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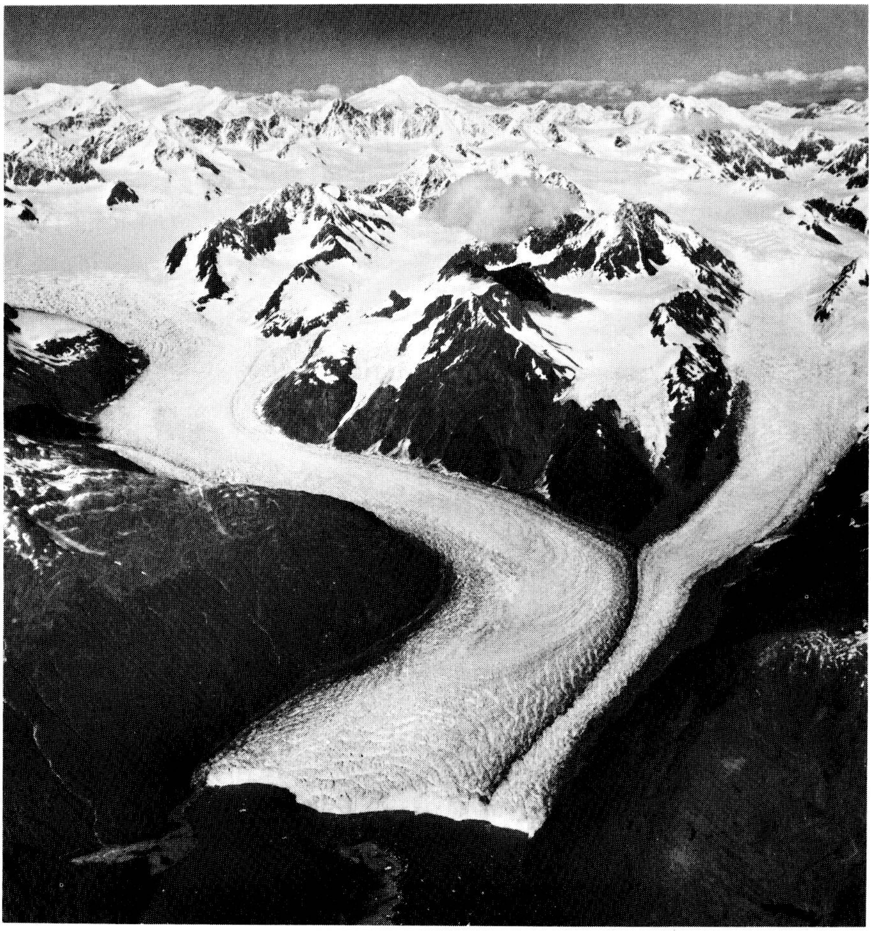

Meares Glacier, Unikwak Bay, Prince William Sound, Alaska. The flow of the glacier ice is an example of a non-Newtonian fluid in motion. (Photograph by Austin Post, University of Washington.)

PREFACE

This book is intended as a text for a first course in fluid mechanics offered to engineering students. We feel that an economy of time and effort has been achieved by the arrangement and selection of subject matter as presented. Increased needs and interests in presenting the various engineering sciences have encouraged us to consider the following objectives:

1. To introduce vector field operations and theorems early in the text, and use these where they confer advantages with respect to brevity and generality of formulations.
2. To preserve a consistent method of deriving control-volume equations from corresponding system equations.
3. To emphasize the need of constitutive relations as well as the basic equations of fluid mechanics in the solution of physical problems.

The book begins with an introduction to basic definitions, field concepts, and pertinent field theorems. Chapters 2 through 6 present basic laws such as conservation of matter, momentum, and energy as well as the concept of state of stress and its relation to forces in a fluid field. Chapter 7 discusses the motion of an ideal fluid, while Chapter 8 introduces a simplified approach to the solution of problems through an introduction to dimensional analysis and similarity. The latter chapter also presents the idealization of one-dimensional flows and their utility in solving simple fluid flow problems of engineering relevance.

The remainder of the text is concerned with some beginning concepts of turbulent and boundary-layer flows. Fluid machinery, controls, and instrumentation have not been included except as examples and assigned problems. The coverage of fluid statics, dimensional analysis, and simplified frictional flows has been reduced relative to that found in existing texts.

