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IMPACT AND OPTIMUM TRANSMISSION OF WAVES

SOME THEORETICAL AND EXPERIMENTAL STUDIES

BY

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D *Optimization of wave transmitting joints*, University of Luleå, Technical Report No. 1979:80T. Also presented at Svenska Mekanikdagar, Göteborg, Sweden, May 11-12, 1979.

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CONTENTS

KEY-WORDS

Impact, optimization, elastic, wave, energy, transmission, impedance, joint.

IMPACT AND OPTIMUM TRANSMISSION OF WAVES

INTRODUCTION

Impact is said to occur when two bodies collide. Instances of undesired collisions are numerous and well known. In many engineering applications, however, impact is intended and used to advantage. Central longitudinal impact of slender bodies has a number of important technical applications due to (i) the large forces generated through impact and (ii) transformation of the mechanical energy produced by the impact. Some examples are pile driving, riveting and percussive rock drilling.

The classical theory of impact of rigid masses is insufficient for explaining the various processes occuring in these applications. An improved theory (a travelling wave theory or pulse theory) which takes into account the elasticity of the colliding bodies was given about a century ago by Neumann [1] and de Saint-Venant [2,3]. Since then the subject has been treated by a large number of authers, see, e.g.,[4-6]. Further improvements have been made where the various three-dimensional effects are taken into account [7,8]. In engineering applications of the pulse theory, however, the analyses are often restricted to the simplest cases of one-dimensional impact situations. The cross-sectional properties of the impacting rods are assumed to be either constant or varying in steps along the rod lengths as in [9,10]. This is because of the difficulties involved in the analytical treatment of the case of arbitrarily varying cross-sections [11]. The case of a finite conical rod has been studied in some detail in paper A. For different degrees of conicalness, comparisons are made between the one-dimensional analytical, experimental and three-dimensional finite element results. The results provide insights about the range of validity of the one-dimensional model and about the usefulness of the finite element method for treating impact problems.

A substantial amount of work is currently being undertaken in the field of optimized structural design. This is evident from the recent literature. See, e.g., [12,13] for a review of the field. More specifically, in the area of continuous elastic vibrating systems two kinds of problems have received considerable attention, (i) Maximizing the lowest characteristic value (eigenvalue) [14-16], and (ii) minimizing the dynamic response for various applied loadings [17,18].

Some applications of longitudinally vibrating rods have been mentioned earlier. In view of such applications another problem of interest is to maximize the efficiency of energy transmission, in other words to minimize the losses of energy due to reflections from inhomogeneities in the vibrating rods. This is the subject of papers C, D and E. In these papers the transmission of elastic wave energy through a joint between two uniform rods is studied. The efficiency of energy transmission (defined as the ratio of transmitted to incident wave energy) is maximized. The interest in these problems is mainly due to their application in percussive drilling. However, the results can be directly interpreted to some other fields like electromagnetic waves in transmission lines and shallow water waves. The concept of characteristic impedance, recapitulated in paper B facilitates such interpretations.

Like the impact problem treated in paper A, the optimization problems treated in papers C to E also concern the propagation of longitudinal elastic waves in rods with variable cross-sections. The motion of such rods is governed by the Webster horn equation which is thoroughly discussed in [19].

In percussive drilling [20], the kinetic energy of a hammer is transformed through impact into elastic stress wave energy. Hammers and drill rods of cylindrical shapes are commonly employed in modern drilling machines. Therefore, a rectangular stress pulse is generated by the impact. This stress pulse propagates along the drill rods which are often connected by (cylindrical) joints. Losses in the energy transmission occur due to reflections at the rod-joint interfaces. One way of minimizing these losses is to optimize the shape of the incident pulse for a given joint. Such optimum shapes have been obtained in paper C, for pulses with a fixed duration. The efficiencies of energy transmission are evaluated for the optimally shaped pulses as well as for the corresponding rectangular pulses. The improvements in the efficiency turn out to be generally small. Moreover, it may not be possible to realize such pulses through impact [21].

Another way of improving the efficiency is to optimize the shape of the joint for a fixed (say rectangular) incident pulse. This is the subject of paper D where optimum shapes (or impedance distributions) are obtained for a joint with a fixed mass and length. The efficiencies for the optimally shaped joints are compared with those for the corresponding cylindrical joints. Significant improvements in the efficiency turn out to be possible through joint optimization. For a case studied in detail improvements of up to 30 per cent are obtained.

The optimum joint shapes turn out to include large and abrupt changes in impedance over the joint length. Impedance ratios of up to 90 are encountered for the case mentioned above. For such joint shapes the three-dimensional effects may become important and hence there may be doubts about the validity of the one-dimensional model employed. Therefore, experiments were performed on some optimally shaped joints as well as on cylindrical joints. Results of these experiments are reported in paper E. The results support the validity of the theoretical model used in papers C and D. Usefulness of the one-dimensional model is clearly demonstrated even in cases where relatively large and abrupt changes occur in the impedance of a wave transmitting rod.

Next follows the detailed summaries of papers A to E.

