Multiple Access Communications

FOUNDATIONS FOR EMERGING TECHNOLOGIES

Edited by **NORMAN ABRAMSON**

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Norman Abramson
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Part 1
Introduction

1.1 Scope of the Collection

THE TERM *multiple access communications* may be interpreted in a variety of ways in different areas of communications. One interpretation that encompasses certain theoretical questions dealing with channel capacity is common among workers in information theory. In the area of spread spectrum, multiple access usually refers to the ability of certain kinds of signals to coexist in the same frequency and time space with an acceptable level of mutual interference. A third area in telecommunications research that is concerned with another view of multiple access involves (usually digital) networks composed of large numbers of intermittent transmitters trying to share a common communication channel, often based on variations of a contention protocol. And, finally, since 1970 those involved in satellite communications have devoted considerable effort to questions related to the use of more than one satellite earth station accessing a single transponder by means of frequency division or time division (FDMA or TDMA) techniques.

In this collection of papers on multiple access communications I have made an editorial decision to include the first three areas listed above but not the fourth. The three areas included have the common characteristic that they all deal with or are directly related to statistical properties of communication channels. The papers in this collection then are connected by that common theme; and in that common theme the reader can find connections from one area to another, and even insights that are of value in one area derived from results published in another. The canonical problem that served as a guide to the selection of papers in this collection is shown in Fig. 1.

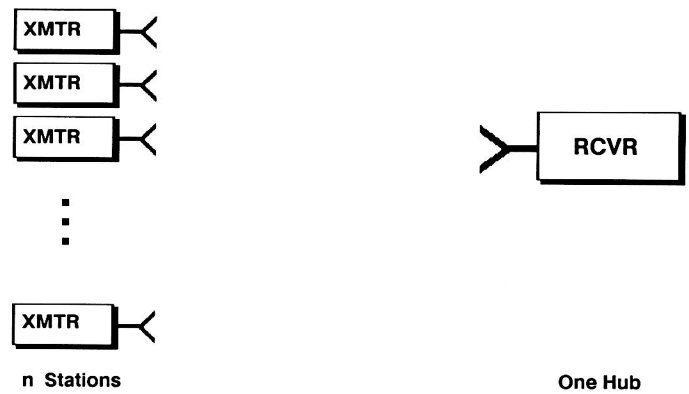

Fig. 1. Model of a multiple access channel.

The n transmitters on the left of Fig. 1 might represent n users in a packet radio LAN (local area network) trying to communicate with the host receiver on the right. The n transmitters might also represent n separate very small aperture terminals (VSATs) transmitting packet data in a satellite network. Finally, the transmitters might provide a useful model for the kind of digital cellular personal communication networks (PCNs) that have been referred to as *third-generation wireless networks*. In all of these cases there are certain common characteristics of the network model I would like to emphasize.

1. The network is composed of independent users attempting to communicate with a single common channel or perhaps even multiple channels—but the number of channels is much smaller than the number of possible transmitters.
2. The data from each of the users accessing the network is bursty—that is, the transmissions from a single user are separated by periods of silence (usually of durations much greater than the periods of transmission).
3. The number of users active on the network at any given time is a random variable, usually unknown to the receiver.

Not all of the papers in this collection deal with models that are consistent with all three of the assumptions listed above; but each of the papers provides insight that can teach us something about such models. And a better understanding of these models is essential for the development of the packet radio LANs, VSAT networks, and digital cellular networks that have assumed increasing importance in the telecommunications of today.

1.2. Organization

The same objectives that dictated the scope of this volume serve as a guide to its organization. My objectives are to provide a collection of papers that can be of value to those involved in the study of the theory of multiple access channels as well as to those interested in the design and analysis of real networks, based on multiple access channels. In Part 2, then, I have placed a collection of papers dealing with the fundamental properties of signals used in multiple access channels and in the fundamental information theoretic properties that limit the operation of those channels.