The text includes more material than would be covered normally in a course of three semester hours. Chapters 1 through 6 would be a necessary part of any elementary course in fluid mechanics presented from a vector point of view. Since there is little interdependence of Chapters 7, 8, and 9, these may be used to satisfy a given course objective.

The text is written for students who have had courses in vectorial mechanics and thermodynamics. A number of examples are included and these should be considered as an *essential part* of the presentation. There are adequate problems and self-study questions at the end of each chapter to provide different assignments for several semesters.

The authors wish to acknowledge with gratitude the assistance given to them by the faculty and administration of their college. Special appreciation is due to Dr. Daniel F. Jankowski, a colleague, for valuable assistance and suggestions, and Dean Lee P. Thompson for continued encouragement and support. Any shortcomings of this work are due entirely to the authors.

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LIST OF MOST USED SYMBOLS

a	linear acceleration
A	cross-sectional area
A	a field variable
A^*	reference area at $N_M = 1$
B (**b**)	an extensive continuous vector function (specific quantity)
B (b)	a continuous scalar function (specific quantity)
B_i (b_i)	ith component of extensive vector variable (specific quantity)
c	sonic celerity
c_v, c_p	specific heat at constant volume, at constant pressure
C_f	surface-resistance coefficient
C_D	drag coefficient
curl	curl operator
D	diameter
div	divergence operator
div **grad**	laplacian operator
D_H	hydraulic diameter
D/Dt	material derivative operator
e	surface-roughness height
e_{ii}	component of longitudinal rates of strain
e, exp	exponential
E (e)	energy (specific quantity)
E	modulus of elasticity
f	friction factor
$\mathbf{F}_B$ ($\mathbf{f}_B$)	body force (specific quantity)
$\mathbf{F}_S$	surface force
$\mathbf{F}_w$	wall force
g	gravitational force per mass
grad	gradient operator

h_L	head loss
h	specific enthalpy
$\mathbf{H}$	moment of momentum
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in x, y, z directions, respectively
K_L	loss coefficient
$d\mathbf{l}$	differential line vector
l	mixing length
l, m, n	direction cosines
L	length
L_{crit}	choking length for Fanno flow
$L^{\dagger}$	transition length
m	mass
$\dot{m}$	mass rate of flow
$\mathbf{M}_B$	moment of body forces
$\mathbf{M}_S$	moment of surface forces
$\mathbf{n}$	outward normal unit vector to an area
n	roughness parameter
n	flow-behavior index
N_C	Cauchy number
N_E	Euler number
N_F	Froude number
N_M	Mach number
N_R	Reynolds number
N_S	Strouhal number
p	pressure
p_0	isentropic pressure stagnation
Δp_L	pressure loss due to friction
p^*	reference pressure at $N_M = 1$
P	perimeter
$\mathbf{P}$ ($\mathbf{p}$)	linear momentum vector (specific quantity)
$\dot{\mathbf{q}}$	rate-of-heat-transfer vector per area
Q	quantity of heat transfer
r, θ, z	cylindrical coordinates
$\mathbf{r}$	position vector
R_H	hydraulic radius
R	radius
R	specific gas constant
R	function of r only
$d\mathbf{S}$	differential surface-area vector
S	surface-area magnitude
S_0	slope
$\$$ ($\cancel{s}$)	entropy (specific quantity)
t	time
$\mathbf{t}$	unit tangent vector
T	absolute temperature
T_0	stagnation temperature
U (u)	internal energy (specific quantity)
V_x, V_y, V_z	components of velocity in x,y,z
V_r, V_θ, V_z	components of velocity in r,θ,z
$\dot{\cancel{V}}$	volume flow rate
$\cancel{V}$	volume
V_*	shearing stress velocity
$\mathbf{V}$	velocity
$\bar{V}$	time-mean average speed
$\tilde{V}$	time-fluctuating speed
V_{SA}	space-average velocity
W	work
$\frac{dW_s}{dt}$	rate of shaft work
x, y, z	spatial coordinates
y_{ls}	laminar sublayer thickness
$\bar{z}$	z coordinate of centroid of an area
γ	specific-heat ratio
γ_{ij}	component of rate of shear strain
Γ	circulation
δ	boundary-layer thickness
δ^*	boundary-layer-displacement thickness
Δ	finite quantity or change of a quantity
$\varepsilon_r, \varepsilon_\theta, \varepsilon_z$	unit vectors in cylindrical coordinates

