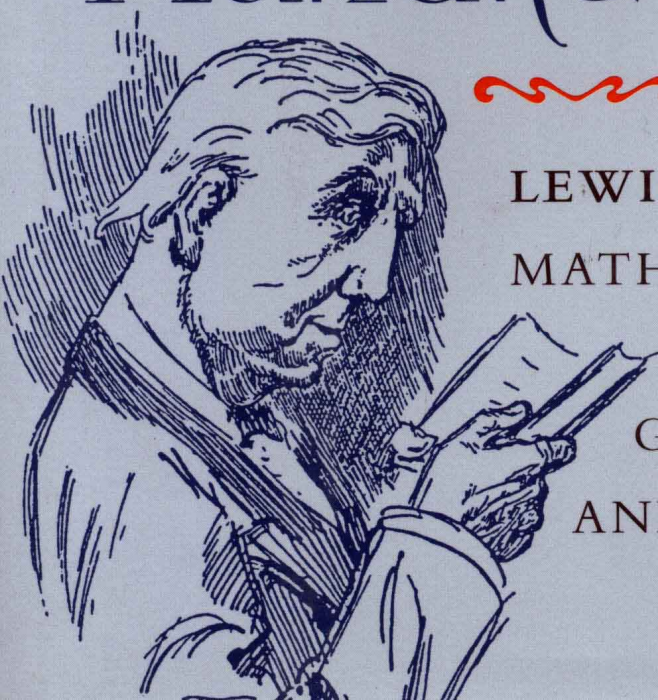# MARTIN GARDNER

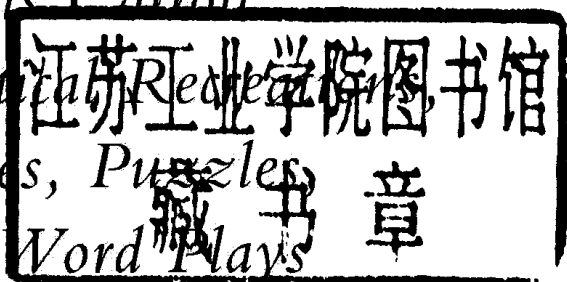## The Universe in a Handkerchief

LEWIS CARROLL'S
MATHEMATICAL
RECREATIONS,
GAMES, PUZZLES,
AND WORD PLAYS.

The Universe in a Handkerchief

Lewis Carroll's
Mathematical Recreations,
Games, Puzzles,
and Word Plays

MARTIN GARDNER

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The
Universe
in a
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To Clarkson N. Potter, friend and former publisher,
who in 1960 had the foresight to think it worthwhile
to annotate Lewis Carroll's *Alice* books.

Preface

I must confess that I did not become interested in Lewis Carroll until my undergraduate days at the University of Chicago. As a child, my greatest reading delights were the fantasies of L. Frank Baum. I tried hard to read the *Alice* books, but was put off by their abrupt transitions, the lack of a consistent story line, and the unpleasant characters in Alice's two dreams. And of course I missed all of Carroll's subtle jokes, word play, logic paradoxes, and philosophical implications. Unlike many Carrollians, I still believe that the *Alice* books should not be read by children, at least not by American children, until they are well into their teens.

When I reread the *Alice* books in my twenties, I was astounded by what I had missed. Two decades later, while writing a column on recreational mathematics for *Scientific American*, I discovered that Carroll not only shared my enthusiasm for play mathematics (puzzles, paradoxes, games, and so on), he also shared my hobby of conjuring. The more I learned about his life and opinions, the more I came to feel a spiritual kinship with him.

It occurred to me some 35 years ago that it was impossible for an American reader today, so far removed from Victorian England in both time and space, to appreciate fully the hundreds of hidden jokes in the *Alice* books without the aid of footnotes. I proposed the idea of an *Annotated Alice* to several

publishers. They found the notion ridiculous. Scholarly notes on two simple children's books? What is there to say?

Clarkson Potter, then with Dial Press, was the first editor who did not think my proposal absurd. When he left Dial to form his own company, Clarkson Potter, Inc. (now a subdivision of Crown), he took my manuscript with him. *The Annotated Alice* was an instant success and has remained in print ever since. In 1990 I followed it with *More Annotated Alice*, with all new notes, with illustrations by Peter Newell instead of John Tenniel to distinguish the book's format from its predecessor.