The multiple access capability of real networks exhibiting the three characteristics listed in section 1.1 has ordinarily been implemented by means of either a spread-spectrum–based protocol or an ALOHA–based protocol. Recent results, however, have shown that these two methods, with different origins and a different literature, are in fact simply two ways of viewing the same basic signals. Part 3 contains a set of papers dealing with spread spectrum, emphasizing the use of spread spectrum for multiple access communications. Part 4 provides a similar collection of papers on using ALOHA channels for multiple access, and this part concludes with a paper dealing with the equivalence of the spread spectrum and ALOHA techniques. My hope is that this juxtaposition will serve to emphasize the common theoretical foundation of two fields that have been developed independently to solve the same problems.

Part 5 contains a collection of papers that illustrate the wide variety of applications of multiple access technology in new telecommunications structures. The applications are of interest in their own right, but they also provide valuable feedback to the theorist interested in this field, because the most effective way to generate communication network models that are relevant to real communication networks is to build a familiarity with the real-world problems that must be analyzed. The papers in Part 5 can provide valuable insight into the motivation leading to the theoretical results in Parts 2, 3, and 4.

1.3. Comments and Additional References

1.3.1 General References

There are several previous volumes in the IEEE PRESS Selected Reprint Series as well as special issues of IEEE journals that deal with some of the topics treated in this volume. Among those that are closest in coverage to the topics covered in this volume are

1. Robert C. Dixon (Ed.), *Spread Spectrum Techniques.* New York: IEEE PRESS, 1976.
2. Special Issue on Computer Communications, R. L. Pickholtz (Ed.), *IEEE Trans. Communications*, vol. COM-25, no. 1, Jan. 1977.
3. Special Issue on Spread Spectrum Communications, L. A. Gerhardt and R. C. Dixon (Eds.), *IEEE Trans. Communications*, vol. COM-25, no. 8, Aug. 1977.
4. Special Issue on Spread-Spectrum Communications, C. E. Cook, F. W. Ellersick, L. B. Milstein, and D. L. Schilling (Guest Eds.), *IEEE Trans. Communications*, vol. COM-30, no. 5, Pt. 1, May 1982.
5. C. E. Cook, F. W. Ellersick, L. B. Milstein, and D. L. Schilling (Eds.), *Spread Spectrum Communications.* New York: IEEE PRESS, 1983.
6. Special Issue on Random-Access Communications, James L. Massey (Guest Ed.), *IEEE Trans. Information Theory*, vol. IT-31, no. 2, March 1985.
7. Special Issue on Packet Radio Networks, B. M. Leiner, D. L. Nielson, and F. A. Tobagi (Guest Eds.), *Proceedings of the IEEE*, vol. 75, no. 1, Jan. 1987.
8. Special Issue on Performance Evaluation of Multiple Access Networks, Victor O. K. Li (Guest Ed.), *IEEE J. Selected Areas Communications*, vol. SAC-5, no. 6, July 1987.
9. Special Issue on Spread Spectrum Communications, D. L. Schilling, R. L. Pickholtz, and L. B. Milstein (Guest Eds.), *IEEE J. Selected Areas Communications*, Parts I and II, vol. 8, nos. 4 and 5, May and June 1990.

Another reference that has a good deal of overlap with this volume is the proceedings of a NATO Advanced Study Institute held in August, 1980.

J. K. Skwirzynski (Ed.), *New Concepts in Multi-User Communications.* Rockville, MD: Sitjhoff and Nordhoff, 1981.

In May, 1991, the Institute of Electronics, Information and Communication Engineers of Japan published a special issue of the *IEICE Transactions on Communications* on spread spectrum. The papers in this special issue are printed in English in order to disseminate Japanese work in this field to an international audience.

Special Issue on Spread Spectrum Techniques and Its Applications, Gen Marubayashi (Ed.), *IEICE Trans. Communications*, vol. J74-B-I, no. 5, May 1991.

1.3.2 General Channel Considerations

The material in Part 2 addresses problems in multiple access channels, multiple access protocols, and multiple access signals. The papers dealing with the capacity of multiple access channels in this part are concerned primarily with multiple access in the additive Gaussian noise channel. The Gaussian channel model has received considerable attention in the multiple access literature because of the importance of this model in the case of single-transmitter single-receiver channels.