η	defined nondimensional variable
η_p	plastic viscosity
Θ	function of θ only
κ	thermal conductivity
μ	absolute viscosity
ν	kinematic viscosity
ρ	mass density
ρ_0	isentropic stagnation density
σ	normal stress
$\bar{\sigma}$	mean bulk stress
τ	shearing stress
τ_0	yield shear stress; also, wall shear stress
ϕ	velocity potential
Φ_B	specific body-force potential
ψ	stream function
$\boldsymbol{\omega}$	angular velocity
$\dot{\boldsymbol{\omega}}$	angular acceleration
$\boldsymbol{\Omega}$	vorticity

Special notation

$\simeq$	approximately equal
$(\bar{\ })$	time-mean average (used with p, $\mathbf{V}$, $\mathbf{f}_B$, and ρ)
$(\ ')$	time-fluctuating part of a variable in turbulent flow
$\lvert\ \rvert$	absolute value
$\oint_S$	integration over a closed surface S
$\oint_C$	integration around a closed path C in direction shown
$0(\ )$	order of magnitude
$\sum$	summation

CONTENTS

1

BASIC DEFINITIONS AND INTRODUCTION TO FIELD CONCEPTS

1.1 Introduction

A basic knowledge of fluid mechanics is essential to engineers and applied scientists because they will likely become involved directly or indirectly in problems involving the flow of fluids. A diversity of application is evident in the following list of prediction and design activities associated with fluids in motion:

aerodynamic surfaces for desired lift
structural surfaces to withstand temperatures and forces of a fluid
propulsion systems
energy conversion systems
transport of fluids
bioengineering
fluid control systems
fluid computers

climatology
oceanography

Although these examples are by no means exhaustive, they do serve to establish the varied applications of a single fascinating discipline.

The study begins with some basic definitions and concepts used to represent the observed behavior of fluids; emphasis is placed on the physical meaning of each representation made in mathematical language.

1.2 Fluids and Continuum Concepts

A *fluid* is defined as any substance deforming continuously when subjected to a shear stress regardless of how small the shear stress may be. This means that fluids will "flow" when subjected to a shear stress; and, conversely, flowing fluids will generally exhibit the presence of shear stresses. (A detailed discussion of stresses is deferred until Chapter 3.) It is of interest to note the difference between a fluid and a solid deformed in the elastic range. A solid deformed by a shear stress is capable of resisting deformation within limits by generating an internal stress proportional to the deformation. If the externally applied shear stress is not too great (so that it does not exceed the limit of elastic behavior for the material), then the internal stress developed can become large enough to equalize the external stress. In such a case deformation ceases; that is, the solid does not deform continuously. The above definition could be used as a criterion to distinguish fluids from "nonfluids" by experiment.

Since this study will not consider fluid behavior from the point of view of kinetic theory, a conceptual model is needed that can be used in a physical as well as a mathematical sense. This model is referred to as the *continuum*. Instead of defining a continuum at the outset, a hypothetical experiment is proposed. Consider a rather large and finite region in a fluid whose mass is not uniformly distributed in a given space, and let this region be completely bounded by a spatial volume $\mathcal{V}_1$. Next imagine a point to be fixed in position inside this volume. The density ρ_1 of the fluid inside this space may be thought of as simply the mass m_1 of this fluid divided by $\mathcal{V}_1$. Such a density would obviously be a gross or average description. At one instant of time one may imagine taking successively smaller regions (that is, $\mathcal{V}_1 > \mathcal{V}_2 > \mathcal{V}_3$, etc.) with each region still surrounding the point but lying wholly inside the previous region, as shown in Fig. 1.1. As sample size is decreased, one would witness an approach to a limiting value because the sample would tend to become more uniform in mass distribution. After reaching a certain volume $\mathcal{V}_s$, any reduction in the sample size beyond this value would yield fluctuations in the calculated value of the gross density. If $\mathcal{V}$ were permitted to approach zero, i.e., reduced to the point it surrounds, the density might approach a very