Two books have been published about Carroll's mathematical and verbal play: *The Magic of Lewis Carroll*, by magician John Fisher, and *Lewis Carroll's Games and Puzzles*, by Edward Wakeling. Although there is overlap in what is offered in those two books and this one, I have organized the topics differently and included, as the other two books do not, the full texts of Carroll's privately published pamphlets and leaflets. I have also covered in detail the recreational aspects of Carroll's fiction, verse, letters, and magazine articles.

Literature about Carroll shows no signs of abating. Morton Cohen's long-awaited biography, issued in 1995 by Knopf, is packed with startling new revelations. One continues to be amazed by how much there is yet to learn about the life and writings of this shy, stammering teacher of mathematics, who for so long was regarded as little more than a scribbler of outlandish nonsense tales for children, an author too unimportant for scholars to take seriously.

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Fiction and Verse

harles Lutwidge Dodgson, who taught mathematics at Christ Church, Oxford, was a competent mathematician though not a great creative one. His original contributions were mainly in the recreational field. His strong sense of mathematical beauty became intertwined with a delight in play that found expression in a fondness for mathematical games, puzzles, logic paradoxes, magic tricks, riddles, and every variety of word play, especially puns, anagrams, and acrostic verse, published under the name Lewis Carroll.

Carroll's interest in card games and chess, as we all know, provided the background for his two immortal *Alice* books. Cards and chess pieces both have their kings and queens. In

the first *Alice* book, the royalty and the Knave of Hearts, even the palace gardeners, are playing cards. Alice shouts at the court just before she awakens from her dream, "You're nothing but a pack of cards!" In the second *Alice* book, the kings, queens, and knights are chess pieces. The book's plot follows the moves of a whimsical, unorthodox chess game that culminates when Alice, who has the role of a white pawn, reaches the board's final rank to be crowned a queen.

Alice's Adventures in Wonderland swarms with word play, mostly obvious puns, although some are not so obvious. An example of puns so well concealed that they were long unrecognized are the three "littles" in the prefatory poem. They refer to the three Liddell sisters (Liddell rhymes with fiddle) who are in the boat with Carroll as they row up the Thames.

> All in the golden afternoon
>   Full leisurely we glide;
> For both our oars, with little skill,
>   By little arms are plied,
> While little hands make vain pretence
>   Our wanderings to guide.

The first *Alice* book also contains the Mad Hatter's notorious riddle about the raven and the writing desk. Carroll confessed that he introduced the riddle without having any answer in mind, though he later supplied one: "Because it can produce a few notes, tho they are *very* flat; and it is nevar put with the wrong end in front." Carroll deliberately misspelled "never" to make it "raven" backward. Scores of clever answers to the riddle have since been suggested by others.

Through the Looking-Glass is even richer in mathematical humor than the first *Alice* book. This is partly due to its pervasive chess themes, but also to the fact that Alice's journey into

the reversed world behind the mirror allowed Carroll to in-
dulge in all sorts of bizarre reversals of space and time. Left and
right symmetries and asymmetries abound. The Tweedle broth-
ers, for instance, are mirror images of each other. The White
Queen's memory works both forward and backward in time.
She screams with pain *before* the pin of her brooch pricks her
finger.

Mathematicians are always losing their way in endless laby-
rinths. The dozing Red King dreams about Alice, who is asleep
and dreaming about the Red King. In both dreams, each dreams

*The Tweedle brothers are what geometers call enantiomorphs—mirror re-
flections of each other. Here they are about to fight a battle. Note how
John Tenniel, the illustrator, has carefully drawn them as mirror images.*

of the other, forming a pair of infinite regresses. The book
ends with Alice considering the "serious question" of which
of them dreamed the other.

Linguistic play (which can be considered a branch of com-

Peter Newell's picture of the Red King as he dreams about Alice, while Alice in turn is dreaming about the sleeping Red King.

binatorics) and logical paradoxes pervade the second *Alice* book even more than the first. It swarms with puns and closes with a poem that is an acrostic on Alice's full name. The book also contains an unanswered conundrum; only this time Carroll knew the answer. The White Queen recites a riddle poem about a fish. The answer—an oyster—was not disclosed in print until it appeared anonymously in the magazine *Fun* (October 30, 1878).

In the two *Sylvie and Bruno* books, in which the real world and a fantasy realm are cleverly interwoven, Carroll's use of recreational mathematics, logic, and word play reaches still greater heights. The first volume's prefatory poem takes up where the terminal poem of *Through the Looking-Glass* leaves off. The three words that end its first stanza repeat, in reverse order, the three words that end the last lines of the earlier poem. Like the former poem, it too is an acrostic, an ingenious one. Not only do the first letters of the lines spell "Isa Bowman," one of Carroll's cherished child-friends, but the first three letters of each stanza are Isa, Bow, and Man.