In spite of this importance, the Gaussian model has some shortcomings that have not yet been adequately covered in the literature on multiple access channels. In most of the applications described in Part 5, the number of active users on the network is a random variable, since any single network transmitter may or may not transmit at any given time. This means that we would like to be able to handle the model where the number of users on the network at any one time is a random variable (characteristic (3) of section 1.1) and where any given user transmits a Gaussian signal, but only conditioned on the fact that it is transmitting some signal. Perhaps this kind of model can be analyzed by generalizing the more common Gaussian assumption for channels with a single transmitter. Related to the question of the suitability of the Gaussian model for the kind of networks covered in this volume is the question of address information within the network. For if we really do want to analyze a network where the number of possible transmitters is large and the number of active transmitters is much less than the number of possible transmitters, then the question of protocol information within the network would seem to be relevant. The paper by Gallager in Part 2 addresses part of this issue, but considerable work remains to be done.

An extensive bibliography on information theory and multiple access channels covering the period through 1975 may be found in:

E. C. van der Meulen, "A survey of multi-way channels in information theory: 1961–1976," *IEEE Trans. Information Theory*, vol. IT-23, no. 2, Jan. 1977.

A shorter bibliography covering work from 1976 to 1980 is given at the end of the Gamal and Cover paper in Part 2.

1.3.3 Spread Spectrum Multiple Access (CDMA)

Spread spectrum multiple access communications is the subject of the articles selected for Part 3. This form of multiple access is often referred to as CDMA (code division multiple access), especially when different spreading sequence codes are assigned to different users. Recent results (see Part 4) have shown that different spreading sequences may not always be necessary or even desirable for spread spectrum multiple access. Although there is no clear agreement on the nomenclature in the technical literature, the term CDMA is preferred only for those cases where different spreading sequences are present.

Spread spectrum has its origins in a variety of military applications, and much of the early work in spread spectrum deals with the transmission of continuous signals rather than the bursty signals of interest in this volume. Again, the reader should note the same caution about many of the papers in the spread spectrum literature that was mentioned in section 1.3.2 when dealing with information theoretic models. When spread spectrum is used for multiple access of continuously transmitting information sources, a channel model where the number of transmitters and the identity of transmitters is known is an appropriate model. But if the number of transmitters is not known and if the address information of the transmitting sources is significant, then this model of spread spectrum should be reexamined.

Several textbooks cover spread spectrum in detail, although many aspects of the general spread spectrum problem are of only peripheral interest for applications in multiple access networks. In particular, synchronization, anti-jam applications, and frequency hopping are not emphasized in this selection of papers. The comprehensive text by Simon et al. includes a wealth of historical material collected by Robert Scholtz and Robert Price that is rarely seen in discussions of technical topics. A similar, but smaller, history of spread spectrum communications in Japan may be found in an article by Mitsuo Yokoyama in the special issue of the *IEICE Transactions on Communications* referenced at the end of Section 1.3.1.

1. Jack K. Holmes, *Coherent Spread Spectrum Systems.* New York: John Wiley and Sons, 1982.
2. Robert C. Dixon, *Spread Spectrum Systems*, 2nd Edition. New York: John Wiley and Sons, 1984.
3. Rodger E. Ziemer and Roger L. Peterson, *Digital Communications and Spread Spectrum Systems.* New York: Macmillan Publishing Company, 1985.
4. Marvin K. Simon, Jim K. Omura, Robert A. Scholtz, and Barry K. Levitt, *Spread Spectrum Communications*, Vols. 1, 2, and 3. Rockville, MD: Computer Science Press, 1985.
5. George R. Cooper and Clare D. McGillem, *Modern Communications and Spread Spectrum*. New York: McGraw-Hill Book Company, 1986.
6. R. Skaug and J. F. Hjelmstad, *Spread Spectrum in Communication.* London: Peter Peregrinus, Ltd., 1985.

