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# Fluid Mechanics



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# FLUID MECHANICS

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## FLUID MECHANICS

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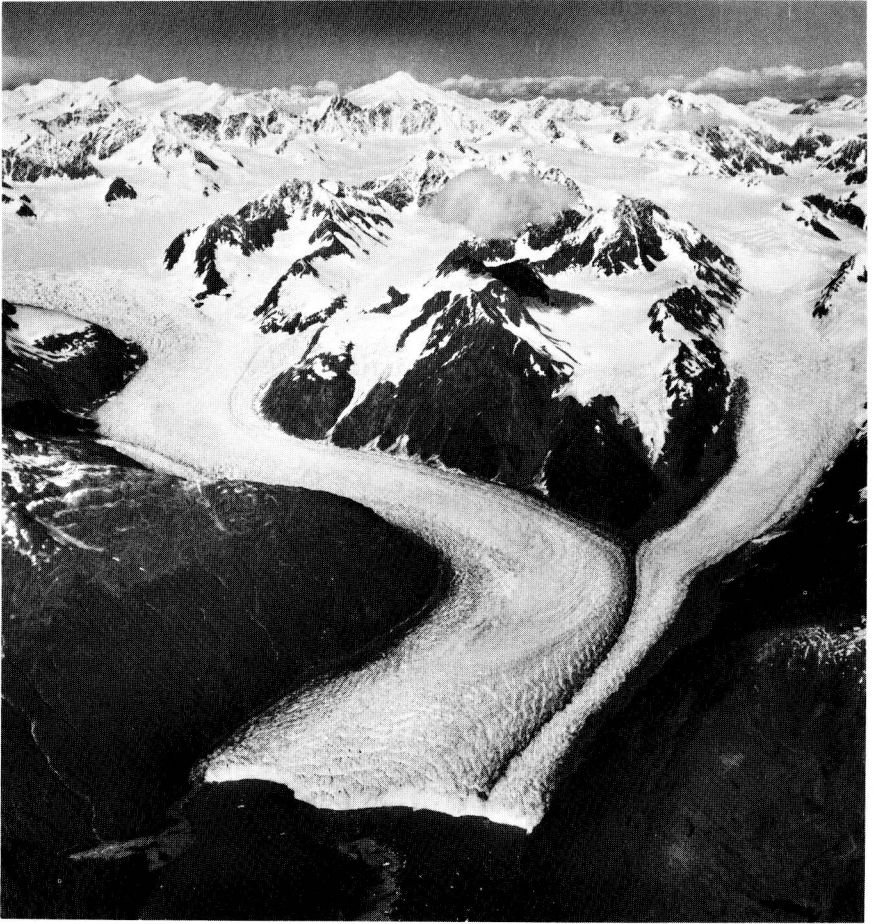
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*Meares Glacier, Unikwak Bay, Prince William Sound, Alaska. The flow of the glacier ice is an example of a non-Newtonian fluid in motion. (Photograph by Austin Post, University of Washington.)*

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# PREFACE

This book is intended as a text for a first course in fluid mechanics offered to engineering students. We feel that an economy of time and effort has been achieved by the arrangement and selection of subject matter as presented. Increased needs and interests in presenting the various engineering sciences have encouraged us to consider the following objectives:

1. To introduce vector field operations and theorems early in the text, and use these where they confer advantages with respect to brevity and generality of formulations.
2. To preserve a consistent method of deriving control-volume equations from corresponding system equations.
3. To emphasize the need of constitutive relations as well as the basic equations of fluid mechanics in the solution of physical problems.

The book begins with an introduction to basic definitions, field concepts, and pertinent field theorems. Chapters 2 through 6 present basic laws such as conservation of matter, momentum, and energy as well as the concept of state of stress and its relation to forces in a fluid field. Chapter 7 discusses the motion of an ideal fluid, while Chapter 8 introduces a sim-

plified approach to the solution of problems through an introduction to dimensional analysis and similarity. The latter chapter also presents the idealization of one-dimensional flows and their utility in solving simple fluid flow problems of engineering relevance.

The remainder of the text is concerned with some beginning concepts of turbulent and boundary-layer flows. Fluid machinery, controls, and instrumentation have not been included except as examples and assigned problems. The coverage of fluid statics, dimensional analysis, and simplified frictional flows has been reduced relative to that found in existing texts.

The text includes more material than would be covered normally in a course of three semester hours. Chapters 1 through 6 would be a necessary part of any elementary course in fluid mechanics presented from a vector point of view. Since there is little interdependence of Chapters 7, 8, and 9, these may be used to satisfy a given course objective.