SUMMARY OF APPENDED PAPERS

Paper A: Longitudinal elastic impact between a finite conical rod and a long cylindrical rod is studied (i) experimentally, (ii) analytically, by using one-dimensional wave theory to obtain a closed-form solution, and (iii) numerically, by using a three-dimensional axisymmetric finite element model. The results from (i)-(iii) are compared for four conical rods. To obtain increasingly three-dimensional behaviour, conical rods with half apex-angles of 5°, 10°, 15° and 25° are investigated. The one-dimensional model accurately predicts the response for the 5°-cone. The discrepancies between one-dimensional analytical results and experimental or finite element results increase for increasing cone angles. The agreement between the experimental and the finit element results is quite good in general. The effect of contact conditions between the impacting surfaces is also investigated. Friction between the impacting surfaces is found to have negligible effects on the strain response. Details regarding the one-dimensional analytical results are given in [22].

Paper B: The purpose of this brief note is to point out a frequently overlooked role of the concept of impedance in one-dimensional wave propagation problems. For the case of longitudinal waves in rods, it is emphasized that the influence of variations in the cross-sectional area A, Young's modulus E, and density ρ can be combined in a single parameter called the impedance $Z = A\rho c = AE/c$, where $c = (E/\rho)^{1/2}$ is the wave speed. Thus, any combinations of A, ρ and E which correspond to the same Z should yield the same results. This means that in paper A the results on cones are also valid for other combinations of A, ρ and E which yield a quadratic variation of the impedance (same as that of the cross-sectional areas of the cones).

Paper C: The problem treated in this paper concerns the optimization of the shape of an incident pulse of given duration such that the energy transmitted through a given rod-joint system is maximized. The interest in the problem is mainly due to its applications in percussive drilling. However, by making use of the concept of impedance discussed in paper B, the results can be directly interpreted in other fields like electromagnetic waves in transmission lines and shallow water waves.

The optimization problem is formulated in general terms applicable to all such fields. The optimum wave shapes are obtained for two special cases: (i) a joint with constant impedance and (ii) a joint with concentrated mass. (i) leads to a matrix eigenvalue problem and a non-unique solution, whereas, (ii) leads to an eigenvalue problem for an integral equation and a unique solution. In both cases the efficiencies of energy transmission for the optimum wave shapes are compared with those for the corresponding rectangular waves (as the latter are commonly employed in modern percussive drilling machines) [23]. The gains in the efficiency through such an optimization generally turn out to be small (a few per cent). However the results clarify how much the efficiencies can be improved by optimizing the wave shape.

Paper D: An alternative way of improving the efficiency of energy transmission for the rod-joint problem treated in paper C is to optimize the shape of the joint rather than the shape of the incident pulse. This is the subject of paper D. The incident pulse is assumed to be of rectangular shape with a given duration. Optimum shape(s) (i.e. impedance distribution(s)) are determined for a joint having a given mass and length. The method employed is as follows. By dividing the joint length into N segments, the joint impedance function Z_0 is discretized into N constant impedances $Z_0^{(1)}$, $Z_0^{(2)}$, ..., $Z_0^{(N)}$. Using the theory developed in paper C, the efficiency of energy transmission η is expressed as a non-linear function of the N variables, i.e.,

$$\eta = \eta(Z_0^{(1)}, Z_0^{(2)}, \ldots\ldots, Z_0^{(N)}) \qquad (1)$$

The constant mass constraint implies a linear relation between the N variables

$$Z_0^{(1)} + Z_0^{(2)} + \ldots\ldots + Z_0^{(N)} = \text{constant}. \qquad (2)$$

Also since the impedances must be positive we have

$$Z_0^{(1)} > 0,\ Z_0^{(2)} > 0,\ \ldots\ldots,\ Z_0^{(N)} > 0. \qquad (3)$$

Thus the problem is to maximize the function η given by (1) and subjected to the constraints (2) and (3). This non-linear programming problem with linear constraints is then solved numerically using the reduced gradient method which is a slightly modified form of the method of steepest descents.

The results show that quite significant improvements in the efficiency can be achieved by optimizing the joint impedance distributions. For a case studied in detail the improvement is about 30 per cent. However, the optimally shaped joints turn out to include large and abrupt changes in the joint impedances over their lengths. Impedance ratios of about 30 to 90 are encountered for the case studied in detail.

<u>Paper E</u>: Occurrence of large and abrupt changes of impedance in the optimum joints of paper D raises the question of validity of the one-dimensional model employed. Thus experimental investigations were needed. This paper presents results of such experiments.

Tests were performed on the optimum joints corresponding to N = 1, 3 and 5 in paper D. Incident stress pulses were produced through longitudinal impact between a hammer and a rod, employing a compressed air gun. Using strain gauges at convenient locations, the incident, reflected and transmitted waves were recorded on a transient recorder. The experimental values of the efficiency could thus be obtained. Comparisons with the theoretical values show a difference of less than 5 per cent in all cases. Also the complete forms of the experimentally obtained incident, reflected and transmitted waves were compared with the corresponding curves according to the one-dimensional model. The agreement between theory and experiment clearly supports the validity of the one-dimensional model and confirms the results obtained in paper D.

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A

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ELASTIC IMPACT BETWEEN A FINITE CONICAL ROD AND A LONG CYLINDRICAL ROD

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