1.3.4 ALOHA Multiple Access

The first use of ALOHA channels for multiple access took place in the ALOHANET local area packet radio network at the University of Hawaii in 1971. The original paper analyzing the ALOHA channel capacity in 1970 used a model that requires some comment in two separate areas: collision intervals and channel power constraints.

In the original ALOHA model it is assumed that two packets that overlap even a small amount result in two lost packets that must then be retransmitted over the channel. Later it was realized that the criterion of overlap in the channel was unduly restrictive and that the key question of correct packet reception was more productively addressed in terms of the overlap of packets at the output of a receiver. This distinction is particularly important in the case of packet receivers that compress the received packets in some way, since the total period of packet vulnerability can be much less than the period of possible overlap in the channel. The difference then led to the introduction of spread ALOHA–a form of ALOHA multiple access equivalent to direct-sequence spread spectrum multiple access, but which can be implemented with a simpler receiver.

In the original ALOHA model, the throughput of the channel was normalized with respect to the throughput of a transmitter sending data at the same rate as the packet-burst data rate. This normalization, therefore, compared the throughput of an ALOHA channel to the throughput of a channel transmitting continuously, even though the continuous channel might transmit more power (typically 10 dB or more) than the intermittent ALOHA transmitter. For the case of the original ALOHANET, where the constraint on the transmitter's power was a peak power constraint, such a normalization was appropriate. For other applications, however, such as VSAT networks or battery-operated transmitters in a PCN, a more appropriate normalization would compare the throughput of the ALOHA channel to that of a channel with the same average power. And as might be expected, when proper allowances are made for a 10-dB or more power mismatch, the ALOHA channel throughput looks considerably more attractive.

There are a number of Ph.D. theses and reports dealing with ALOHA channels that could not be included in this collection because of space limitations. The first careful analysis of multi-channel ALOHA networks was an informal ALOHA system report by George Turin in 1976. This work was subsequently expanded upon in the Berkeley thesis of Wing-Po Yung.

Wing-Po Yung, "Analysis of multi-channel ALOHA systems," Ph.D. thesis, University of California at Berkeley, Nov. 1978.

The same general topic was treated in a different and imaginative way by George Thomas of the Indian Space Research Organization. An abbreviated version of the results by Thomas is included in Part 4, but the interested reader might wish to consult the complete report.

George Thomas, "Multiclass spread-spectrum slotted ALOHA: A new approach for high efficiency in very large packet networks," Tech. Rep SCPO-TR-3, Satellite Communications Programmes Office, Indian Space Research Organisation Headquarters, Dept. of Space, Government of India, Bangalore 560 009, India.

Finally, note that the question of the spatial capacity of ALOHA transmitter networks is included in several of the papers in Part 4. But considerable additional results in this area are available in the reports listed below.

1. John Silvester, "On the spatial capacity of packet radio networks," UCLA, Computer Science Dept. Rep. No. UCLA-ENG-8021, May 1980.
2. Deepak Sant, "Some models of packet broadcasting networks," ALOHA Systems Tech. Rep. B81-1, University of Hawaii, Honolulu, Feb. 1981.
3. Hideaki Takagi, "Analysis of throughput and delay for single and multi-hop packet radio networks," UCLA, Computer Science Dept. Rep. No. CSD-830523, May 1983.

1.3.5 Applications

The general architecture of the worldwide telephone network assumed its present form in North America at the end of the 19th century. This architecture is based upon the use of point-to-point telecommunications channels, connected by a hierarchy of switches to provide for interconnection of large numbers of voice users. During the last two decades this telecommunications architecture has been augmented by an increasingly diverse set of special purpose telecommunications structures, motivated in many but not all cases by the need for a new telecommunications architecture better suited to digital traffic. Many of these new structures, such as local area networks (LANs), packet radio networks, VSAT networks, and digital personal communication networks (PCNs), are based on the use of a multiple access channel. Part 5 contains a collection of papers that looks at some of these new telecommunications structures with particular emphasis on the use of multiple access and this new telecommunications architecture.