The text is written for students who have had courses in vectorial mechanics and thermodynamics. A number of examples are included and these should be considered as an *essential part* of the presentation. There are adequate problems and self-study questions at the end of each chapter to provide different assignments for several semesters.

The authors wish to acknowledge with gratitude the assistance given to them by the faculty and administration of their college. Special appreciation is due to Dr. Daniel F. Jankowski, a colleague, for valuable assistance and suggestions, and Dean Lee P. Thompson for continued encouragement and support. Any shortcomings of this work are due entirely to the authors.

*Theodore Allen Jr.*  
*Richard L. Ditsworth*

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# LIST OF MOST USED SYMBOLS

<b>a</b>	linear acceleration	<b>curl</b>	curl operator
<i>A</i>	cross-sectional area	<i>D</i>	diameter
<i>A</i>	a field variable	div	divergence operator
<i>A*</i>	reference area at $N_M = 1$	div <b>grad</b>	laplacian operator
<b>B (b)</b>	an extensive continuous vector function (specific quantity)	<i>D<sub>H</sub></i>	hydraulic diameter
<i>B (b)</i>	a continuous scalar function (specific quantity)	<i>D/Dt</i>	material derivative operator
<i>B<sub>i</sub> (b<sub>i</sub>)</i>	<i>i</i> th component of extensive vector variable (specific quantity)	<i>e</i>	surface-roughness height
<i>c</i>	sonic celerity	<i>e<sub>ii</sub></i>	component of longitudinal rates of strain
<i>c<sub>v</sub>, c<sub>p</sub></i>	specific heat at constant volume, at constant pressure	<i>e, exp</i>	exponential
<i>C<sub>f</sub></i>	surface-resistance coefficient	<i>E (e)</i>	energy (specific quantity)
<i>C<sub>D</sub></i>	drag coefficient	<i>E</i>	modulus of elasticity
		<i>f</i>	friction factor
		<b>F<sub>B</sub> (f<sub>B</sub>)</b>	body force (specific quantity)
		<b>F<sub>S</sub></b>	surface force
		<b>F<sub>w</sub></b>	wall force
		<b>g</b>	gravitational force per mass
		<b>grad</b>	gradient operator



$h_L$	head loss	$R$	function of $r$ only
$h$	specific enthalpy	$dS$	differential surface-area vector
$\mathbf{H}$	moment of momentum	$S$	surface-area magnitude
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in $x, y, z$ directions, respectively	$S_0$	slope
$K_L$	loss coefficient	$\$ (\$)$	entropy (specific quantity)
$d\mathbf{l}$	differential line vector	$t$	time
$l$	mixing length	$\mathbf{t}$	unit tangent vector
$l, m, n$	direction cosines	$T$	absolute temperature
$L$	length	$T_0$	stagnation temperature
$L_{\text{crit}}$	choking length for Fanno flow	$U (u)$	internal energy (specific quantity)
$L^\dagger$	transition length	$V_x, V_y, V_z$	components of velocity in $x, y, z$
$m$	mass	$V_r, V_\theta, V_z$	components of velocity in $r, \theta, z$
$\dot{m}$	mass rate of flow	$\dot{V}$	volume flow rate
$\mathbf{M}_B$	moment of body forces	$V$	volume
$\mathbf{M}_S$	moment of surface forces	$V_*$	shearing stress velocity
$\mathbf{n}$	outward normal unit vector to an area	$\mathbf{V}$	velocity
$n$	roughness parameter	$\bar{V}$	time-mean average speed
$n$	flow-behavior index	$\tilde{V}$	time-fluctuating speed
$N_C$	Cauchy number	$V_{SA}$	space-average velocity
$N_E$	Euler number	$W$	work
$N_F$	Froude number	$\frac{dW_s}{dt}$	rate of shaft work
$N_M$	Mach number	$x, y, z$	spatial coordinates
$N_R$	Reynolds number	$y_{1S}$	laminar sublayer thickness
$N_S$	Strouhal number	$\bar{z}$	$z$ coordinate of centroid of an area
$p$	pressure	$\gamma$	specific-heat ratio
$p_0$	isentropic pressure stagnation	$\gamma_{ij}$	component of rate of shear strain
$\Delta p_L$	pressure loss due to friction	$\Gamma$	circulation
$p^*$	reference pressure at $N_M = 1$	$\delta$	boundary-layer thickness
$P$	perimeter	$\delta^*$	boundary-layer-displacement thickness
$\mathbf{P} (\mathbf{p})$	linear momentum vector (specific quantity)	$\Delta$	finite quantity or change of a quantity
$\dot{\mathbf{q}}$	rate-of-heat-transfer vector per area	$\epsilon_r, \epsilon_\theta, \epsilon_z$	unit vectors in cylindrical coordinates
$Q$	quantity of heat transfer		
$r, \theta, z$	cylindrical coordinates		
$\mathbf{r}$	position vector		
$R_H$	hydraulic radius		
$R$	radius		
$R$	specific gas constant		