The prefatory poem of *Sylvie and Bruno Concluded* is another unusual acrostic. The third letters of each line spell "Enid Stevens." The dedicatory poem of *The Nursery Alice* is still another acrostic, its lines' second letters spelling "Marie Van Der Gucht." Carroll wrote dozens of acrostic poems on children's names, which he sent to them in letters or inscribed in gift books. You'll find some of them gathered in a section on acrostics in the Modern Library edition of Carroll's writings.

Recreational mathematics is most explicit in the inventions of the Professor in the first *Sylvie and Bruno* book and of his counterpart, Mein Herr, a German professor in the sequel. In

A boat, beneath a sunny sky
Lingering onward dreamily
In an evening of July—

Children three that nestle near,
Eager eye and willing ear,
Pleased a simple tale to hear—

Long has paled that sunny sky;
Echoes fade and memories die;
Autumn frosts have slain July.

Still she haunts me, phantomwise,
Alice moving under skies
Never seen by waking eyes.

Children yet, the tale to hear,
Eager eye and willing ear,
Lovingly shall nestle near.

In a Wonderland they lie,
Dreaming as the days go by,
Dreaming as the summers die:

Ever drifting down the stream—
Lingering in the golden gleam—
Life, what is it but a dream?

Carroll's best-known acrostic poem, which he wrote as an epilogue to Through the Looking-Glass. *The first letters of the lines spell the real Alice's name.*

the sequel's Chapter 7, Mein Herr explains to Lady Muriel how a Möbius strip has only one side and one edge. He then teaches her how to sew together two handkerchiefs to make a

Mein Herr shows Lady Muriel how to fold and sew a handkerchief to make a closed surface that has no outside or inside. The drawing is by Harry Furniss for Carroll's Sylvie and Bruno Concluded.

three-dimensional one-sided surface known to topologists today as a projective plane. (It is a close cousin of the better-known one-sided surface called a Klein bottle.) Mein Herr calls it Fortunatus's Purse because, having neither outside nor inside, it can be said to contain the entire universe. Carroll drew a sketch of the purse in a letter to the book's illustrator, Harry Furniss, who copied it exactly for his picture of the scene.

In the same chapter, Mein Herr describes a plan for running trains entirely by gravity. The track goes through a straight tunnel between two widely separated locations. Gravity pulls the train down to the tunnel's center, giving it sufficient momentum to continue up to the other end. Curiously, if friction and air resistance are ignored, the train will go from one end of the tunnel to the other in about 42 minutes regardless of the
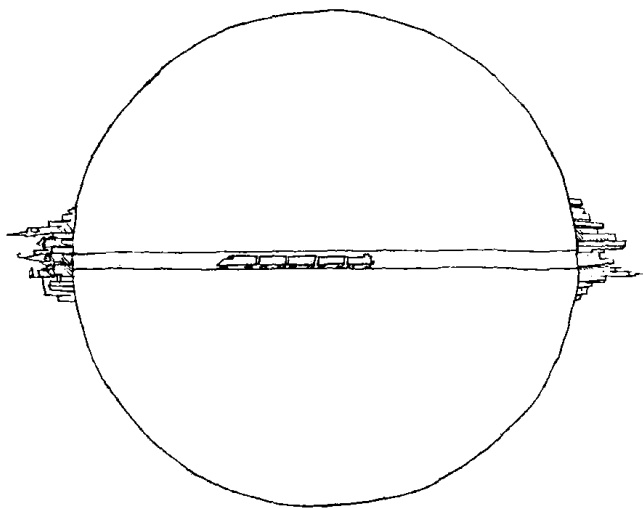
The gravity-operated train invented by the German professor in Carroll's Sylvie and Bruno Concluded. *From Martin Gardner's* Space Puzzles *(Simon and Schuster, 1971).*

tunnel's length. As we shall see, 42 had for Carroll some sort of special significance.

In Chapter 11 of *Sylvie and Bruno Concluded*, Mein Herr describes a map drawn on a scale of a mile to a mile:

> "It has never been spread out, yet," said Mein Herr: "the farmers objected; they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well."

Count Alfred Korzybski, founder of general semantics, liked to say "The map is not the territory." On the Professor's planet the two become identical.

Mein Herr goes on to describe a planet visited by a friend— a world so small that one can walk around it in twenty minutes:

> "There had been a great battle, just before his visit, which had ended rather oddly: the vanquished army ran away at