The definitive reference on the early days of modern computer networks is the Harvard Ph.D. thesis of Robert Metcalfe, published as a Project MAC report at MIT.

Robert Melancton Metcalfe, "Packet communication," Rep. MAC TR-114, Project MAC, Massachusetts Institute of Technology, Cambridge MA, Dec. 1973.

After the demonstration of the effectiveness of a multiple access communications architecture in the ALOHANET in the early 1970s, the commercial exploitation of this technology was retarded by the limitations imposed by conventional frequency allocation procedures. With the exception of the use of multiple access architectures employed in the international MARISAT system in 1976 and in several VSAT networks during the late 1980s, the only possibility for commercial exploitation of this technology was in the use of multiple access procedures in cable-based local area networks. But in 1983 the ability to fully develop the commercial potential of multiple access channels was provided with an important boost by the decision of the United States Federal Communications Commission to authorize three separate multiple access frequency bands for shared use by commercial interests.

Finally, it should be noted that the large-scale investments in telecommunications based on conventional point-to-point architectures common in North America, Europe, and Japan do not exist, or exist in a much more limited form, in most of the rest of the world. The consequence of this fact is that in many of the developing countries of the world, the new telecommunications structures, based on multiple access channels, assume an even greater importance than they do in developed countries. Thus digital cellular radio, VSATs, and packet radio local area networks may provide an option to telecommunication planners in developing countries that will allow them to develop new, appropriate telecommunications structures more suited to their needs than those structures that are the norm in developed countries. The general background for these developments is addressed in:

Norman Abramson, "Satellite data networks for national development," *Telecommunications Policy*, vol. 8, no. 1, March 1984.

One specific proposal to apply this technology to meet the needs of developing countries by means of a digital network of 100,000 VSATs is described in:

Mahesh Kumar Goel, "Some studies on a low cost VSAT satellite mesh network," Ph.D. Thesis in the Dept. of Electrical Engineering, Indian Institute of Technology, Delhi, New Delhi 110 016, India, Dec. 1989.

1.4 Acknowledgment

The editor is grateful for the assistance of Robert Griffin in the collection and evaluation of relevant papers for an early version of this collection.

Part 2
General Channel Considerations

Poisson, Shannon, and the Radio Amateur*

J. P. COSTAS†, SENIOR MEMBER, IRE

Summary—**Congested band operation as found in the amateur service presents an interesting problem in analysis which can only be solved by statistical methods. Consideration is given to the relative merits of two currently popular modulation techniques, SSB and DSB. It is found that in spite of the bandwidth economy of SSB this system can claim no over-all advantage with respect to DSB for this service. It is further shown that there are definite advantages to the use of very broadband techniques in the amateur service.**

The results obtained from the analysis of the radio amateur service are significant, for they challenge the intuitively obvious and universally accepted thesis that congestion in the radio frequency spectrum can only be relieved by the use of progressively smaller transmission bandwidths obtained by appropriate coding and modulation techniques. In order to study the general problem of spectrum utilization, some basic results of information theory are required Some of the significant work of Shannon is reviewed with special emphasis on his channel capacity formula. It is shown that this famous formula, in spite of its deep philosophical significance, cannot be used meaningfully in the analysis and design of practical, present day communications systems. A more suitable channel capacity formula is derived for the practical case.

The analytical results thus obtained are used to show that broadband techniques have definite merit for both civil and military applications. Furthermore, such techniques will result in far more efficient spectrum utilization in many applications than any practical narrow-band, frequency-channelized approach. Thus broad-band techniques can, in many cases, increase the number of available "channels." With regard to military communications it is shown that the ability of a communication system to resist jamming varies in direct proportion to the transmission bandwidth for a given data rate. Thus narrow-band techniques lead progressively to more expensive communications systems and less expensive jammers. It is concluded that in the military field broad-band techniques are not only desirable but also often mandatory.