$\eta$	defined nondimensional variable
$\eta_p$	plastic viscosity
$\Theta$	function of $\theta$ only
$\kappa$	thermal conductivity
$\mu$	absolute viscosity
$\nu$	kinematic viscosity
$\rho$	mass density
$\rho_0$	isentropic stagnation density
$\sigma$	normal stress
$\bar{\sigma}$	mean bulk stress
$\tau$	shearing stress
$\tau_0$	yield shear stress; also, wall shear stress
$\phi$	velocity potential
$\Phi_B$	specific body-force potential
$\psi$	stream function
$\omega$	angular velocity

$\dot{\omega}$	angular acceleration
$\Omega$	vorticity

### *Special notation*

$\simeq$	approximately equal
$\overline{(\quad)}$	time-mean average (used with $p$ , $\mathbf{V}$ , $\mathbf{f}_B$ , and $\rho$ )
$(\cdot)$	time-fluctuating part of a variable in turbulent flow
$  $	absolute value
$\oint_S$	integration over a closed surface $S$
$\oint_C$	integration around a closed path $C$ in direction shown
$O(\quad)$	order of magnitude
$\sum$	summation

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# 1

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## BASIC DEFINITIONS AND INTRODUCTION TO FIELD CONCEPTS

### *1.1 Introduction*

A basic knowledge of fluid mechanics is essential to engineers and applied scientists because they will likely become involved directly or indirectly in problems involving the flow of fluids. A diversity of application is evident in the following list of prediction and design activities associated with fluids in motion:

- aerodynamic surfaces for desired lift
- structural surfaces to withstand temperatures and forces of a fluid
- propulsion systems
- energy conversion systems
- transport of fluids
- bioengineering
- fluid control systems
- fluid computers



climatology  
oceanography

Although these examples are by no means exhaustive, they do serve to establish the varied applications of a single fascinating discipline.

The study begins with some basic definitions and concepts used to represent the observed behavior of fluids; emphasis is placed on the physical meaning of each representation made in mathematical language.

## 1.2 Fluids and Continuum Concepts

A *fluid* is defined as any substance deforming continuously when subjected to a shear stress regardless of how small the shear stress may be. This means that fluids will “flow” when subjected to a shear stress; and, conversely, flowing fluids will generally exhibit the presence of shear stresses. (A detailed discussion of stresses is deferred until Chapter 3.) It is of interest to note the difference between a fluid and a solid deformed in the elastic range. A solid deformed by a shear stress is capable of resisting deformation within limits by generating an internal stress proportional to the deformation. If the externally applied shear stress is not too great (so that it does not exceed the limit of elastic behavior for the material), then the internal stress developed can become large enough to equalize the external stress. In such a case deformation ceases; that is, the solid does not deform continuously. The above definition could be used as a criterion to distinguish fluids from “nonfluids” by experiment.

Since this study will not consider fluid behavior from the point of view of kinetic theory, a conceptual model is needed that can be used in a physical as well as a mathematical sense. This model is referred to as the *continuum*. Instead of defining a continuum at the outset, a hypothetical experiment is proposed. Consider a rather large and finite region in a fluid whose mass is not uniformly distributed in a given space, and let this region be completely bounded by a spatial volume  $V_1$ . Next imagine a point to be fixed in position inside this volume. The density  $\rho_1$  of the fluid inside this space may be thought of as simply the mass  $m_1$  of this fluid divided by  $V_1$ . Such a density would obviously be a gross or average description. At one instant of time one may imagine taking successively smaller regions (that is,  $V_1 > V_2 > V_3$ , etc.) with each region still surrounding the point but lying wholly inside the previous region, as shown in Fig. 1.1. As sample size is decreased, one would witness an approach to a limiting value because the sample would tend to become more uniform in mass distribution. After reaching a certain volume  $V_s$ , any reduction in the sample size beyond this value would yield fluctuations in the calculated value of the gross density. If  $V$  were permitted to approach zero, i.e., reduced to the point it surrounds, the density might approach a very