point binary formats, particularly in reducing the ill effects of data overflow.

Chapter 10 provides a collection of *tricks of the trade* that the professionals often use to make their digital signal processing algorithms more efficient. Those techniques are compiled into a chapter at the end of the book for two reasons. First, it seems wise to keep our collection of tricks in one chapter so that we'll know where to find them in the future. Second, many of these schemes require an understanding of the material from the previous chapters, so the last chapter is an appropriate place to keep our collection of clever tricks. Exploring these techniques in detail verifies and reiterates many of the important ideas covered in previous chapters.

The appendices include a number of topics to help the beginner understand the mathematics of digital signal processing. A comprehensive description of the arithmetic of complex numbers is covered in Appendix A, while Appendix B derives the often used, but seldom explained, closed form of a geometric series. Appendix C strives to clarify the troubling topics of complex signals and negative frequency. The statistical concepts of mean, variance, and standard deviation are introduced and illustrated in Appendix D, and Appendix E provides a discussion of the origin and utility of the logarithmic decibel scale used to improve the magnitude resolution of spectral representations. In a slightly different vein, Appendix F provides a glossary of the terminology used in the field of digital filters.

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If you're still with me this far into the Preface, I end by saying that I had a ball writing this book and hope you get some value out of reading it.

CONTENTS

CHAPTER ONE

Discrete Sequences and Systems

Digital signal processing has never been more prevalent or easier to perform. It wasn't that long ago when the fast Fourier transform (FFT), a topic we'll discuss in Chapter 4, was a mysterious mathematical process used only in industrial research centers and universities. Now, amazingly, the FFT is readily available to us all. It's even a built-in function provided by inexpensive spreadsheet software for home computers. The availability of more sophisticated commercial signal processing software now allows us to analyze and develop complicated signal processing applications rapidly and reliably. We can now perform spectral analysis, design digital filters, develop voice recognition, data communication, and image compression processes using software that's interactive in both the way algorithms are defined and how the resulting data are graphically displayed. Since the mid-1980s the same integrated circuit technology that led to affordable home computers has produced powerful and inexpensive hardware development systems on which to implement our digital signal processing designs.[†] Regardless, though, of the ease with which these new digital signal processing development systems and software can be applied, we still need a solid foundation in understanding the basics of digital signal processing. The purpose of this book is to build that foundation.

In this chapter we'll set the stage for the topics we'll study throughout the remainder of this book by defining the terminology used in digital signal

[†] During a television interview in the early 1990s, a leading computer scientist stated that had automobile technology made the same strides as the computer industry, we'd all have a car that would go a half million miles per hour and get a half million miles per gallon. The cost of that car would be so low that it would be cheaper to throw it away than pay for one day's parking in San Francisco.