I. INTRODUCTION

MOST common usage of the radio frequency spectrum involves operation at specified frequencies as assigned by the appropriate regulatory agencies in the various countries. In contrast, the radio amateur service is assigned various bands of frequencies and properly licensed stations are permitted to operate at any frequency within these bands. This freedom of choice of frequency is necessitated by the obviously impossible administrative problem of assigning specific frequencies to specific stations and, furthermore, the available bandwidths fall short by several orders of magnitude of providing exclusive channels to each authorized station. Thus, as one might suspect, the situation in the amateur bands is a chaotic one in terms of mutual interference. There is very little tendency to "channelize" for several reasons. The crowded conditions normally leave no empty spaces in frequency so that a station starting operation has no choice but to transmit "in between" two strong stations or on top of a weaker station. Furthermore, at the higher HF frequencies, the ionospheric "skip" makes it impossible to choose a good operating frequency by listening, since the signal situation will be radically different between two points spaced many miles apart. Thus, the very nature of the amateur service would lead one to expect that any meaningful analysis of this problem must be based on a statistical approach.

A mathematical study of amateur radio communications can be of use in other important areas. Consider, for example, military communications where allocation of frequencies cannot possibly prevent interference due to the use of the same frequencies by the opposing forces. It is not hard to imagine that under such conditions each operator will shift frequency and take other appropriate action in order to get his message through. Thus, in a combat area we might well expect to find the very same chaos in the communications services that we observe in the amateur bands today. Certainly in such situations interference cannot be eliminated by allocation; interference will exist and we must simply learn to live with it. We are not speaking here of intentional jamming but rather of the casual interference which is inevitable when two opposing military forces (which today depend heavily on radio) attempt to operate independently and use the same electromagnetic spectrum. The problem of intentional jamming will be treated in detail in Section VI.

In the analysis of the radio amateur problem which follows, three modes of operation are compared. It is first assumed that all stations employ suppressed-carrier single-sideband (SSB). Then exclusive use of suppressed-carrier AM (DSB) is assumed. Finally, a frequency diversity system is examined in which each station transmits a large number of identical signals at randomly selected frequencies in the band. Intuitively we might suspect that SSB would be superior to DSB because of the two-to-one difference in signal bandwidths. The frequency diversity system is intuitively ridiculous because it apparently "wastes" bandwidth rather indiscriminantly. As we shall see, intuition is a poor guide in these matters. The feeling that we should always try to "conserve bandwidth" is no doubt caused by an environment in which it has been standard practice to share the RF spectrum on a frequency basis. Our emotions do not alter the fact that bandwidth is but one dimension of a multidimensional situation.

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† General Electric Co., Syracuse, N. Y.

Reprinted from *Proc. IRE*, vol. 47, no. 12, pp. 2058–2068, Dec. 1959.

II. Congested Band Analysis

SSB Case

We shall first consider the case of exclusive use of SSB. The spectral situation is shown in Fig. 1 as it might

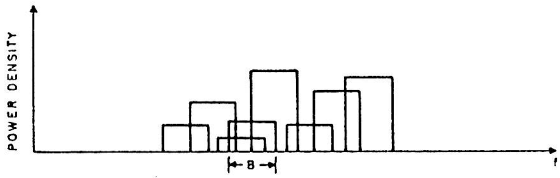

Fig. 1—Power density spectra—SSB case.

appear to a particular receiver. Each signal occupies a bandwidth B (equal to the baseband bandwidth for SSB), has a location in frequency independent of all other signal locations, and has an amplitude of power density independent of all other signal amplitudes. The signal amplitudes will have a probability distribution which will be specified at a later time. While the frequency locations of the various signals are distributed at random, it can be said that, on the average, there are a given number of signals per given unit of bandwidth. Thus, we may specify the density of loading of the band by a quantity k which represents the average number of signals per unit bandwidth. It happens that we shall need to know the probability of having a given number of signals ν falling in a bandwidth B. This, of course, is given for the conditions specified by the celebrated distribution of Poisson as

$$P(\nu, B) = \frac{(kB)^\nu}{\nu!} e^{-kB}, \tag{1}$$

where $P(\nu, B)$ is the probability of having ν signals in the bandwidth B if there are k signals per unit bandwidth on the average.

The choice of the distribution function for the signal power densities is somewhat arbitrary and, as far as the final results are concerned, apparently not particularly critical. It is physically reasonable and mathematically convenient to choose the chi-squared distribution[1]

$$p_\nu(x) = \frac{x^{\nu/2-1}e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)} \qquad (x \geq 0), \tag{2}$$

where $p_\nu(x)$ is the probability density function of the spectral amplitude which results from the summation of ν independent signals. For $\nu = 1$ the distribution has a mean of unity. This specifies that the average signal strength at the receiver is unity which results in no loss of generality for this application.

For convenience only, we shall assume that we are receiving a signal of average strength and want to find the probability that the Signal-to-Noise Ratio at the receiver output will equal or exceed a specified value. For SSB operation, the SNR at RF is the same as the SNR at the receiver output. We shall estimate the effective noise level at the receiver input by noting the interference level at the center of the pass band. We shall now determine the probability that the interference level will be less than or equal to J, which means that the SNR at the receiver output will be equal to or greater than $1/J$, since the desired signal is assumed to be of average strength of unity. Let $P_{SSB}(\text{SNR} \geq 1/J)$ be this probability. Then

$$P_{SSB}(\text{SNR} \geq 1/J) = P(0, B) + P(1, B)\int_0^J p_1(x)dx + P(2, B)\int_0^J p_2(x)dx + P(3, B)\int_0^J p_3(x)dx + \cdots, \tag{3}$$

which states that the event will occur if there are no signals in B, if there is one signal in B with amplitude less than J, if there are two signals in B the sum of whose amplitudes is less than J, etc. It should be clear that if an interfering signal is to contribute to the measurement of interference, its lowest frequency must fall somewhere within a frequency band extending from the center of the pass band to B cycles below. It is to this event that the terms $P(\nu, B)$ in (3) refer. Substituting (1) and (2) into (3) one obtains

$$P_{SSB}(\text{SNR} \geq 1/J) = e^{-kB}\left[1 + \sum_{\nu=1}^{\infty} \frac{(kB)^\nu}{\nu!}\int_0^J \frac{x^{\nu/2-1}e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)}dx\right]. \tag{4}$$

Evaluation of (4) for a fixed J and variable k will give the probability of exceeding a certain receiver output SNR as a function of band loading. For example, for $J=1$ the expression gives the probability of exceeding a 0-db SNR when receiving a signal of average strength, or of exceeding a +3-db SNR when receiving a signal of twice (power) average strength, etc. Fortunately, the integral function in (4) is tabulated[2] and the series converges rather rapidly, so that the numerical work involved in evaluating (4) is not too difficult.

DSB Case

As might be suspected, the analysis of the case involving exclusive use of DSB is quite similar to the SSB analysis. There are two important differences to be noted. First, since all transmitted signals have twice the baseband bandwidth it is to be expected for a given band loading there will be more interfering signals involved than in the case of SSB. In the DSB analysis then, we will be concerned with the probability of having ν interfering signals in a bandwidth $2B$, using the same estimate of effective receiver input noise level as before.

[1] H. Cramer, "Mathematical Methods of Statistics," Princeton University Press, Princeton, N. J., ch. 18; 1946.

[2] C. D. Hodgman, "Mathematical Tables," Chemical Rubber Publishing Co., Cleveland, Ohio, p. 257; 1946.

Thus the Poisson distribution $P(\nu, 2B)$ must be used in the equation equivalent to (3) for the DSB analysis. This represents a loss caused by increased transmission bandwidth; there is a compensating gain as will be seen. The second difference between the SSB and DSB analysis involves the relationship between the predetector and postdetector SNR's. In SSB these two ratios are the same. In DSB the postdetector SNR is 3 db better than the predetector value. This difference arises because of the coherent addition of upper and lower sideband components of the signal and incoherent addition of the corresponding interference components in the synchronous detector. Thus, for identical output SNR's the interference power density will be two times as great relative to desired signal density in DSB as compared to SSB. Consequently, in the equation equivalent to (3) the upper limit on all integrals must be changed from J to $2J$ in order that J have the same meaning in both cases.

When the two changes discussed above are made, the probability of exceeding an output SNR of $1/J$ for a desired signal of mean strength (unity) becomes

$$P_{\text{DSB}}(\text{SNR} \geq 1/J)$$
$$= e^{-2kB}\left[1 + \sum_{\nu=1}^{\infty} \frac{(2kB)^{\nu}}{\nu!} \int_0^{2J} \frac{x^{\nu/2-1}e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)}\, dx\right]. \quad (5)$$

A comparison of (4) and (5) shows that the increased bandwidth of DSB has in some ways been detrimental ($2kB$ in place of kB in the Poisson distribution), and in other ways beneficial ($2J$ in place of J in the integral expression). As later calculations show, the increased bandwidth of DSB does not affect the relative congested band performance as compared to SSB in any significant manner. We might begin to suspect that the efficient use of broader bandwidths in a congested operating band is not necessarily a bad idea. The broader bandwidth signals will increase the tendency of frequency overlap and tend, in a sense, to cause more interference. This is obvious. *What is not so obvious is the fact that the increased bandwidth gives to the receiving system an increased ability to discriminate between the desired signal and the interference.* In order to investigate further the effects of increasing transmission bandwidth, a rather simple form of broad-band technique will now be analyzed.

Frequency Diversity Case

For this example we shall use the SSB mode of transmission (although the DSB mode would yield identical results), in a somewhat unusual manner. Each station will transmit not one but M (where M is a large number) identical signals at randomly chosen frequencies in the congested band. The receiver must know these frequency locations so that all M signals may be received, detected, and added coherently to produce the receiver output signal. With each station transmitting M identical signals, the interference spectrum amplitude will, with nearly unit probability, be very nearly equal to a constant value at all frequencies for sufficiently large M. This value may be determined quite easily by inspection.

Consider first the normal SSB situation without diversity. The received signals are distributed in amplitude of power density about a mean of unity. Thus, the average received power is B watts per station. Since there are k stations per cycle on the average, the mean interference power density will be kB watts per cycle. Going from one transmission to M transmissions per station (assuming the power of each station is now split evenly between the M signals) does not alter the value of the average interference power density. In the diversity case this *average* value will be very nearly the *actual* value of interference density level which will exist at all frequencies and at all times. The diversity receiver output SNR may now be easily calculated.

Each of the M signals will have a power B/M (for the average signal strength case) and the noise power accepted in receiving each of the M signals will be kB^2. The RF SNR at each of the M frequencies will be $1/MkB$ and coherent addition of M such signals will yield an output SNR of $1/kB$. So then

$$(\text{SNR})_{\text{Div}} = \frac{1}{kB} \quad (6)$$

on a power basis for a desired signal of mean strength. Note that in (6) we are able to specify the precise SNR, while in the SSB and DSB cases of (4) and (5) we can only predict the probability or the percentage time the SNR will exceed a given value.

III. Results and Discussion—Congested Band

The results represented by (4)–(6) may be interpreted in many different ways. For the purposes of this discussion let us assume that voice communications is involved and that message reception will be considered successful if the receiver output SNR equals or exceeds unity or 0 db. Keep in mind that this is not a commerical service but rather a service where the operator is willing to exert some effort in order to understand what is being said. Thus, the 0-db choice is probably reasonable with regard to sentence intelligibility where the interference is of an incoherent nature. The three equations will then be used to calculate the circuit reliability for signals at 0, +3, +6, and +9 db relative to mean signal strength as a function of kB, the band loading expressed in average number of stations per audio bandwidth. The resulting graphs are shown in Figs. 2 through 5. Turning first to Fig. 2, which assumes a received signal of mean strength, we note that the circuit reliability drops rather rapidly with band loading for both SSB and DSB. SSB shows some advantage, but of a small amount, at loadings which result in a reasonable reliability percentage. An estimate of the increased number of users for the same performance which results from SSB use may be obtained by drawing a line horizonally from any given